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Performance Analysis of Fuzzy Logic Based PD, PI and PID Controllers

Er. Simranjeet Singh#, Er. Jasbir Singh*

#Student, M.Tech, 2nd year, Regular, ECE Department, Yadavindra College of Engineering, Talwandi Sabo
*Assistant Professor, ECE Department, Yadavindra College of Engineering, Talwandi Sabo

Abstract— Fuzzy logic control has emerged as one of the active area for research. It has evolved as an alternative to the conventional control strategies in various engineering areas. The way is applying fuzzy logic to the design of PID controllers is termed as Fuzzy PID controllers. In this paper, fuzzy logic based PID controllers are compared with fuzzy logic based PD and PI controllers on the basis of transient response (such as rise time, peak time, maximum overshoot and settling time in step response) and steady state response (such as sum square error, integral absolute error (IAE) and integral absolute time multiplied error (IATE)) for first order and second order systems.

Keywords— Unit step signal, Fuzzy controller, Transfer function, Discrete filter

I. INTRODUCTION

The structure of the Fuzzy PD controller is given in Figure 1.1. In this, the derivation is made at the input of the fuzzy block error *e*. For the fuzzy PD controller, the following relation in the z-domain:

$$u(z) = \widetilde{c_u}[x_e(z)+x_{de}(z)] = \widetilde{c_u}[c_e+c_{de}\frac{z-1}{hz}]e(z)$$

With this relation the transfer function results:

$$H_{RF}(z) = \frac{u(z)}{e(z)} = \widetilde{c_u}(c_e + c_{de}\frac{z-1}{hz})$$

For the PD linear controller we take the transfer function:

$$H_{RG}(s) = K_{RG}(1+T_D s)$$

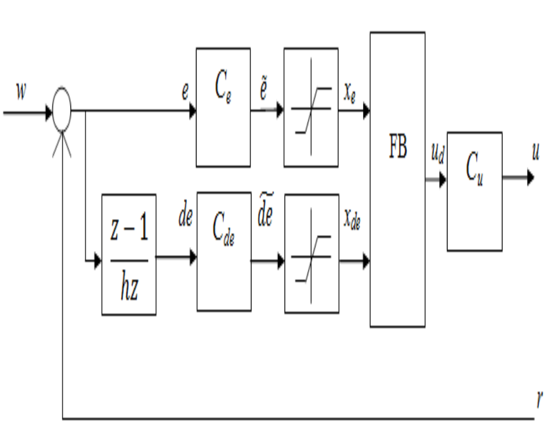

Figure 1.1 The block diagram of the Fuzzy PD controller with scaling coefficients

The structure of the Fuzzy PI controller with integration at its output is presented in Fig 1.2. The controller is working after the error *e* between the input variable reference and the feedback variable *r*. In this structure we may notice that two filter were used. One of them is placed at the input of the fuzzy block and the other at the output of the fuzzy block. In the approach of the fuzzy PID controllers the concepts of integration and derivation are used for describing that these filters have mathematical models obtained by discretization of a continuous time mathematical models for integrator and derivative filters. The structure of the linear PI controller may be presented in a modified block diagram in Figure 1.3.

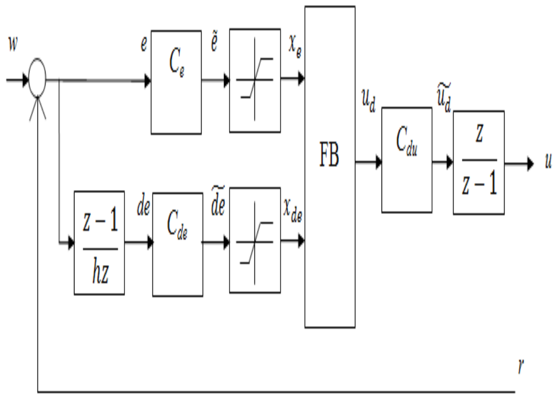

Figure 1.2 Structure of a Fuzzy PI controller

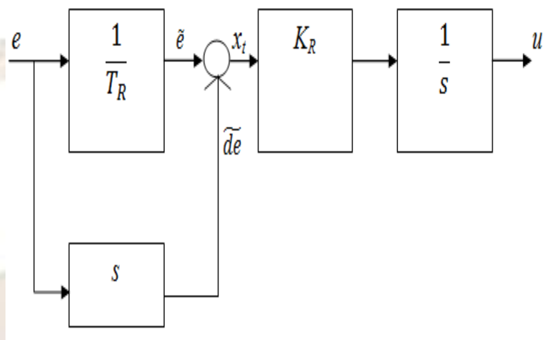

Figure 1.3 The modified block diagram of the linear PI controller

For this structure the following modified form of the transfer function may be written:

$$u(s) = K_R\frac{1}{s}\left(s+\frac{1}{T_R}\right)e(s) = K_R\frac{1}{s}x_t(s)$$

Where

$$x_t = \tilde{e} + \widetilde{de}$$

$$\tilde{e} = \frac{1}{T_R}e$$

$$\widetilde{de} = s.e$$

The fuzzy block may be described using its input-output transfer characteristics, its variable gain and its gain in origin as a linear function around the origin ($\tilde{e}=0$, $\widetilde{de}=0$, $u_d=0$).

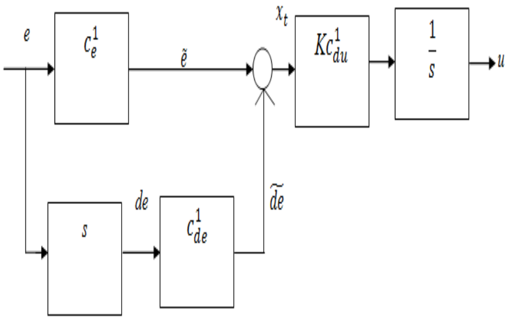

Figure 1.4 The block diagram of the linear PI controller with scaling coefficients

The block diagram of the linear PI controller may be put similar as the block diagram of the fuzzy PI controller as in Figure 1.4. For the transfer function of the linear PI controller with scaling coefficients the following relation may be written:

$$H_R(s) = K_R.\frac{1}{s}(s + \frac{1}{T_R}) = K.c^l_{du}.\frac{1}{s}.(c^l_e + c^l_{de}s)$$

The derivation, integration are made in discrete time and specific scaling coefficients are introduced. The saturation elements are introduced because the fuzzy block is working on scaled universes of discourse [-1,1]. The filter from the controller input placed on the low channel takes the operation of digital derivation at its output we obtain the derivative *de* of the error *e*:

$$de(t) = \frac{d}{dt}e(t)$$

$$de(z) = \frac{z-1}{hz}e(z)$$

where *h* is the sampling period. In the domain of discrete time the derivative block has the input-output model:

$$de(t+h) = \frac{1}{h}e(t+h) - \frac{1}{h}e(t)$$

That shows us that the digital derivation is there accomplished based on the information of error at the time moments $t = t_k = k.h$ and $t_{k+1} = t_k + h$:

$$e_k = e(kh)$$

$$e_{k+1} = e((k+1)h)$$

So, the digital equipment is making in fact the substraction of the two values. The error *e* and its derivative *de* are scaled with two scaling coefficients c_e and c_{de}, as it follows:

$$\tilde{e}(t) = c_e e(t)$$

$$\widetilde{de}(t) = c_{de}\, de(t)$$

The variables x_e and x_{de} from the inputs of the fuzzy block are obtained by a superior limitation to 1 and an inferior limitation to –1, of the scaled variables *e* and *de*. This limitation is introduced because in general case the numerical calculus of the inference is made only on the scaled universe of discourse [-1, 1]. The Fuzzy block offers the defuzzified value of the output variable u_d. This value is scaled with an output scaling coefficient c_{du}:

$$\widetilde{u_d} = c_{du}\, u_d$$

In the case of the fuzzy PI controller with integration at the output the scaled variable $\widetilde{u_d}$ is the derivative of the output variable '*u*' of the controller. The output variable is obtained at the output of the second filter which has an integrator character and it is placed at the output of the controller:

$$u(t) = \int_0^t \widetilde{u_d}(\tau)d\tau$$

$$u(z) = \frac{z}{z-1}\widetilde{u_d}(z)$$

The input-output model in the discrete time of the output filter is:

$$u(t+1) = u(t) + \widetilde{u_d}(t+1)$$

The above relation shows that the output variable is computed based on the information from the time moments *t* and *t* + *h*:

$$u_{k+1} = u((k+1)h)$$

$$u_k = u(kh)$$

$$\tilde{u}_{dk+1} = \tilde{u}_d((k+1)h)$$

From the above relations we may notice that the "integration" is reduced in fact at a summation:

$$u_{k+1} = u_k + \tilde{u}_{dk+1}$$

This equation could be easily implemented in digital equipments. Due to this operation of summation, the output scaling coefficient c_{du} is called also the increment coefficient. The controller presented above could be called Fuzzy controller with summation at the output and not with integration at the output.

The structure of the Fuzzy PID controller is presented in Figure 1.5. In this, the derivation and integration is made at the input of the fuzzy bock on the error *e*. The Fuzzy block has three input variables x_e, x_{ie} and x_{de}. The transfer function of the PID controller is obtained considering a linearization of the fuzzy block around the origin, for $x_e=0$, $x_{ie}=0$, $x_{de}=0$, $u_d=0$ with a relation of the following form:

$$u_d = K_0(x_e + x_{ie} + x_{de})$$

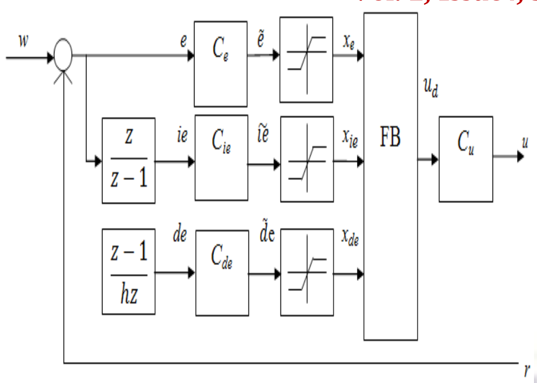

Figure 1.5 The block diagram of the Fuzzy PID controller

The Fuzzy block from the PID controller which has 3 input variables may describe is :

$$K_{BF}(x_t; x_{de}, x_{ie} = 0) = \frac{u_d}{x_t}, x_t \neq 0$$

where:

$$x_t = x_e + x_{ie} + x_{de}$$

The value K_0 is the limit value in origin of the characteristics of the function:

$$K_o = lim_{x \to 0} K_{BF}(x_t; x_{de}, x_{ie} = 0)$$

Taking account of the correction made on the fuzzy block with the incremental coefficient c_u, the characteristic of the fuzzy block corrected and linearized around the origin is given by the relation:

$$u = c_u K_o (x_e + x_{ie} + x_{de})$$

We are denoting:

$$\widetilde{c_u} = c_u K_o$$

For the fuzzy controller with the fuzzy block, the following input output relation in the z domain may be written:

$$u(z) = \widetilde{c_u}[x_e(z) + x_{ie}(z) + x_{de}(z)] = \widetilde{c_u}[c_e + c_{ie}\frac{z}{z-1} + c_{de}\frac{z-1}{hz}]e(z)$$

With these observations the transfer function of the fuzzy PID controller becomes:

$$H_{RF}(z) = \frac{u(z)}{e(z)} = \widetilde{c_u}(c_e + c_{ie}\frac{z}{z-1} + c_{de}\frac{z-1}{hz})$$

For the linear PID controller, the following relation for the transfer function is considered:

$$H_{RG}(s) = K_{RG}(\left(1 + T_D s + \frac{1}{T_1 s}\right)$$

II. Implementation

In **Fuzzy Based PD controller**, Proportional gain K_p and Derivative gain K_d are used to produce controlled output. For the designing of fuzzy based PD controller for first order and second order systems, the model as shown in figure 1.6 is used. C_e and C_{de} are the input gains and C_u is output gain that are used to produce proportional and derivative control action.

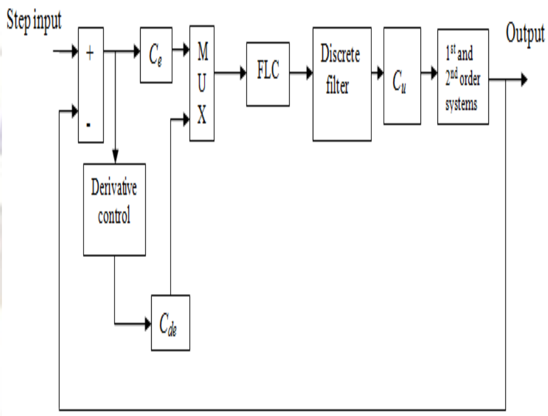

Figure 1.6 Block Diagram of Fuzzy Based PD Controller

In **Fuzzy Based PI controller**, Proportional gain K_p and Integral gain K_i are used to produce controlled output. For the designing of fuzzy PI controller for first order and second order systems, the model as shown in figure 1.7 is used. In this model C_e and C_{de} are the input gains and C_{du} is output gain that are used to produce proportional and integral control action.

In **Fuzzy Based PID controller**, Proportional gain K_p, Integral gain K_i and Derivative gain K_d are used to produce controlled output. For the designing of fuzzy PID controller for first order and second order systems, the model as shown in figure 1.8 is used. C_e, C_{ie} and C_{de} are the input gains and C_u is output gain that are used to produce proportional, integral and derivative control action.

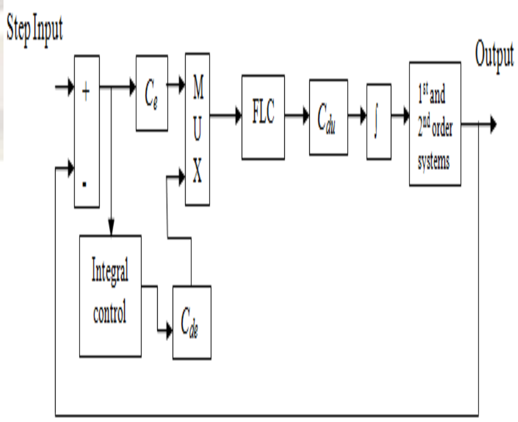

Figure 1.7 Block Diagram of Fuzzy Based PI Controller

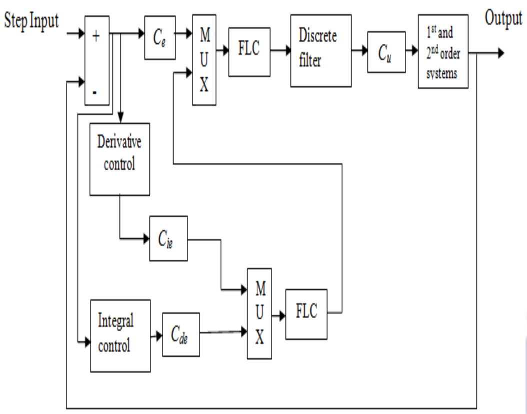

Figure 1.8 Block Diagram of Fuzzy Based PID Controller

In fuzzy based PD, PI, PID controllers, Proportional gain K_p, Integral gain K_i and Derivative gain K_d are used to produce controlled output. There are two inputs error (E) and change in error (CE) to fuzzy controller and a single output U. The universe of discourse each variable E, CE and U is [-2,2]. For input and output variables there are seven membership functions named as NL (Negative Large), NM (Negative Medium), NS (Negative Small), ZR (Zero), PS (Positive Small), PM (Positive Medium), PL (Positive Large) respectively. A change in gain factor affects all fuzzy rules. Gain factors for fuzzy PD controller are calculated in section 1 in terms of C_e, C_{de} and C_u. From these gain variables proportional and derivative gains calculated. Gain factors for fuzzy PI controller are calculated in section 1 in terms of C_e, C_{de} and C_{du}. From these gain variables proportional and Integral gains calculated. Gain factors for fuzzy PID controller are calculated in section 1 in terms of C_e, C_{ie}, C_{de} and C_u. From these gain variables proportional, Integral and derivative gains calculated. The fuzzy controller uses the set of 49 rules to produce the controlled output. The output is again multiplied by the output gain and then applied to the first order and second order systems. The discrete filter is used to obtain the change in error signal. The integrator is used to perform the integral action. The final output y is calculated up to the time period of 50 sec, then its characteristics like rise time, peak time, maximum overshoot, settling time, sum square error, integral absolute error (IAE) and integral of time multiplied absolute error (ITAE) are calculated. The output is calculated by hit and trial method.

III. RESULTS

In results, fuzzy based PD, PI and PID controllers for first and second order systems are compared on the basis of terms like rise time, peak time, maximum overshoot, settling time, sum square error, integral absolute error (IAE) and integral absolute time multiplied error (IATE) are used. K_p, K_i and K_d made variable to get the undamped transient response of the system.

Integral Absolute Error (IAE) is the magnitude of the error at every instant of time given by equation as under

$$IAE = \sqrt{(R-y)^2}$$

Sum Square Error (SSE) is simply sum of magnitude of Integral Absolute Error (IAE) values at every instant of time is given by the equation as under

$$SSE = \sum_{t=1}^{n} \sqrt{(R-y)^2}$$

Integral Absolute Time Multiplied Error (IATE) is sum of the Integral Absolute Error (IAE) multiplied with corressponding instant of time given by equation as under

$$IATE = \sum_{t=1}^{n} \left[\left(\sqrt{(R-y)^2} \right) * t \right]$$

First Order System

a) Fuzzy Logic Based PD Controller

The transient response of the system is shown in figure 1.9. It is clear from the response that rise time of the system is 3.00 sec. The response reaches to its maximum value at 5.00 sec. The maximum overshoot produced by the response is 0.54 and settled down at 34 sec.

The steady state response of the system is shown in the figure 1.10. This plot shows the variation of sum squared error with respect to time. In initial conditions the sum square error is one then it reduces steadily but after about 3 seconds it increases again due to undershoot. Finally, the overshoot decreases and error reduces to almost zero and system reaches its steady state response when time reaches to 44 sec. The integral absolute error (IAE) is 11.50 and integral absolute time multiplied error (IATE) is 42.00.

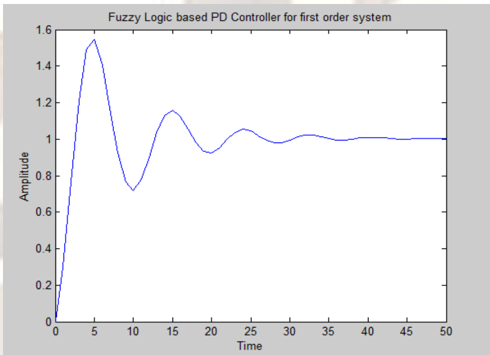

Figure 1.9 Transient Response of Fuzzy Based PD Controller

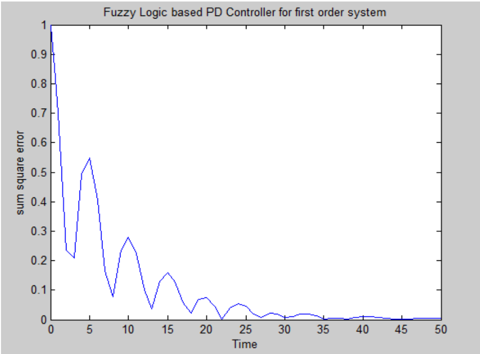

Figure 1.10 Steady State Response of Fuzzy Based PD Controller

b) Fuzzy Logic Based PI Controller

The transient response of the system is shown in figure 1.11. It is clear from the response that rise time of the system is 3.00 sec. The response reaches to its maximum value at 5.00 sec. The maximum overshoot produced by the response is 0.38 and settled down at 19 sec.

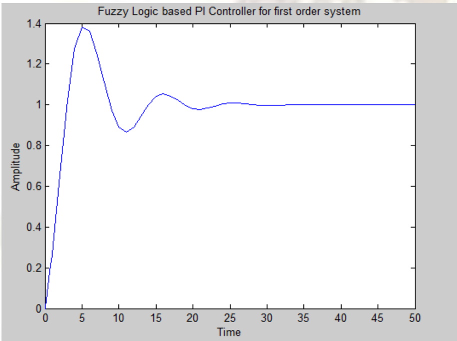

Figure 1.11 Transient Response of Fuzzy Based PI Controller

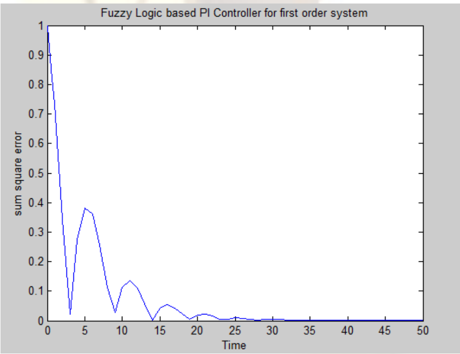

Figure 1.12 Steady State Response of Fuzzy Based PI Controller

The steady state response of the system is shown in the figure 1.12. This plot shows the variation of sum squared error with respect to time. In initial conditions the sum square error is one then it reduces steadily but after about 3 seconds it increases again due to undershoot. Finally, the overshoot decreases and error reduces to almost zero and system reaches its steady state response after 32 seconds. The integral absolute error (IAE) is 9.96 and integral absolute time multiplied error (IATE) is 20.27.

c) Fuzzy Logic Based PID Controller

The transient response of the system is shown in figure 1.13. It is clear from the response that rise time of the system is 3.00 sec. The response reaches to its maximum value at 4.00 sec. The maximum overshoot produced by the response is 0.17 and settled down at 9.00 sec.

The steady state response of the system is shown in the figure 1.14. This plot shows the variation of sum squared error with respect to time. In initial conditions the sum square error is one then it reduces steadily but after about 3 seconds it increases again due to undershoot. Finally, the overshoot decreases and error reduces to almost zero and system reaches its steady state response when time reaches to 10 sec. The integral absolute error (IAE) is 8.58 and integral absolute time multiplied error (IATE) is 16.4.

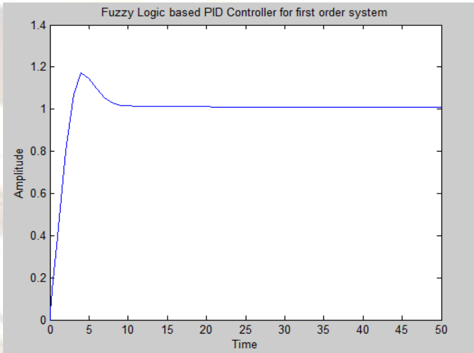

Figure 1.13 Transient Response of Fuzzy Based PID Controller

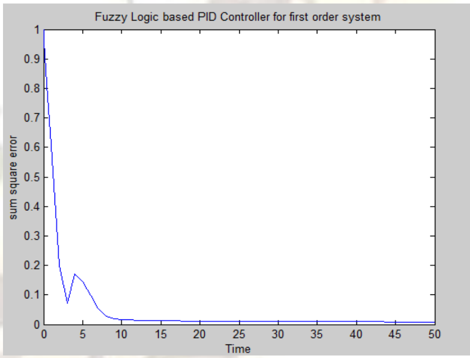

Figure 1.14 Steady State Response of Fuzzy Based PID Controller

Comparative Terms	Fuzzy Logic Based PD Controller	Fuzzy Logic Based PI Controller	Fuzzy Logic Based PID Controller
Integral Absolute Error (IAE)	11.50	9.96	8.58
Integral Absolute Time Multiplied Error (IATE)	42.00	20.27	16.4
Rise Time	3.00	3.00	3.00
Peak Time	5.00	5.00	4.00
Maximum Overshoot	.54	.38	.17
Settling Time	34.00	19.00	9.00

Table 1.1 Comparison of Performance of Controllers for First Order System

Second Order System

a) Fuzzy Logic Based PD Controller

The transient response of the system is shown in figure 1.15. It is clear from the response that rise time of the system is 5.9 sec. The response reaches to its maximum value at 10.00 sec. The maximum overshoot produced by the response is 0.57 and settled down at 46.74 sec.

The steady state response of the system is shown in the figure 1.16. This plot shows the variation of sum squared error with respect to time. In initial conditions the sum square error is one then it reduces steadily but after about 5.5 seconds it increases again due to undershoot. Finally, the overshoot decreases and error reduces to almost zero and system reaches its steady state response when time reaches to 50 sec. The integral absolute error (IAE) is 36.22 and integral absolute time multiplied error (IATE) is 395.78.

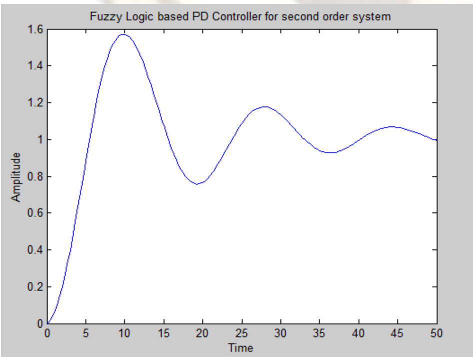

Figure 1.15 Transient Response of Fuzzy Based PD Controller

b) Fuzzy Logic Based PI Controller

The transient response of the system is shown in figure 1.17. It is clear from the response that rise time of the system is

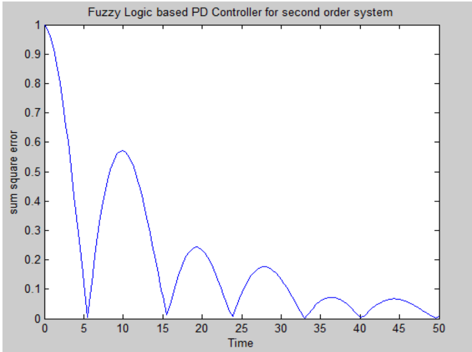

Figure 1.16 Steady State Response of Fuzzy Based PD Controller

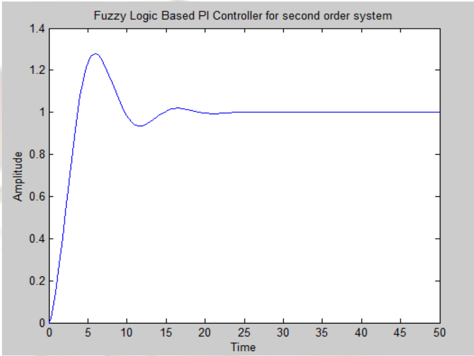

Figure 1.17 Transient Response of Fuzzy Based PI Controller

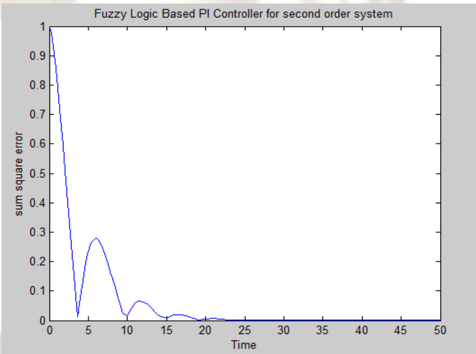

Figure 1.18 Steady State Response of Fuzzy Based PI Controller

4.00 sec. The response reaches to its maximum value at 6.00 sec. The maximum overshoot produced by the response is .27 and settled down at 9.39 sec.

The steady state response of the system is shown in the figure 1.18. This plot shows the variation of sum squared error with respect to time. In initial conditions the sum square error is one then it reduces steadily but after about 4 seconds it increases again due to undershoot. Finally, the overshoot decreases and error reduces to almost zero and system reaches its steady state response when time reaches to 22.5 sec. The integral absolute error (IAE) is 15.5 and integral absolute time multiplied error (IATE) is 33.6.

c) Fuzzy Logic Based PID Controller

The transient response of the system is shown in figure 1.19. It is clear from the response that rise time of the system is 3.78 sec. The response reaches to its maximum value at 4.82 sec. The maximum overshoot produced by the response is 0.09 and settled down at 6.38 sec.

The steady state response of the system is shown in the figure 1.20. This plot shows the variation of sum squared error with respect to time. In initial conditions the sum square error is one then it reduces steadily but after about 3 seconds it increases again due to undershoot. Finally, the overshoot decreases and error reduces to almost zero and system reaches its steady state response when time reaches to 12 sec. The integral absolute error (IAE) is 12.90 and integral absolute time multiplied error (IATE) is 31.84.

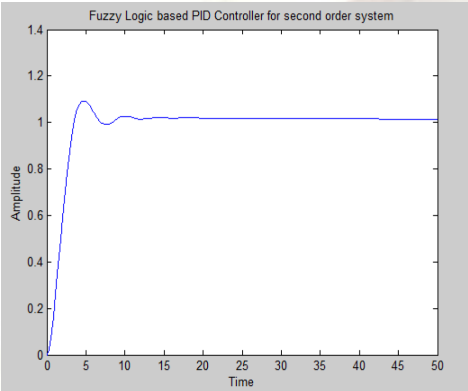

Figure 1.19 Transient Response of Fuzzy Based PID Controller

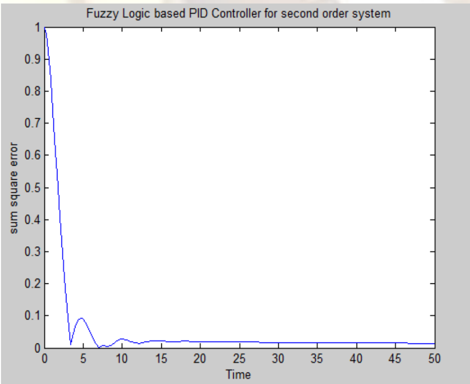

Figure 1.20 Steady State Response of Fuzzy Based PID Controller

Comparative Terms	Fuzzy Logic Based PD Controller	Fuzzy Logic Based PI Controller	Fuzzy Logic Based PID Controller
Integral Absolute Error (IAE)	36.22	15.5	12.90
Integral Absolute Time Multiplied Error (IATE)	395.78	33.6	31.84
Rise Time	5.9	4.00	3.78
Peak Time	10.00	6.00	4.82
Maximum Overshoot	.57	.27	.09
Settling Time	46.74	9.39	6.38

Table 1.2 Comparison of Performance of Controllers for Second Order System

IV. CONCLUSION

It is concluded that fuzzy based PID controllers designed with integral and derivative control actions and a set of 49 rules improves the transient and steady state responses for first order and second order systems. In case of first order system, The Integral absolute error (IAE), Integral absolute time multiplied error (IATE), Peak time, Maximum overshoot, Settling time are reduced in fuzzy based PID controller as compared to fuzzy based PD and PI controllers but Rise time for fuzzy based PD, PI and PID controllers are same. In case of second order system, The Integral absolute error (IAE), Integral absolute time multiplied error (IATE), Rise time, Peak time, Maximum overshoot and Settling time are reduced in fuzzy based PID controller as compared to fuzzy based PD and PI controllers. In both first order and second order systems, the maximum overshoot is controlled by controlling the value of rise time, peak time and settling time by using of adjustable gain factors. So, from the results, it is clear that fuzzy based PID controllers are more effective than fuzzy based PD and PI controllers for first order and second order systems.

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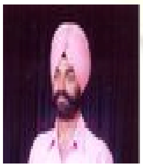
Simranjeet Singh Currently persuing M.Tech degree in "Electronics and Communication Engineering" department from Yadavindra College of Engineering, Talwandi Sabo under Punjabi University, Patiala. He has done his B.Tech in E.C.E department from GTBKIET, Chhapianwali (Malout) under Punjab Technical University, Jalandhar. His area of interest is Fuzzy Logic.



Jasbir Singh is working as Assistant Professor in E.C.E department, Yadavindra College of Engineering, Talwandi Sabo. He has done M.E. in Electronics Product Design and Technology from PEC, Chandigarh. His areas of interest include Fuzzy Logic and Electronics Product Design. He has published 4 papers in national conferences.