Alok Pandey, L.D. Malviya, Vineet Sharma / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 2, Issue 3, May-Jun 2012, pp.1584-1587

Comparative Study of LMS and NLMS Algorithms in Adaptive Equalizer

Alok Pandey*, L.D. Malviya **, Vineet Sharma***

*(Department of Electronics and Communication, S.G.S.I.T.S, Indore)
** (Department of Electronics and Communication, S.G.S.I.T.S, Indore)
*** (Department of Electronics and Communication, S.G.S.I.T.S, Indore)

Abstract—In this paper we provide a thorough ser(symbol error rate) analysis of two well known adaptive algorithms for equalization based on a novel least squares reference model that allows to treat the equalizer problem equivalently as system identification problem. An adaptive algorithm is a procedure for adjusting the parameters of an adaptive filter to minimize a cost function chosen for the task at hand. Here we firstly proposed a noise-robust optimal-step-size frequency domain LMS (least mean square) algorithm for estimating the equalizer coefficients and after the modified LMS algorithm which is an extension of the standard LMS (least mean square) algorithm which bypasses this issue by calculating maximum step size value. The proposed algorithms conclude that the step-size ambiguity of the LMS (least mean square) algorithm is solved by the NLMS (normalized mean square) algorithm, which gives faster convergence speed as compared to the LMS (least mean square) algorithm. Computer simulation results a represented to show its improved performance for trained adaptive equalization. This paper focuses on the use of these two proposed algorithms to reduce this unwanted echo, thus increasing communication quality.

Keywords: ***Adaptive filters, Adaptive algorithms, channel equalization, LMS(least mean square), NLMS(normalized least mean square).***

1. INTRODUCTION

In modern digital communications, it is well known that channel equalization plays an important role in compensating channel distortion. Unfortunately, various channels have time varying characteristic and their transfer functions change with time. Furthermore, time-varying multipath interference and multiuser interference are two major limitations for high speed digital communications. Usually, adaptive equalizers are applied in order to cope with these issues [1]. For adaptive channel equalization, we need a suitable filter structure and proper adaptive algorithms. High-speed digital transmissions mostly suffer from inter-symbol interference (ISI) and additive noise. The adaptive equalization algorithms recursively determine the filter coefficients in order to eliminate the effects of noise and ISI. We consider only uncoded and quadrature amplitude modulation (4-QAM).

The most popular design strategy in this setting is reduce the mean-squared-error (MSE) using suitable adaptive algorithm [3] . However, as recognized in, a better strategy is to choose the equalizer coefficients so as to minimize the error probability or symbol error rate (SER). Minimum-BER equalization first appeared in which among the numerous algorithms that can be used for adaptive filtering, the Least Mean Square (LMS) algorithm has enjoyed widespread popularity because of its simplicity in computation and implementation. However, it is well known that the least mean square (LMS) type algorithms can only minimize the current estimate error to some extent. It is known that a variable step size algorithm has to be applied to make a trade-off between the convergence rate and the steady-state mis adjustment.

Our objective in this paper is firstly to compare a scatter results of both proposed algorithms in the generic adaptive filter and then after we compare their ser characteristics. The first adaptive LMS (least mean square) algorithm for approximating the minimum-BER equalizer was proposed in, where receiver estimates of the channel, noise power, and noiseless channel output were used to approximate a stochastic gradient algorithm [2]. This algorithm is significantly have slow convergence and poor tracking as compare to the the normalized least-mean-square (NLMS) algorithm, and even with perfect knowledge of the channel and noise power would be susceptible to mis convergence. By optimally selecting the step size during the adaptation, we can obtain both fast convergence rate and low steady state mean square error.

2. ADAPTIVE EQUALIZATION

It is very difficult for estimating both the channel order and the distribution of energy among the taps and even it is very difficult to predict the effect of the environment on these taps. Hence it is a must for the equalization process to be adaptive. The equalizer need to be adapted very frequently with the changing environment. This includes two phases [3]. Firstly the equalizer needs to be trained with some known samples in the presence of some desired response (Supervised Learning). After training the weights and various parameters associated with the equalizer structure is frozen to function as a detector. These two processes are frequently implemented to keep the equalizer adaptive. We call the Equalizer is Frozen, if we keep the adaptable parameters of the equalizer constant. Figure.1 depicts how the equalization process is adaptive, After the initial training period (if there is one), the coefficients of an adaptive equalizer may be continually adjusted in a decision-directed manner.

In this mode, the error signal $e_k = z_k - x_k$ is derived from the final (not necessarily correct) receiver estimate $\{x_k\}$ of the transmitted sequence $\{x_k\}$. In normal operation, the receiver decisions are correct with high probability, so that the error estimates are correct often enough to allow the adaptive equalizer to maintain precise equalization. The larger the step size, the faster the equalizer tracking capability[5]. However, a compromise must be made between fast tracking and the excess mean-square error of the equalizer. The excess MSE is that part of the error power in excess of the minimum attainable MSE (with tap gains frozen at their optimum settings). This excess MSE, caused by tap gains wandering around the optimum settings, is directly proportional to the number of equalizer coefficients, the step size, and the channel noise power[6].The step size that provides the fastest convergence results in an MSE which is, on the average, 3 dB worse than the minimum square error.

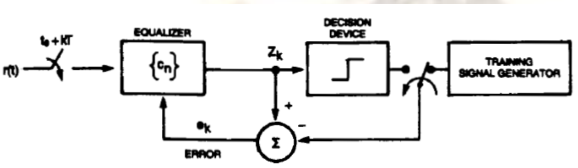

Fig.1 Adaptive equalizer

3. Gradient based Adaptive algorithm

An adaptive algorithm is a procedure for adjusting the parameters of an adaptive filter to minimize a cost function chosen for the task at hand[7]. In this section, we describe the general form of many adaptive FIR filtering algorithms and present a simple derivation of the LMS(least mean square) adaptive algorithm. In our discussion, we only consider an adaptive FIR filter structure in Figure.2 Such systems are currently more popular than adaptive IIR filters because

(1) The input-output stability of the FIR filter structure is guaranteed for any set of fixed coefficients, and

(2) The algorithms for adjusting the coefficients of FIR filters are simpler in general than those for adjusting the coefficients of IIR filters.

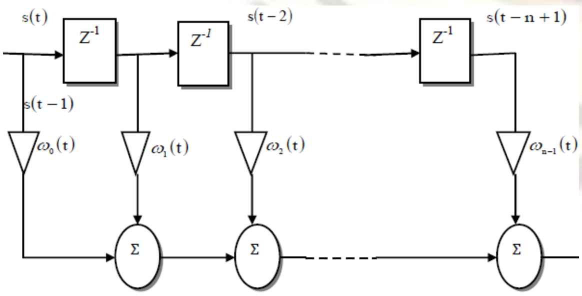

Fig2. Structure of an FIR filter

Figure.2 shows the structure of a direct-form FIR filter, also known as a tapped- delay-line or transversal filter, where z-1 denotes the unit delay element and each ω(t) is a multiplicative gain within the system. In this case, the parameters in ω(t) correspond to the impulse response values of the filter at time n. We can write the output signal y(t) as,

$$y(t) = \sum_{i=0}^{n-1} \omega_i(t)\ s(t-i) = W^T(t)\ S(t) \quad (1)$$

Where,

$$S(t) = [s(t), s(t-1), \ldots\ldots\ldots\ldots\ldots\ldots s(t-n+1)]^t \quad (2)$$

denotes the input signal vector and T denotes vector response

$$\omega_i(t) = [\omega_0(t), \omega_1(t) \cdots, \omega_{n-1}(t)]^T \text{ is } \{\omega_i(t)\},\ 0 \le i \le n-1 \quad (3)$$

Are the n parameter of the system at time t. The general form of adaptive algorithm algorithm is

$$W(t+1) = W(t) + \mu(t)\, G\,(e(t)\, s(t)\psi(t)) \quad (4)$$

where G(φ) is a particular vector-valued nonlinear function, μ(*t*) is a step size parameter, *e*(*t*) and s(*t*) are the error signal and input signal vector, respectively, and is a vector of states that store pertinent information about the characteristics of the input and error signals. In the simplest algorithms, ψ(t) is not used[3].

The form of G(φ) in (4) depends on the cost function chosen for the given adaptive filtering task. The Mean-Squared Error **(**MSE) cost function can be define as,

$$J_{MSE}(t) = \frac{1}{2}\int_{-\infty}^{\infty} e^2(t)\, p_t(e(t))\, d\, e(t)$$
$$= \frac{1}{2} E\{e^2(t)\} \quad (5)$$

Where, p_t (*e* (*t*)) represents the probability density function of the error at time *t* and E{●} is the expectation integral on the right-hand side of (5).

3.1 LMS Algorithm

The LMS algorithm changes (adapts) the filter tap weights so that *e(n)* is minimized in the mean-square sense. When the processes *x(n)* & *d(n)* are jointly stationary, this algorithm converges to a set of tap-weights which, on average, are equal to the Wiener-Hopf solution[6].

The LMS algorithm is a practical scheme for realizing Wiener filters, without explicitly solving the Wiener-Hopf equation. This is shown in Figure 3.

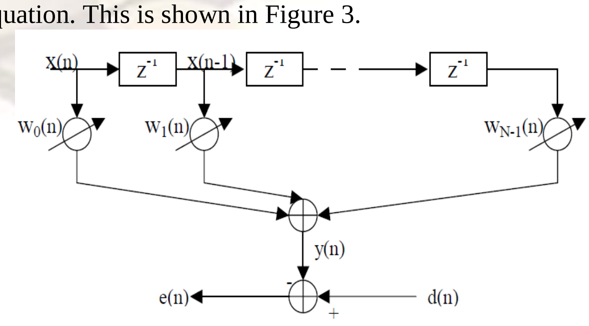

Fig.3 N tap transversal adaptive filter

$$y(n) = \sum_{i=0}^{N-1} w_i(n)x(n-i) \quad (6)$$

$$e(n) = d(n) - y(n) \quad (7)$$

The cost function J(t) chosen for the steepest descent algorithm of eq.(5) determines the coefficient solution obtained by using adaptive filter. If the MSE cost function in (5) is chosen, the resulting algorithm depends on the statistics of s(t) and d(t) because of the expectation operation that defines this cost function. One such cost function is the least-squares cost function given by

$$J_{LS}(t) = \sum_{i=0}^{t} \alpha(i)(d(i) - W^T(t)S(i))^2 \quad (8)$$

The weight update equation for LMS can be represented as

$$W(t+1) = W(t) + \mu e(t)S(t) \quad (9)$$

Where μ is learning factor, equation (9) requires only multiplications and additions to implement. In fact, the number and type of operations needed for the LMS algorithm is nearly the same as that of the FIR filter structure with fixed coefficient values and hence LMS has become very popular [5].

In effect, the iterative nature of the LMS coefficient updates is a form of time-averaging that smoothes the errors in the instantaneous gradient calculations to obtain a more reasonable estimate of the true gradient.

3.2 NLMS Algorithm

The NLMS algorithm has been implemented in Matlab. As the step size parameter is chosen based on the current input values, the NLMS algorithm shows far greater stability with unknown signals[4]. This combined with good convergence speed and relative computational simplicity make the NLMS algorithm ideal for the real time adaptive echo cancellation system.

As the NLMS is an extension of the standard LMS algorithm, the NLMS algorithms practical implementation is very similar to that of the LMS algorithm. Each iteration of the NLMS algorithm requires these steps in the following order [7].

1. The output of the adaptive filter is calculated

$$y(n) = \sum_{i=0}^{N-1} w(n)x(n-i) = \mathbf{w}^T(n)\mathbf{x}(n) \quad (10)$$

2. An error signal is calculated as the difference between the desired signal and the filter output

$$E(n) = d(n) – y(n)$$

3. The step size value for the input vector is calculated

$$\mu(n) = \frac{1}{\mathbf{x}^T(n)\mathbf{x}(n)} \quad (11)$$

4. The filter tap weights are updated in preparation for the next iteration.

$$W(n+1) = W(n) + \mu(n)e(n)x(n)$$

Each iteration of the NLMS algorithm requires 3N+1 multiplications, this is only N more than the standard LMS algorithm. This is an acceptable increase considering the gains in stability and echo attenuation achieve.

4. RESULTS FOR LMS & NLMS ALGORITHM

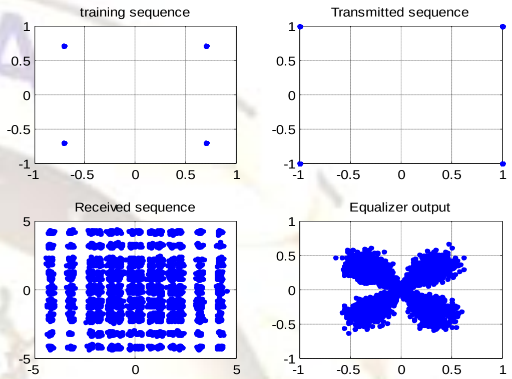

Fig4. Scattering fig of LMS algorithm

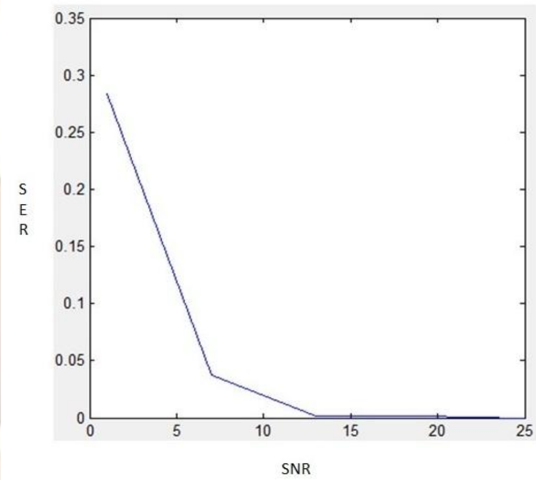

Fig5. SER Performance of LMS algorithm

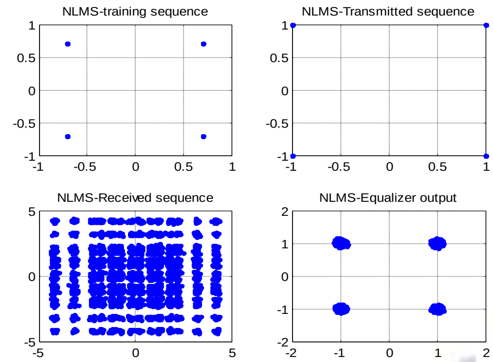

Fig6. Scattering fig of NLMS algorithm

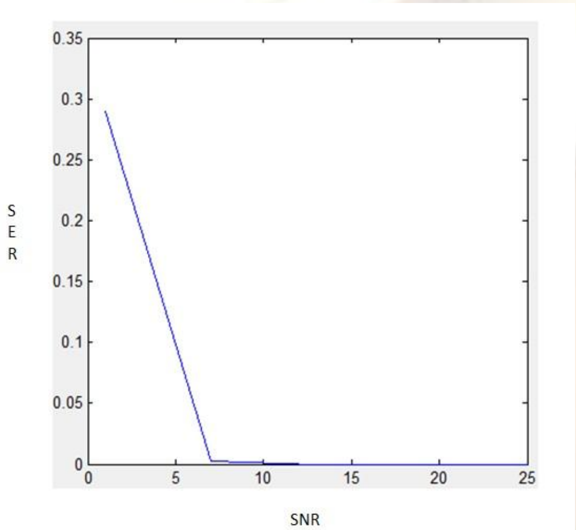

Fig7. SER performance of NLMS algorithm

5. CONCLUSION

In these algorithms, the LMS algorithm is the most popular adaptive algorithm, because of their low computational complexity. However, the LMS algorithm suffers from slow and data dependent convergence behavior. The NLMS algorithm, an equally simple, but more robust variant of the LMS algorithm, exhibits a better balance between simplicity and performance than the LMS algorithm. Due to its good characteristics the NLMS has been largely used in real-time applications.

6. REFERENCES

[1] Haykin. S. Digital Communication. Singapore: John WilSons Inc, 1988

[2] Haykin. S. Adaptive Filter Theory. Delhi: 4^{th} Ed, Pearson Education, 2002.

[3] Qureshi. S. U. H, "Adaptive equalization," Proc. IEEE, vol. 73, no.9, pp.1349-1387, 1985

[4] Lee, K.A.; Gan,W.S; "Improving convergence of the NLMS algorithm using constrained subband updates," Signal Processing Letters IEEE, vol. 11, pp. 736-739, Sept. 2004.

[5] Tandon, A.; Ahmad, M.O.; Swamy, M.N.S.; "An efficient, low-complexity, normalized LMS algorithm for echo cancellation", IEEE workshop on Circuits and Systems, 2004. NEWCAS 2004, pp. 161-164, June 2004

[6] Soria, E.; Calpe, J.; Chambers, J.; Martinez, M.; Camps, G.; Guerrero, J.D.M.; "A novel approach to introducing adaptive filters based on the LMS algorithm and its variants", IEEE Transactions, vol. 47, pp. 127-133, Feb 2008.

[7] D. Morgan and S. Kratzer, "On a class of computationally efficient rapidly converging, generalized NLMS algorithms," IEEE Signal Processing Lett., vol. 3, pp. 245–247, Aug. 1996.

[8] Lucky, R.W., Techniques for adaptive equalization of digital communication systems, *Bell Sys.Tech. J.*, 45, 255–286, Feb. 1966.

[9] Douglas, S.C. and Meng, T.H.-Y., Stochastic gradient adaptation under general error criteria, *IEEE Trans. Signal Processing*, 42(6), 1335–1351, June 1994.

[10] Messerschmitt, D.G., Echo cancellation in speech and data transmission, *IEEE J. Sel. Areas Commun.*, SAC-2(2), 283–297, March 1984.

[11] Mathews, V.J., Adaptive polynomial filters, *IEEE Signal Processing Mag.*, 8(3), 10–26, July 1991

[12] Oppenheim, A.V. and Schafer, A.W., *Discrete-Time Signal Processing*, Prentice-Hall, Englewood Cliffs, NJ, 1989