SINR improvement in MIMO-OFDM systems by channel estimation and Normalized MMSE

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Abstract- Orthogonal frequency division multiplexing (OFDM) technique is becoming the most frequently used technique for wireless systems such as Long Term Evolution (LTE) and enhanced standards are contemplating its combination with multiple input multiple output (MIMO).These systems suffer from inter-channel interference (ICI) introduced by phase noise and channel estimation errors.ICI is also caused due to Quadrature phase shift in the signals which leads to the loss of information. It degrades the Signal to Interference Noise Ratio (SINR) which in-turn decreases the system performance. In this paper we will derive an analytical expression for SINR degradation. The Quadrature phase change can be corrected using channel estimation and normalized Minimum Mean Square Error (MMSE) method.
Keywords— MIMO-OFDM systems, SINR, channel estimation, ZF receiver, MMSE.

1.INTRODUCTION

Orthogonal Frequency Division multiplexing (OFDM) is becoming the most frequently used technique for wireless systems such as Long Term evolution (LTE) and enhanced standards are contemplating its combination with multi input multi out put these systems suffer from inter channel interference(ICI) introduced by phase noise and channel estimation errors. MIMO-OFDM is being considered for communication systems where high throughput and spectral efficiency are important factors. Theoretical capacity calculations show significant capacity/throughput gains from a MIMO-OFDM system. However, to measure the true performance of the system the impact of analog impairments needs to be considered. A spatial multiplexing MIMO-OFDM system transmits independent OFDM modulated data from multiple antennas simultaneously. At the receiver, after OFDM demodulation, MIMO decoding extracts the different transmitted data streams from each of the subcarriers, as long as the subcarriers are mutually orthogonal. If the subcarriers lose their orthogonality due to analog and RF impairments, the performance of the MIMO-OFDM system degrades dramatically.

Phase noise is caused by non-idealities in the local oscillators (LO) of the system causing the power spectral density (PSD) to exhibit skirts around the carrier frequency. In MIMO-OFDM systems, similar to OFDM systems, the interference due to phase noise can be separated into a common phase error (CPE) term and an inter-carrier interference (ICI) term[1]. The extent of CPE, which can be estimated and corrected, depends on a number of architecture and system level factors. As the number of subcarriers increases the CPE term decreases and the ICI term increases. The CPE decreases as the number of antennas increases in a power constrained MIMO-OFDM system. Similarly, when the phase noise is uncorrelated the amount of CPE decreases.

In this paper we discuss MIMO-OFDM systems with uncorrelated phase noise which we believe is an equally common scenario in real-world systems and compare it with systems with correlated phase noise. More importantly, the issue of correlated v/s uncorrelated phase noise is a tradeoff that needs to be borne in mind during the system design phase. We show that in the case of uncorrelated phase noise at the transmitters, in a spatial multiplexing MIMO-OFDM system, the CPE needs to be estimated and corrected independently for the different data streams. In the case of correlated phase noise the CPE is common to the various data streams and the estimates improve due to diversity. In general the amount of CPE, the correctable term, is much higher when the phase noise is correlated compared to the uncorrelated case.
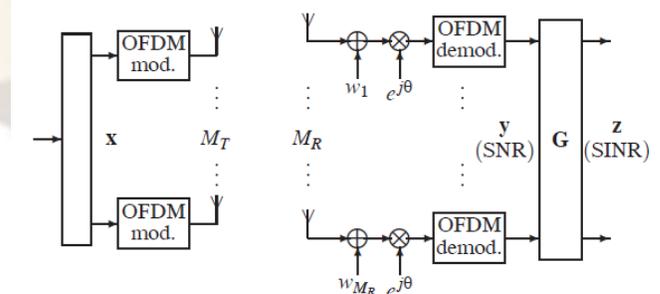
Fig1:Scheme of the MIMO-OFDM system

2.MIMO-OFDM SYSTEM:

The spatial multiplexing MIMO-OFDM system is shown in Fig. 1, where MT independent data streams are OFDM modulated over N sub-carriers and sent to MT transmit antennas. The receiver has MR antennas. The vector of transmitted symbols is $X=[X_0^T,\ldots X_{N-1}^T]$, where each component $X_n=[x_n,1,\ldots x_n,M_T]^T$ groups the symbol transmitted on the nth sub-carrier on all the antennas. For each antenna pair (I,j),I=1,…$M_{R,j=1,\ldots}$ M_T we have a multipath M_R*M_T impulse response $h_m[I,j]$,m=0,…N_{Ch} with length NCh shorter than the cyclic prefix. The elements of hm are randomly distributed with powers determined according to the power delay profile. The spatial correlation is characterized by E $[H_nH_n^H]$ =R$_R$, E $[H_n^HH_n]$ =R$_T$ where $(\cdot)H$ denotes the conjugate transpose. In a separable channel model, RT and RR correspond to the antenna correlations at transmitter and receiver, respectively. The phase noise θ (t) at the receiver, sampled at kT, $\theta_k = \theta$ (kT), coming mainly from the down conversion by high-frequency oscillators, is assumed to be the same for all the antennas. The received signal after the discrete Fourier transform (DFT), y=$[y_0^T,\ldots.yT_{N-1}]^T$ with $y_n=[y_{n,1},\ldots..y_{n,MR}]^T$ grouping all the signals on sub-carrier n is

y=QH$_X$+W …………………(1)

H=diag$[H_0,H_1,\ldots.H_{N-1}]$ is the $MRN \times MTN$ block diagonal channel frequency response, where each block is the nth subcarrier component of the channel DFT, groups the symbols transmitted on the nth sub-carrier on all the antennas. For each antenna pair (i, j), I=1, .M$_R$, j=1, .M$_T$ block diagonal channel frequency response, where each block is the nth subcarrier component of the channel DFT,

$$H_n = \sum_{m=0}^{N_{ch}-1} h_m e^{-j2\pi\frac{mn}{N}} \ldots\ldots\ldots(2)$$

The phase noise matrix Q in (1) is

$$Q = \begin{bmatrix} \Theta_0 & \Theta_{N-1} & \cdots & \Theta_1 \\ \Theta_1 & \Theta_0 & \cdots & \Theta_2 \\ \vdots & \vdots & \ddots & \vdots \\ \Theta_{N-1} & \Theta_{N-2} & \cdots & \Theta_0 \end{bmatrix} \otimes I_{M_R},$$

…………(3)

Where $\otimes$ is the Kronecker product and $\Box n$ is the nth component of the phase noise vector DFT

$$\theta_n = \sum_{k=0}^{N-1} e^{j\theta_k} e^{-j2\pi\frac{kn}{N}} \ldots(4)$$

SINR is Signal to Interference plus Noise Ratio that is calculated as SINR = P / (I + N) where P is signal power, I is interference power and N is noise power.

SINR is commonly used in nwireless communication as a way to measure the quality of wireless connections. Typically, the energy of a signal fades with distance. In wireless networks, this is commonly defined by path loss. But unlike wired networks (where the existence of a wired path between sender s and receiver r determines the correct reception of a message), a wireless communication network has to take a lot of environmental parameters into account (e.g. the background noise, interfering strength of other simultaneous transmission). SINR attempts to create a representation of this aspect [8].We define the SINR after the receiver as the ratio between the useful signal power σ^2_x and the variance of the overall disturbance caused by noise and spatial interference σ^2_0, that is, $SINR = \sigma^2_x / \sigma^2_0$ .In ideal conditions, that is, without phase noise or estimation error, the SINR for the the nth signaling vector at the output of the ZF receiver is

$$SINR_n = \frac{SNR/M_T}{\left[H_{eq}^H H\right]^{-1}_{n,n}} \ldots\ldots\ldots(5)$$

3.Channel estimation:

Complex channel estimation (i.e., estimation of channel gain, which includes phase and amplitude) performed for each individual RAKE fingers is required for coherent detection (Maximal Ratio Combining). Complex channel estimation is performed with the assistance of known transmitted pilot symbols[3]-[4]. The accuracy of the channel estimation is crucial for RAKE receiver performance, and it depends on the pilot channel energy, the channel estimation algorithms, and the environment conditions. In particular, mobile speed is required for a variety of channel estimation algorithms.

The pilot symbols can be transmitted in two basic ways: In the case of *dedicate pilot channel scheme*, system has one physical channel fully dedicated to pilot symbol transmission.[6] E.g. Common Pilot Channel, CPICH, in downlink of WCDMA. Another option is to *insert pilot symbols into the data stream* (time multiplexed pilot symbols). E.g. DPDCH/DPCCH in uplink of WCDMA.

(A)*Dedicate Pilot Channel Scheme*:

One possible phase estimation architecture based on a dedicate pilot channel is shown in the following figure:
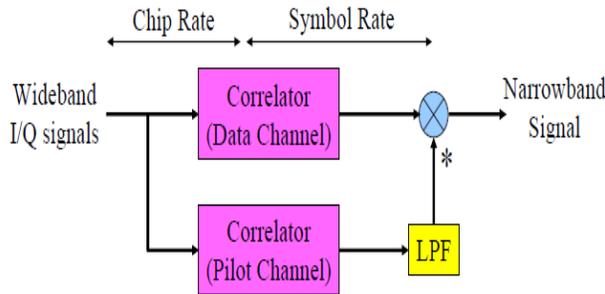
Fig:2 Dedicate Pilot Channel Scheme
The output of the channel estimation is filtered by a low pass filter (LPF), whose bandwidth should be made adjustable to the Doppler frequency.

4.zero-forcing receiver:

Research on the performance analysis of wireless MIMO systems in the majority of cases focuses on Shannon capacity (in particular erotic capacity) and pair wise error probability (PEP) for maximum likelihood receivers. While erotic capacity and PEP are well understood, only little is known about the symbol error rate (SER) performance of low complexity linear MIMO receivers, especially in the presence of fading correlation at the receive antenna array. For uncorrelated Rayleigh fading in the context of smart antenna systems that for zero-forcing (ZF) receivers, the sub channel signal to noise ratio (SNR) (for each user) follows a simple gamma distribution. This result was extended for MIMO systems to cover the case of fading correlation at the transmit antenna array in and independently in. On the other hand, many results are available on the analysis of minimum mean squared error (MMSE) processing (which is termed optimum combining in smart antenna literature) with spatially uncorrelated fading. The exact sub channel SINR distribution for users with different transmit powers was given in based on a statistical result on certain matrix quadratic forms in. For equal-power interferers, an exact SER analysis was presented in , where the eigenvalue probability density function of complex Wishart matrices was used for the derivation . However, to the authors' best knowledge, no general exact analytical SER expressions can be found in literature for the case of spatial fading correlation at the receive antenna array. Available results for MMSE receivers are approximations or are semi-analytic, thus still requiring lengthy Monte-Carlo simulations. For the special case of only two transmit and two receive antennas, exact SER formulas were given in for ZF receivers and in for MMSE receivers based on a random eigenvalue approach for systems with receive as well as transmit correlation. However, these results could not be generalized for an arbitrary number of transmit and receive antennas. In this paper, for the first

time we present fully analytic SER expressions for MIMO ZF receivers and an arbitrary finite number of transmit and receive antennas with arbitrary fading correlation at the transmit as well as the receive antenna array.[2] We emphasize that correlation at the receiver (a practically relevant case also in multi-user beam forming scenarios) can be taken into account, which is not possible with other mathematical approaches. In the course of the derivation, we present expressions for the sub channel SNR moment generating function (MGF) in terms of certain expected values of ratios of random determinants. As it appears that there are no results available in literature for calculating these expected values, we present closed form formulas that are derived by a novel mathematical approach. Specifically, we make use of certain complex Gaussian integrals for the derivation. Based on the MGF, we derive exact formulas for arbitrary moments as well as closed form expressions for PDF and CDF. We show that the SER of ZF receivers in the presence of correlated fading at transmit and receive antenna array can be given in closed form for arbitrary square QAM constellations by using a well-known integral representation of the Gaussian Q function. Moreover, we calculate exact formulas for the mean mutual information (MMI) of the sub channels. In the expression of the ICI variance, for the ZF receiver we have

$$Tr\left\{E\left[H_i^H G_n^H G_n H_i\right]\right\} = Tr\left\{E\left[H_i^H \left(H_n^\dagger\right)^H H_n^\dagger H_i\right]\right\}$$

$$+ Tr\left\{E\left[Z_i^H \left(H_n^\dagger\right)^H H_n^\dagger Z_i\right]\right\} \ldots\ldots\ldots\ldots(6)$$

Because of the independence of **Z** and **H** and $E\left[Z_i Z_i^H\right] = \sigma_{est}^2 \mathbf{I}$, we have

$$Tr\left\{E\left[Z_i^H \left(H_n^\dagger\right)^H H_n^\dagger Z_i\right]\right\} = \sigma_{est}^2 Tr\left\{E\left[\left(H_n^H H_n\right)^{-1}\right]\right\}$$

$$= \sigma_{est}^2 \frac{Tr\left[R_T^{-1}\right]}{M_R - M_T} \ldots\ldots\ldots\ldots(7)$$

For the particular case of Wiener phase noise and spatially in correlated channel with exponential PDP, the variance of the overall phase noise interference after ZF can be summarized in

$$\sigma_{v_n}^2 = \sigma_x^2 \sigma_{CPE}^2 + \frac{\sigma_x^2}{M_T} \sum_{\substack{i=0 \\ i\neq n}}^{N-1} \sum_{k=0}^{+\infty} \frac{2\pi B_0}{1 + \left(\frac{(n-i)^2 / N + k}{B_0}\right)^2}$$

$$\times\left[\frac{\sigma_{est}^2 M_T}{M_R - M_T} + \frac{M_T}{1+4\pi^2(n-i)^2(\Delta f T_{rms})^2} + \frac{4\pi^2(n-i)^2(\Delta f T_{rms})^2}{1+4\pi^2(n-i)^2(\Delta f T_{rms})^2}M_T M_R\right] \qquad \sigma_{v_n}^2 = \sigma_x^2 \sigma_{CPE}^2 + \frac{\sigma_x^2}{M_T}\sum_{\substack{i=0 \\ i\neq n}}^{N-1}\sum_{k=0}^{+\infty}\frac{2\pi B_0}{1+\left(\frac{(n-i)^2/N+k}{B_0}\right)^2}$$

$$(8)$$

5.MinimumMean Square Error (MMSE):

In statistics and signal processing, a minimum mean square error (MMSE) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality. The term MMSE specifically refers to estimation in a Bayesian setting, since in the alternative frequentist setting there does not exist a single estimator having minimal MSE [7]. A somewhat similar concept can be obtained within the frequentist point of view if one requires unbiasedness, since an estimator may exist that minimizes the variance (and hence the MSE) among unbiased estimators. Such an estimator is then called the minimum-variance unbiased estimator.

$$G_{MMSE} = \left(\overset{H}{\overline{H}}_{eq}\overline{H}_{eq} + \frac{1}{SNR}I_{NM_T}\right)^{-1}\overset{H}{\overline{H}}_{eq}$$

$$\dots\dots(9)$$

Also in the case of the MMSE receiver, in the trace of the inner matrix of we can separate the effects of the estimation error and of the sub-carrier correlation,

$$Tr\left\{E\left[H_i^H G_n^H G_n H_i\right]\right\} = \sigma_{est}^2 E\left[Tr\left\{G_n^H G_n\right\}\right]$$

$$+Tr\left\{E\left[H_i^H H_n\left\{\left(H_n^H H_n + \frac{1}{SNR}I_{M_T}\right)^{-1}\right\}^H \times \left(H_n^H H_n + \frac{1}{SNR}I_{M_T}\right)^{-1} H_n^H H_i\right]\right\}$$

$$(10)$$

Where $H_i = \rho H_n + \sqrt{1-\rho H_I}$

$$Tr\left\{E\left[H_n H_n^H H_n H_n^H\right]\right\} = M_T M_R\left[T_R(R_R^2) + \frac{1}{M_T}Tr^2(R_R)\right]$$

$$(11)$$

In the independent fading term, we have the expected trace of the product of independent matrices with

$$Tr\left\{E\left[H_n H_n^H H_I H_I^H\right]\right\} = M_T Tr\left[R_R^2\right]$$

Final expression for MMSE is

$$\times\left[\frac{\sigma_{est}^2 M_T}{M_R - M_T} + \frac{SNR^2 M_T M_R(M_T + M_R)}{1+4\pi^2(n-i)^2(\Delta f T_{rms})^2} + \frac{4\pi^2(n-i)^2(\Delta f T_{rms})^2}{1+4\pi^2(n-i)^2(\Delta f T_{rms})^2}SNR^2 M_T^2 M_R\right]$$

$$(12)$$

6.Simulation results
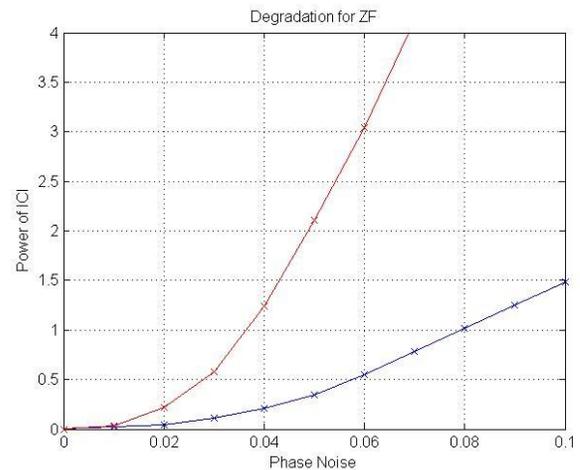
Fig.3.power of the ICI contribution for the ZF receiver with independent fading and ideal channel estimation. As the phase noise increases the ici also increases. The curves of the ICI power vs. *B* show a very good matching, although the first order approximation of is slightly optimistic
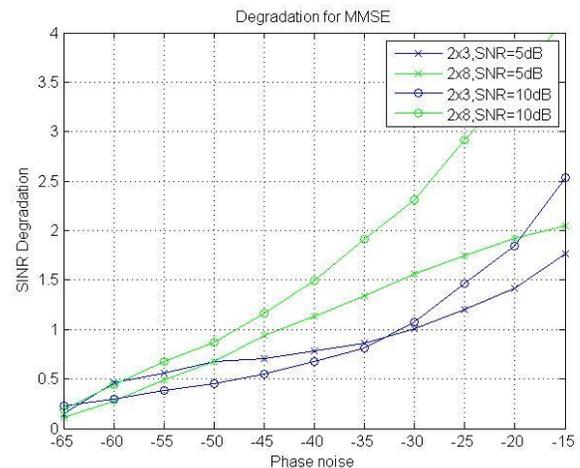
Fig.4.degradation for MMSE as a function of the phase noise at 0.1/T, with exponential T_{rms} =0.1t, estimation error MSE=-30db and refrence SNR=5db and SNR=10db as phase noise increase the SINR degradation more.
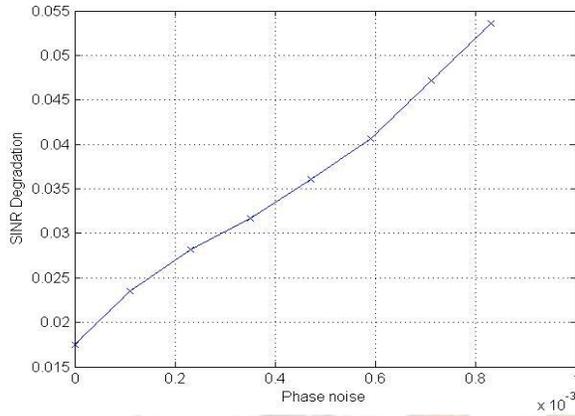
Fig.5.It is the between phase noise and SINR Degradation in that sinr is improved when compared to the other two figures. It is in the MMSE method. It is the theoretical simulation result.
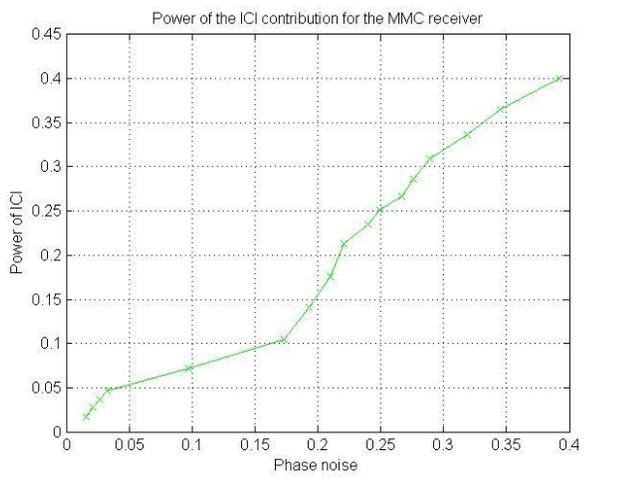
Fig.6.SINR degradation improves in ZF recovering that the phase noise increases power of ICI increases. When compared to the previous simulation results.

7.CONCLUSIONS:

In this paper we present a SINR degradation in linear receiver of ZF and MMSE for MIMO OFDM by considering both channel estimation error and the phase noise with partial CPE compensation. The accuracy of the ZF and MMSE has been compared with phase noise the results obtained here can be used to design efficient system.

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