"Modal dispersion curves of different types of metal-clad trapezoidal optical wave-guides and their comparative study"

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ABSTRACT

In this contribution, we present the modal analysis of a new type of non-conventional optical waveguide having different type of trapezoidal shape cross-section. The characteristic equations have been derived by using Goell's point matching method (GPMM) under weak-guidance approximation. The dispersion curves are also interpreted in each case .The modal properties of proposed structure of optical wave guide are compared with those of standard square wave guide with metallic-cladding. It has been observed that the effect of distortion of a square wave guide into a trapezoidal shape with metallic cladding is to shift the lowest cut-off value (Vc) towards a large value and increase the number of modes.

Keywords-Dispersion curves, integrated optics, Modal analysis, Metal-cladding.

I. INTRODUCTION

Much research work has done during the last forty years in the field of optical fiber technology. This leads to high capacity and high transmission rate system [1-8].Wave guide of unusual structure have generated great interest in comparison of conventional structures of optical wave-guides [9-17]. The materials used for production of optical waveguides like dielectrics, metals chiral, liquid crystal, polymers etc., have also played great role in revolutionized the communication technology[18-21]. Metal-clad optical wave guide have attracted the interest of many researchers, as they are widely used in integrated optics. Attenuation of T.E.modes is less than that of T.M. modes in such types of metal-clad optical waveguides [22-28]. Metal optical wave guides are used as mode filter and mode analyzer. Metal optical waveguide play an important role in directional couplers .Therefore modal analysis of such waveguides is very important.

In this paper, we propose an optical wave-guide having trapezoidal shape cross-section with metallic-cladding. This proposed wave guide is analyzed in five different cases. In each case, all boundaries of proposed wave-guide are surrounded by metal instead of dielectric. For each case modal characteristic equation and corresponding dispersion curves are obtained by using Goell's point matching method (GPMM) [29]. These dispersion curves are compared with the dispersion curves of square waveguide with metallic cladding and some important insight are found which are of technical importance for optical fiber communication.

II.THEORY

Figures 1(b) – (e) shows the geometry of proposed optical waveguide, which has a trapezoidal shape. For such a wave guide with core and cladding of refractive indices n_1 and n_2, respectively and $(n_1 - n_2)/n_1$ can be taken as smaller than unity under weak guidance approximation. The coordinate representation of a general shape of trapezoidal wave guide is shown in figure (2) [15].In order to find the coordinates on the various sides of the trapezoid, we can use the following formula:

$$\theta_1 = \arctan \frac{d_{t.c}}{D} \qquad (1)$$

Here, $d_{t..c}$ and *D are* transverse contraction on one parallel side of trapezoid and side of square, respectively.

Let the coordinate of a point on PS side of trapezoid as shown in figure, (2), is (r, θ).

$$\text{Where, } r = \frac{D}{2}\sec(\pi - \theta) \quad , \pi \le \theta \le \theta_2, \qquad (2)$$

The coordinate on the RS side can be written as,

$$r = \frac{D(D + d_{t.c})}{2(D - d_{t.c})\cos\theta_1 + 4Dd_{t.c} \sin\theta_1} \qquad (3)$$

The coordinate on RN side, can be written as,

$$r = \frac{D}{2}\sec\theta \qquad (4)$$

Where, $0 \le \theta \le \theta_1$

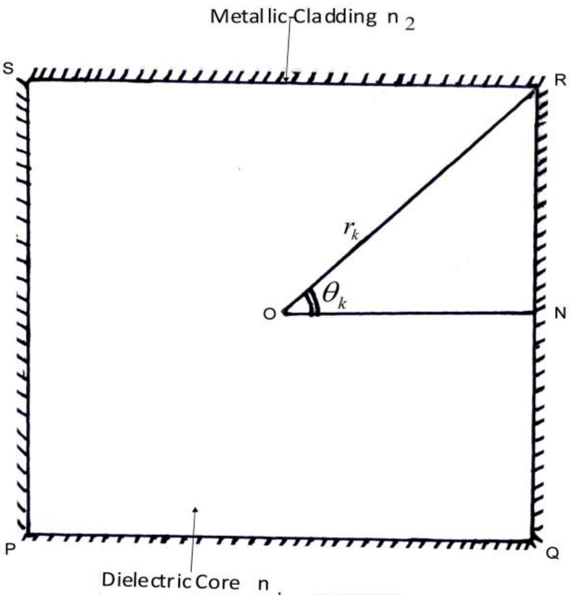


Figure (a) $d_{t.c} = D$

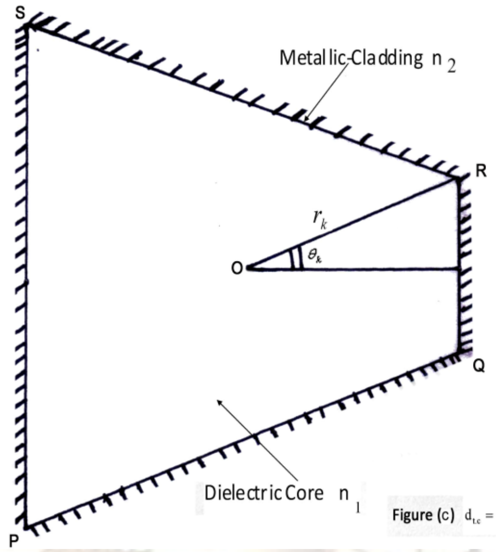


Figure (c) $d_{t.c} = \frac{D}{3}$

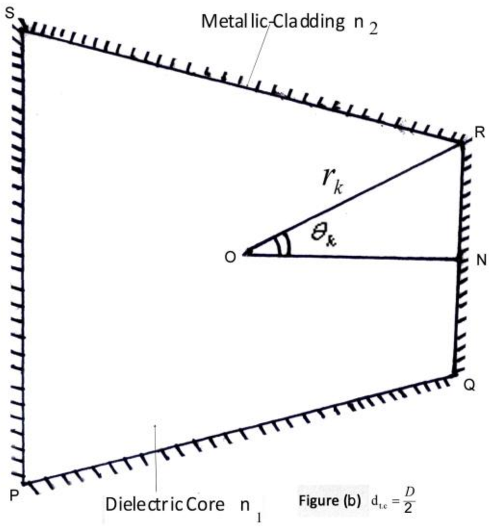


Figure (b) $d_{t.c} = \frac{D}{2}$

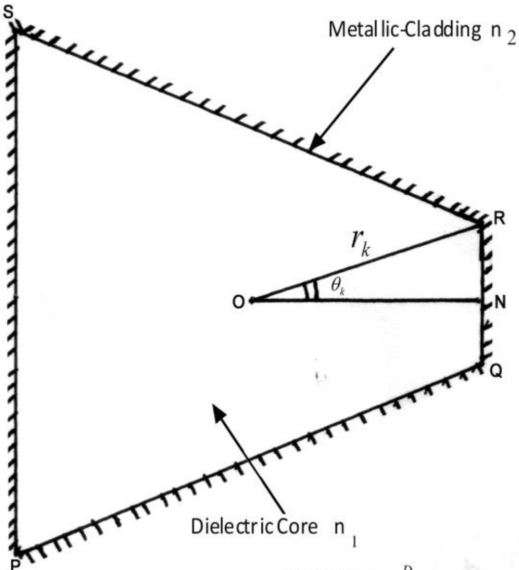
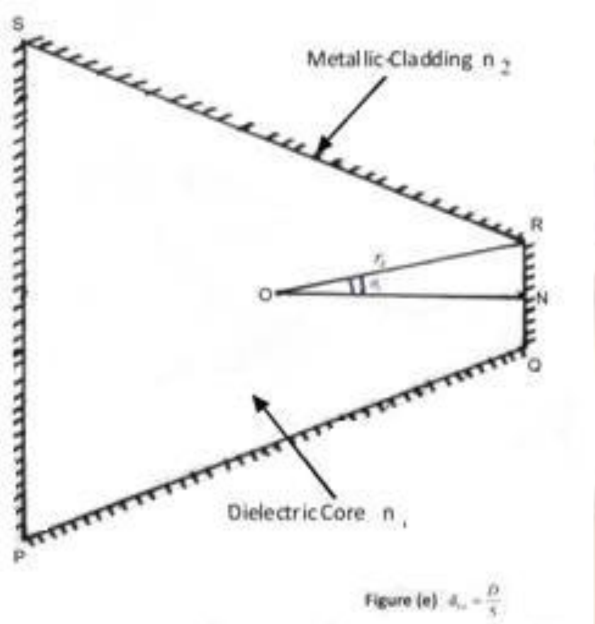


Figure (d) $d_{t.c} = \frac{D}{4}$



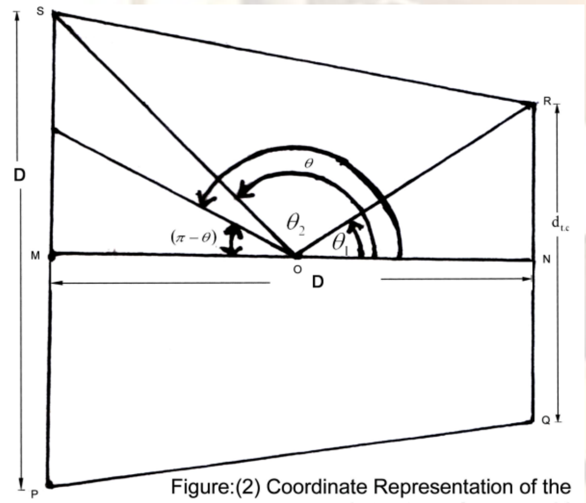


Figure:(2) Coordinate Representation of the proposed structure of wave -guide

1. (a) Square shape waveguide with $d_{t.c}$=D 1.(b)-(e): trapezoidal shape with different values of $d_{t.c}$=D/2,d/3,D/4 and D/5 respectively.

2. Coordinate representation of the proposed structure of optical wave-guide.

To study the proposed structure of optical wave guide, Goell's point matching method (GPMM) is employed. For this scalar wave equation in cylindrical polar co-ordinate(r,θ, z) system may be written as:

$$\nabla^2\psi - \frac{n^2}{c^2}\frac{\partial^2\psi}{\partial^2 t} = 0 \qquad (5)$$

Here n stands for refractive index of the core or cladding region as the case may be and the function ψ stands for the z-component of the electric field (E_Z) or magnetic field (H_Z). Considering harmonic variation of ψ with t and z, it can be written as:

$\psi = \psi_0 \exp\, j(\omega t - \beta z)$.

Now equation (5) takes the form as

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \left(\frac{n^2\omega^2}{c^2} - \beta^2\right)\psi = 0 \qquad (6)$$

Here, ω, β and c are Optical angular frequency, z-Component of the propagation vector, and Velocity of light in free space respectively. If ε and μ are permittivity of and permeability of the medium respectively then equation (6) can be written as:

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \sigma^2\psi = 0 \qquad (7)$$

Such that $\sigma^2 = \omega^2\mu\varepsilon - \beta^2$

To use separation of variables technique function ψ can be given as:

$$\psi(r,\theta,z,t) = f_1(r) f_2(\theta) \exp\, j(\omega t - \beta z) \qquad (8)$$

With this expression equation (7), can be written as

$$\frac{\partial^2 f_1(r)}{\partial r^2} + \frac{1}{r}\frac{\partial f_1(r)}{\partial r}\left(\sigma^2 - \frac{\nu^2}{r^2}\right) f_1(r) = 0 \qquad (9)$$

And, $\dfrac{\partial^2 f_1(\theta)}{\partial\theta^2} + \nu^2 f_2(\theta) = 0 \qquad (10)$

Here, ν is a non-negative integer, is known as order of Bessel's function.

The solution of equation (9) in terms of Bessel function and the solution of equation (10) in term of trigonometric functions like $\cos(\nu\theta)$ or $\sin(\nu\theta)$ can be found.

The solution of the core as well as the cladding region can be taken as the linear combination of product of Bessel's functions and trigonometric functions of various orders. In cladding region, $\left(\sigma^2 - \frac{\nu^2}{r^2}\right) < 0$ and in guiding region, $\left(\omega^2\mu\varepsilon - \beta^2\right) > 0$. The solution can be taken as the sum of a series where each term is the product of Bessel's function of the first kind $J_\nu(x)$ and trigonometric function of same order where as in the outer non-guiding region, the solution can be taken as linear combination of the product of modified Bessel's functions of second kind $K_\nu(x)$ and the trigonometric functions. However, there will be no series corresponding to the metallic, non-guiding region where, the field will be identically vanishes If ψ_1 and ψ_2 represents the solution in the core and cladding regions respectively, then we have

$$\psi_1 = A_0J_0(u\,r) + A_1J_1(u\,r)\cos(\theta) + B_1J_1(u\,r)\sin(\theta) + A_2J_2(u\,r)\cos(2\theta) + B_2J_2(u\,r)\sin(2\theta) + ... \quad (11)$$

and, $$\psi_2 = 0 \quad (12)$$

The Parameters 'r' and 'θ' in the above equations (11) represents the polar-coordinates of the various points on the boundary of proposed structure of optical wave guide. The parameters 'u' *and* 'w' is defined as

$$u^2 = \left(\frac{2\pi n_1}{\lambda}\right) - \beta^2 \text{(Core-region parameter)} \quad (13)$$

$$w^2 = \beta^2 - \left(\frac{2\pi n_2}{\lambda}\right) = 0 \text{ (Cladding-region parameter)} \quad (14)$$

In order to study the proposed structure of an optical wave guide. In Goell's point-matching method, the fields in core region and cladding region are matched at selective points on boundary of the wave guide. In this case twenty eight points are taken on the core-cladding boundary to obtain reasonable results. Matching the field along with their derivatives at chosen points on the boundary of the wave-guide in each case following equations is f

$$\sum_{q=0}^{13} A_qJ_q(u\,r_k)\cos q\theta_k + \sum_{q=0}^{13} B_qJ_q(u\,r_k)\sin q\theta_k - \sum_{q=0}^{13} C_qK_q(w\,r_k)\cos q\theta_k - \sum_{q=0}^{13} D_qK_q(w\,r_k)\sin q\theta_k = 0 \quad (15)$$

Since, in proposed structure of optical wave guide, the non-guiding region (cladding region) is metallic, therefore last two terms in equation (15) will be vanished, and also derivative part has no significance. So equation (15) takes the form as:

$$\sum_{q=0}^{13} A_qJ_q(u\,r_k)\cos q\theta_k + \sum_{q=0}^{13} B_qJ_q(u\,r_k)\sin q\theta_k = 0 \quad (16)$$

For, k= 1, 2,3…….28.

The quantities r_k and θ_k are the polar coordinate of the points on the core-cladding boundary. The equation (16) gives the 28 simultaneous equations involving 28 unknown constants A_q and B_q and their coefficients form a determinant Δ of order 28×28. A non -trivial solution will exist for these sets of equation if ,

$$\Delta = 0 \quad (17)$$

The equation (17) is the characteristic equation, which contains all the information regarding to the modal properties of proposed optical waveguides. The solution of equation (17) gives propagation constants β for sustained modes in core region. In each case dispersion curve is obtained and interpreted.

III. NUMERICAL,ESTIMATION,RESULTS AND DISCUSSION

The modal properties of the proposed optical waveguides can be found explicitly from the characteristics equation (17) on the basis of numerical calculations. We plot β-V Curves (dispersion curves) of the proposed optical wave-guides by which propagation properties of different modes can be analyzed. For plotting the dispersion curves, the eigen values of β are obtained by solving characteristics equation (17). The L.H.S of this characteristics equation contain the propagation constant β , angular frequency ω, permeability μ, side of square waveguide D and ε_1 and ε_2 which represent the relative permittivity of core and cladding regions, respectively. By keeping ω, μ, ε_1, ε_2 and D fixed at proper values, we select a large number of equispaced values of β in propagation region $n_1k_0 \geq \beta \geq n_2k_0$ (where k_0 wave-vector corresponding to free space) and corresponding values of Δ are calculated . In all the five cases of the proposed waveguides for which transverse contraction on one parallel side of proposed waveguide of trapezoidal cross-section

are $d_{t.c} = 0$, $d_{t.c} = \frac{D}{2}$, $d_{t.c} = \frac{D}{3}$, $d_{t.c} = \frac{D}{4}$ and. $d_{t.c} = \frac{D}{5}$.

(15)We have taken $\lambda_0 = 1.55\mu m$,

$n_1 = 1.48$ and $n_2 = 1.46$.

Now, Δ versus β plots gives the graphical representation of the characteristics equation (17), shown in figure (3) and zero crossing are noted which gives the possible values of β for a fixed value of D.This process can be repeated by selecting different values of D.Also, the dimensionless wave guide parameter V is given by as:

$$V = \left(\frac{2\pi}{\lambda_0}\right) D \sqrt{n_1^2 - n_2^2} \quad (18)$$

Here λ_0 is free space wavelength.

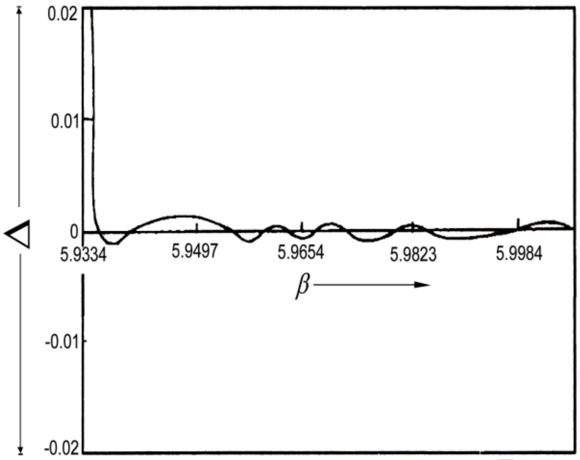


Fig. 3. Graph showing admissible values of β for different modes in proposed structure of optical wave guides.

Thus, we can plot β versus V curves corresponding to the different modes in each case. Figures 4(b)-e) shows dispersion curves of the metal-clad trapezoidal optical waveguide with different values of transverse contraction ($d_{t.c}$) on one parallel side of proposed wave guide of trapezoidal cross-section. Figure 4(a) shows dispersion curves for metal- clad square waveguide, for $d_{t.c}$=0. Here, we have found some important and interesting results after comparative study of obtained dispersion curves 4. (b)- (e) for different cases of the proposed optical waveguide, which are as follows:

1. The lowest cut off values (V_C) of trapezoidal cross-section of the optical waveguide are slightly greater than the lowest cut-off values of square cross-section.
2. The number of modes in trapezoidal shape optical wave guide is larger for V=8 than square shape cross-section of optical wave guide for same value of V. Also, the number of modes increases with increase in transverse contraction ($d_{t.c}$) on one parallel side of the trapezoidal shape of optical waveguides .In this way, we conclude that more power is transmitted through metal-clad trapezoidal optical waveguide than square shaped optical waveguide with a common value of V.
3. It can be noticed that for larger values of transverse contraction ($d_{t.c}$), cut-off values of all the modes become closely spaced and most of them lie between=2.8 to 7.4.

Table-1 binds the obtained interesting results for the different trapezoidal shape optical waveguides.

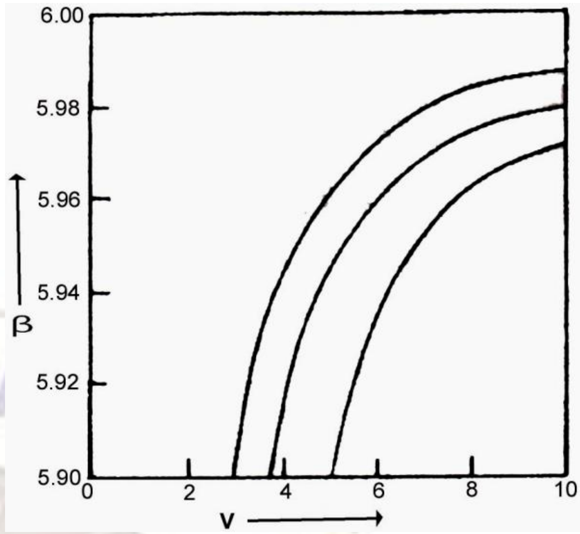


Fig.4. (a)

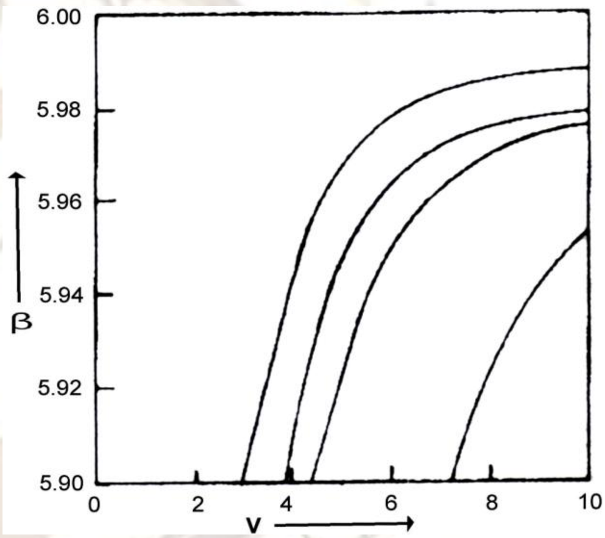


Fig.(b)

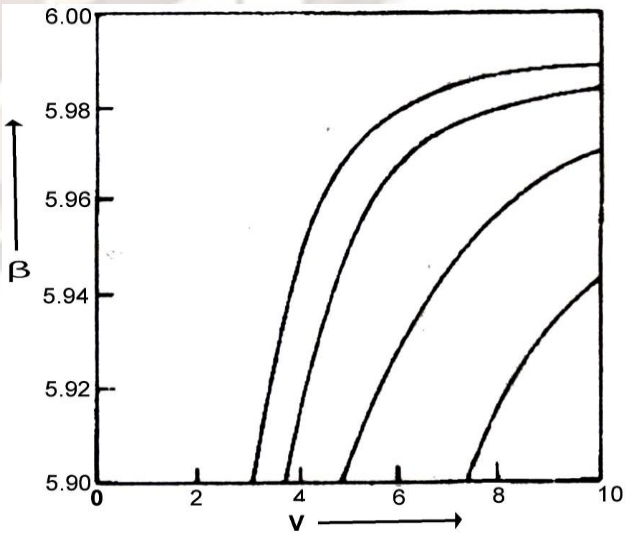


Fig.(c)

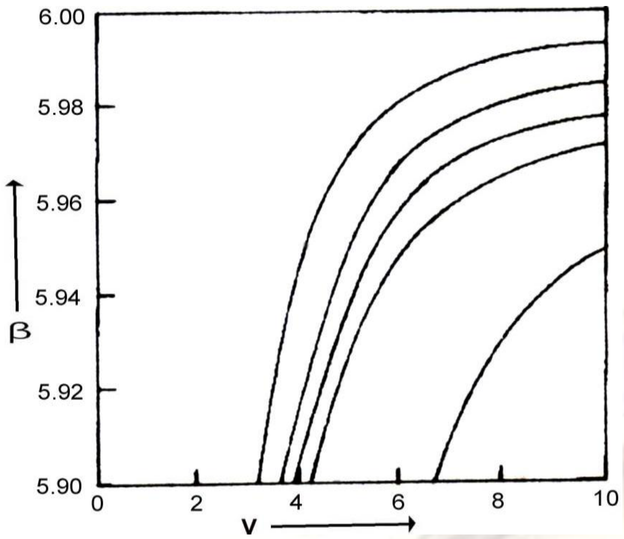


Fig. (d)

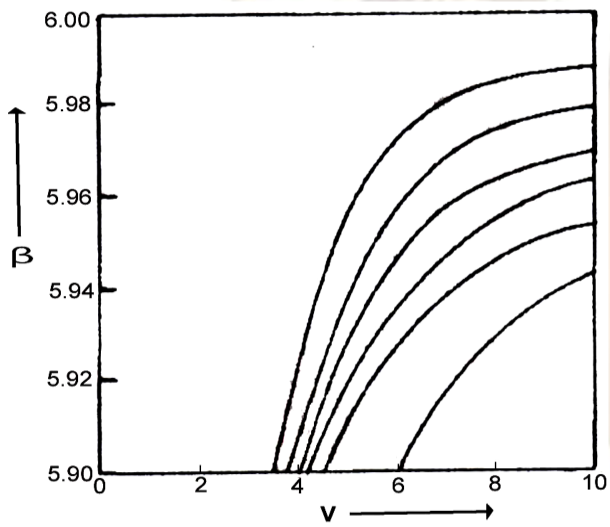


Fig.(e)

4. (a)Dispersion curves for square waveguide with metallic cladding and (b)-(e),shows dispersion curves for trapezoidal optical wave guides with metallic-cladding for different values of transverse contraction on one parallel side of square shape wave guide.

TABLE-1(Effect of transverse contraction ($d_{t.c}$) on the one parallel side of the proposed optical wave guide of trapezoidal cross-section.

S.No	$d_{t.c}$	**Lowest cut-off value(V_C)**	**Number of modes at V=8**
1.	D	≈2.8	3
2.	D/2	≈2.9	4
3.	D/3	≈3.0	4
4.	D/4	≈3.1	5
5.	D/5	≈3.4	6

IV. COCLUSION

The authors hope that the predicated results will be of sufficient interest to induce researchers worldwide take up the experimental verification of the results reported in the present work.

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