# Computational Analysis of Shockwave in Convergent Divergent Nozzle

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## Abstract

The objective of this paper is to computationally analyse shock waves in the Convergent Divergent (CD) Nozzle. The commercial CFD code Fluent is employed to analyse the compressible flow through the nozzle. The analysis includes static pressure, temperature, Mach number and density of the flow for different nozzle pressure ratios (NPR i.e., the ratio between exit pressure of the nozzle to ambient pressure). The results are compared with the analytical results of quasi-one dimensional equation. The flow characteristic before and after the shock is discussed.

Keywords Supersonic flow, Shock wave, convergent divergent nozzle,

### Nomenclature

| | |
|---|---|
| A | = Nozzle area, $m^2$ |
| F | = External body force, N |
| M | = Mach number |
| NPR | = Nozzle Pressure Ratio ($p_e$ / $p_a$) |
| p | = Static pressure, $N/m^2$ |
| r | = Local radius, m |
| v | = Velocity, m/sec |
| ρ | = Density, $kg/m^3$ |

### Subscript

| | |
|---|---|
| 0 | = Stagnation conditions |
| 1 | = Before Shock |
| 2 | = After Shock |
| a | = Ambient conditions |
| c | = critical conditions |
| e | = Exit |
| i | = Inlet |
| r | = Radial coordinate |
| x | = Axial coordinate |

### Superscript

| | |
|---|---|
| * | = Throat |

## Introduction

The study of CD nozzles exhausting supersonic jets is a noteworthy problem in many aerospace applications. The imperfect matching between the pressures $p_a$ and $p_e$ leads to the formation of a complicated shock wave structure. Though experimental, numerical and analytical investigations of the shock structure of the supersonic jets have been reported in early literatures, the subject is quite complicated and yet remains to be clearly understood.

Supersonic nozzle flow separation occurs in CD nozzles at pressure ratios far below their design value that results in shock formation inside the nozzle. In the one-dimensional analysis it is treated that the shock is normal, the flow past the shock stays attached to the wall, and the subsonic flow downstream of the shock expands isentropically to the level of back pressure at the nozzle exit. But in reality the flow detaches from the wall and forms a separation region, subsequently the flow downstream becomes non-uniform and unstable.

Adamson and Nicholls [1], analysed nozzle jets experimentally and presented an analytical method for calculating the position of shock inside the nozzle, whereas Lewis and Carlson [2] experimentally determined the distance of the first Mach disc in under expanded supersonic nozzles issuing gas from the nozzle exit plane. Romine [3] presented the mechanisms of flow separation from the nozzle wall. Back.et.al [4] presented the comparison of measured and predicted flows through conical supersonic nozzles. They also presented the wall static pressure for various conical nozzles in the region of 2-D flow as quasi-one dimensional theory is not applicable. They claim that flow through the transonic region was found to depend essentially on local configuration, i.e., on the ratio of radii $r_c/r_*$.

Supersonic flow separation in a CD nozzle leads to unstable plume in the exit region. Qing Xiao., et. Al., [5] demonstrated that such separation can be efficiently utilized to enhance the jet mixing. Potential applications include propulsion of jet engines, turbofans, turbo jets, spin-stabilised rockets and ramjet engines.

The present work deals with computational investigation for different nozzle NPR values. Shock structure is formed near the exit of the nozzle by varying the exit pressure. The considered NPR values are 1.63, 1.59, 1.55 and 1.52. It is observed that for NPR below 1.52 there is no formation of shock and for NPR above 1.63 reversible flow is detected and the computational solution tends to be more precise.

**The governing equations:**

The flow model considered here is a supersonic flow through the nozzle. Air is considered as the working fluid, flow being inviscid.

The 2-D Navier-stokes equation is given by,

The continuity equation,

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial r}(\rho v_r)+\frac{\rho v_r}{r}=0 \qquad (1)$$

The axial and radial momentum equations are,

$$\frac{\partial}{\partial t}(\rho v_x)+\frac{1}{r}\frac{\partial}{\partial x}(r\,\rho\,v_x v_x)+\frac{1}{r}\frac{\partial}{\partial r}(r\,\rho\,v_x v_r)=-\frac{\partial p}{\partial x}+F_x \qquad (2)$$

$$\frac{\partial}{\partial t}(\rho v_r)+\frac{1}{r}\frac{\partial}{\partial x}(r\,\rho\,v_x v_r)+\frac{1}{r}\frac{\partial}{\partial r}(r\,\rho\,v_r v_r)=-\frac{\partial p}{\partial r}+F_r \qquad (3)$$

The Energy equation,

$$\frac{\partial}{\partial t}(\rho E)+\nabla.\left(\vec{v}(\rho E+p)\right)=0 \qquad (4)$$

**Computational Methods**

The Convergent Divergent nozzle configuration used in this investigation is that proposed for the typical ramjet engine by J.C. Dutton [7]. The area contraction ratio between the nozzle inlet to throat ($A_i/A^*$) is 0.668 with $M_e$ = 2.4682. The axisymmetric nozzle is shown in fig. 1.

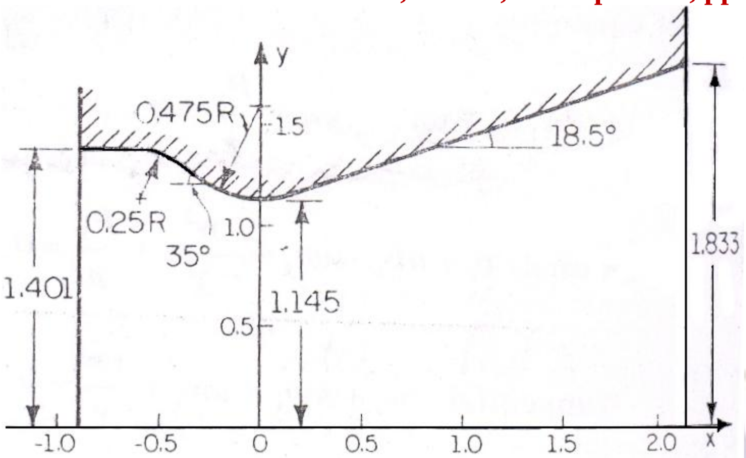


Fig.1. The axi-symmetric nozzle profile.

The set of governing equations is solved on the Gambit generated grid structure of size 240 ×150 with 36000 cells and 71610 faces. The nozzle inlet parameters $P_0$, $a_0$ and $T_0$ and other parameters like γ are assumed suitably according to standards. The inlet boundary condition is considered as pressure inlet and outlet boundary condition is considered as pressure outlet. The outlet pressure is varied to achieve different NPR values.

## Analytical Calculations

In this work the computational results are compared with analytical results. In supersonic flow, when the total pressure at the nozzle exit is reduced, the nozzle becomes chocked and the mass flow rate becomes a fixed value. Further reduction in pressure results with the formation of shock wave at the downstream from the throat. The flow characteristics changes suddenly across the shock wave. Upstream of the shock wave the flow is supersonic and immediately after the shock wave the flow becomes subsonic. Further downstream, the subsonic flow pressure increases and at exit matches with the $p_a$. This is in detail discussed by Anderson J.D [6] in his analytical work. The same analytical equations (equation 9 & 10) are used by the authors to find the pressure and Mach number across the shock.

The one-dimensional isentropic relation for the Mach number variation through the nozzles is,

$$\frac{A^*}{A} = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/(\gamma-1)} \qquad (5)$$

The variation of pressure, density and temperature as functions of Mach number are,

$$\frac{p}{p_0} = \left[1+\frac{\gamma-1}{2}M^2\right]^{-\gamma/(\gamma-1)} \qquad (6)$$

$$\frac{\rho}{\rho_0} = \left[1+\frac{\gamma-1}{2}M^2\right]^{-1/(\gamma-1)} \qquad (7)$$

$$\frac{T}{T_0} = \left[1+\frac{\gamma-1}{2}M^2\right]^{-1} \qquad (8)$$

The Total pressure ratio across the shock, as a function of $M_1$,

$$\frac{p_{0_2}}{p_{0_1}} = \left[ \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2} \right]^{\gamma/(\gamma-1)} \left[ \frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)} \right]^{1/(\gamma-1)} \tag{9}$$

The relation between Mach numbers before and after shock wave,

$$M_2 = \left[ \frac{1+[(\gamma-1)/2] M_1^2}{\gamma M_1^2 - (\gamma-1)/2} \right]^{1/2} \tag{10}$$

## Results and Discussion

In order to validate, the computational results are compared with that of the one dimensional isentropic flow, Fig.2 and Fig.3 show the pressure ratio and Mach number variations for normal supersonic flow. The reason for difference between the analytical and computational is due to the one dimensional approximation.

From the computational results it is observed that the shock is captured inside the nozzle for the four nozzle pressure ratios considered. In all the cases the shock is found downstream of the throat. Fig.4 shows the comparison of pressure ratio for different NPR values. From figure it is evident that by increasing the nozzle exit pressure, the shock structure moves towards the nozzle throat. It is also observed that pressure decreases after passing through the shock wave, and the pressure ratio increases with increase in NPR values. Fig.5 shows the change in Mach number with increase in NPR. The Mach number decreases after passing through the shock wave. From the figure it is clear that the Mach number after the shock became subsonic. Similarly Fig.6 and Fig.7 show the comparison of density ratio and temperature ratio respectively along the nozzle length. It is observed that the density ratio and temperature ratio increases after the shock. The authors observed during the computational work that there is no shock formation below the NPR value of 1.52. It is also observed that with increase in NPR values the exit pressure and Mach number increase. This is plotted in Fig.8 and Fig.9.

The computational result obtained using the commercial tool is shown in Fig.10 for the various NPR values. The figure shows the pressure contours of the CD nozzle. Decrease in pressure across the shock followed by increase in the downstream can be noticed in the figure.

The change in flow characteristics across the shock wave is tabulated in Table 1. The table also compares the flow characteristics of analytical and computational results for various NPR values. The deviation in results from analytical to computational is due to the one-dimensional approximation.

Table 1. Comparison of analytical and computational flow characteristics across the shock wave

| NPR | $M_2/M_1$ | | $p_2/p_1$ | | $T_2/T_1$ | | $\rho_2/\rho_1$ | |
|---|---|---|---|---|---|---|---|---|
| | Analytical | Computational | Analytical | Computational | Analytical | Computational | Analytical | Computational |
| 1.63 | 0.4246 | 0.2932 | 2.7640 | 3.2838 | 1.3777 | 1.6196 | 2.0065 | 2.4700 |
| 1.59 | 0.3786 | 0.2636 | 3.1853 | 4.6233 | 1.4548 | 1.7411 | 2.1895 | 2.6970 |
| 1.55 | 0.3473 | 0.2335 | 3.5500 | 4.9426 | 1.5204 | 1.8174 | 2.3353 | 2.7601 |
| 1.52 | 0.3245 | 0.2208 | 3.8695 | 5.8900 | 1.5770 | 1.9436 | 2.4537 | 3.0354 |

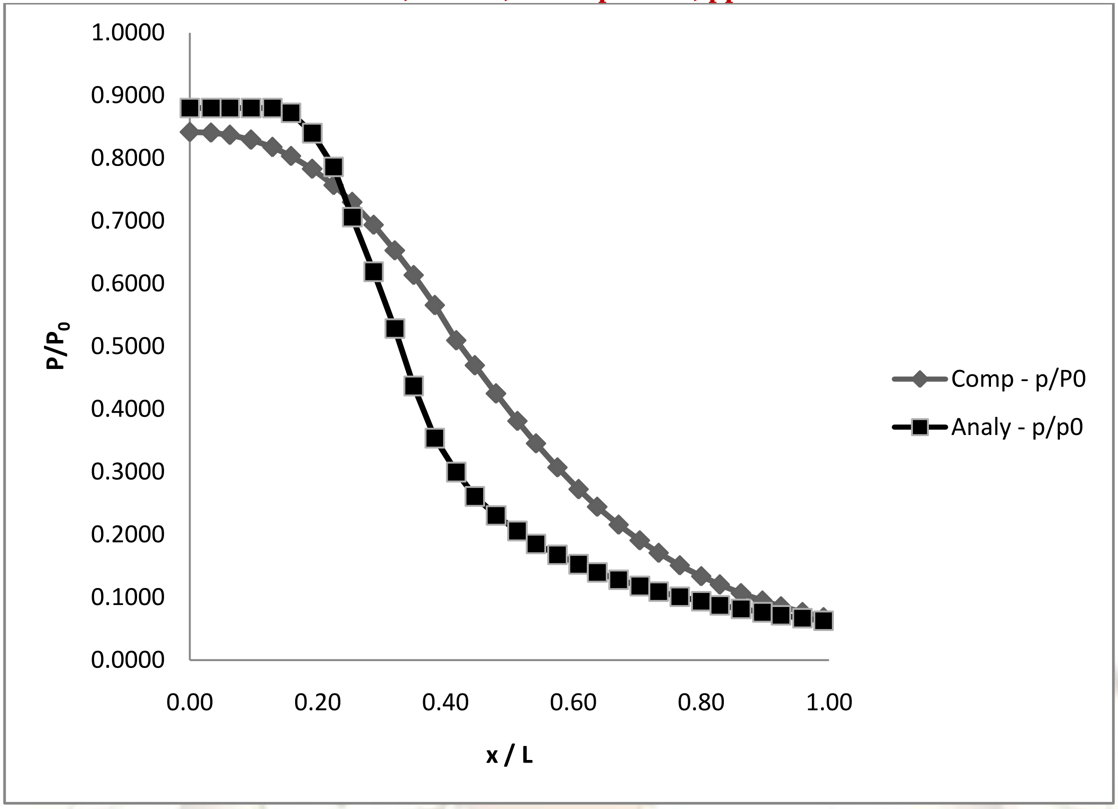


Fig.2 Comparison of pressure ratio of computational results with analytical

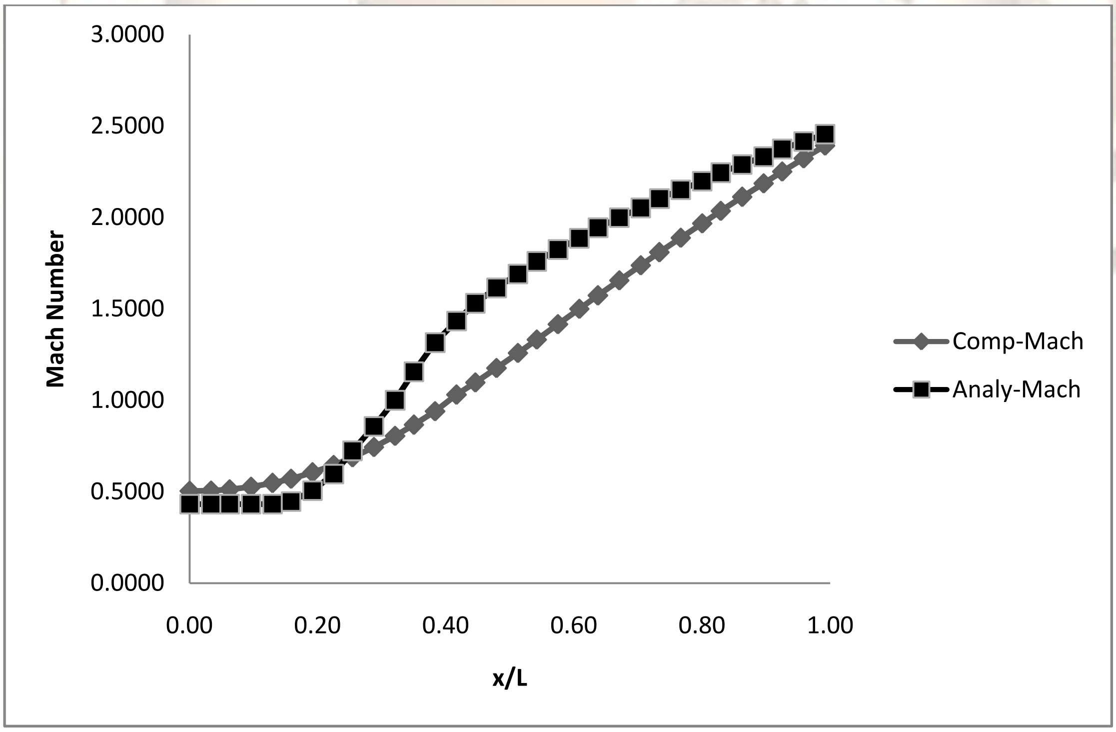


Fig.3 Comparison of Mach number of computational with analytical

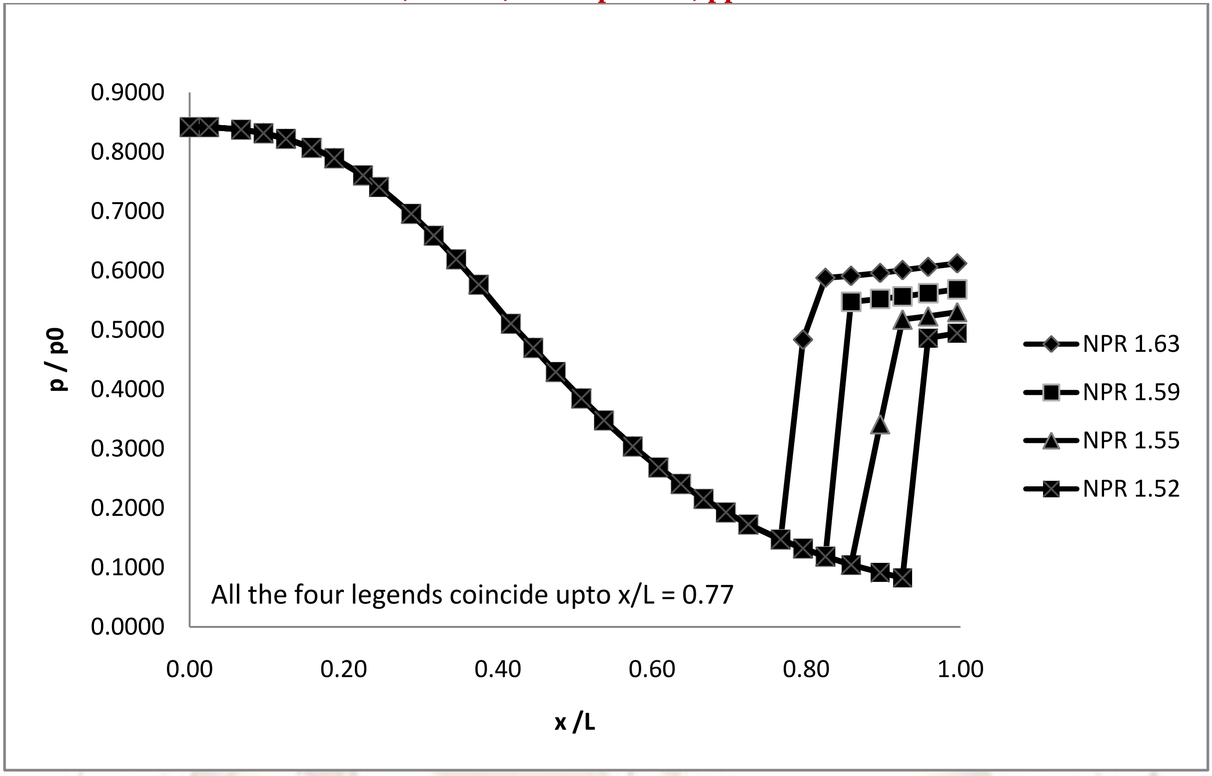


Fig.4 Comparison of pressure ratio along the nozzle length for different NPR values

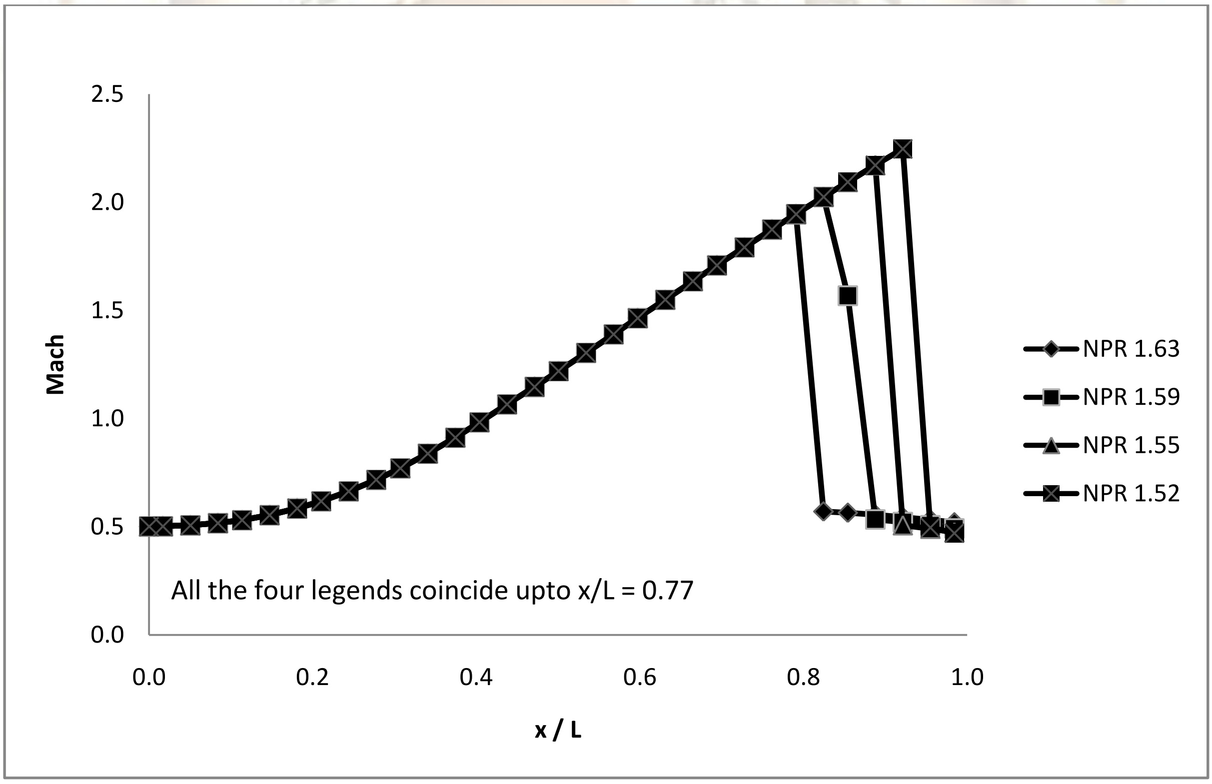


Fig.5 Comparison of Mach number along nozzle length for different NPR values

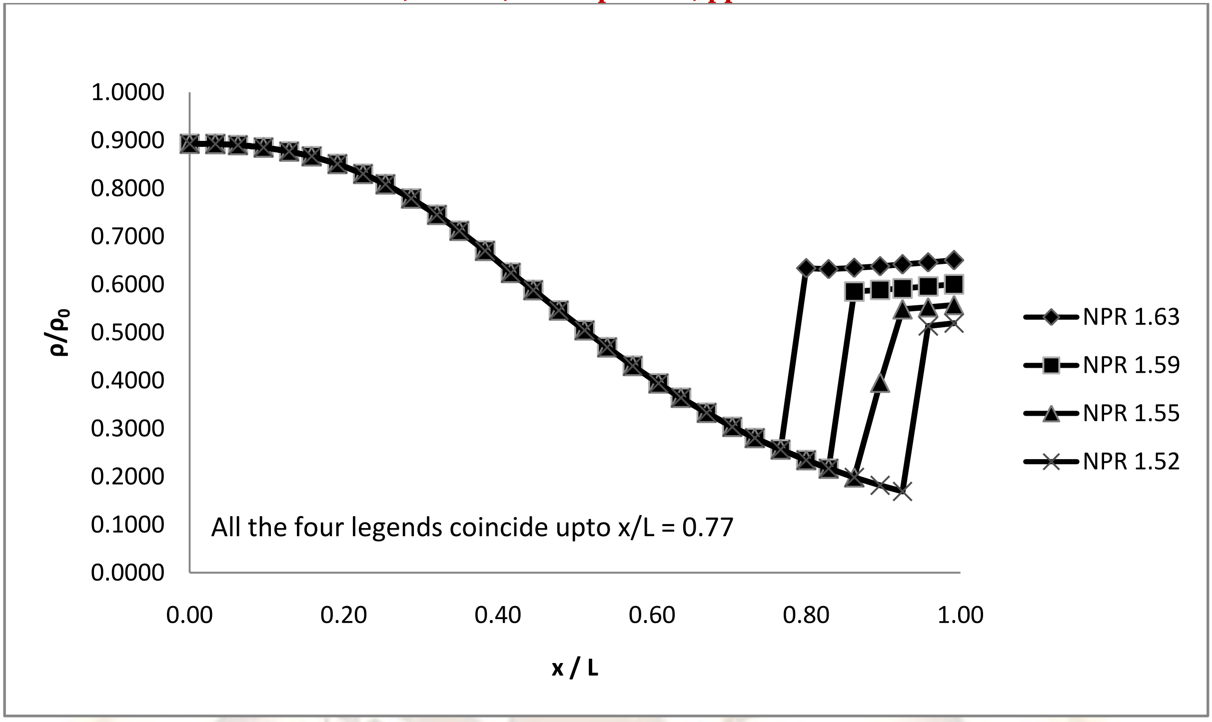


Fig.6 Comparison of density ratio along nozzle length for different NPR values

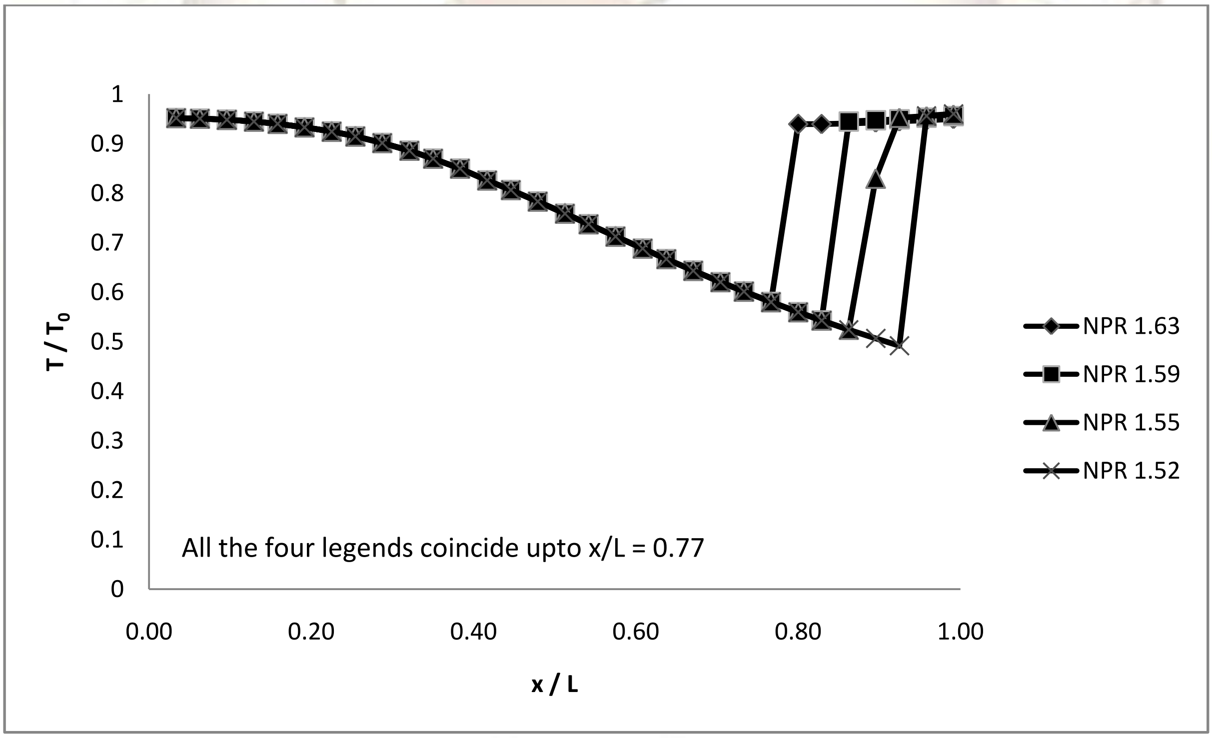


Fig.7 Comparison of temperature ratio along nozzle length for different NPR values

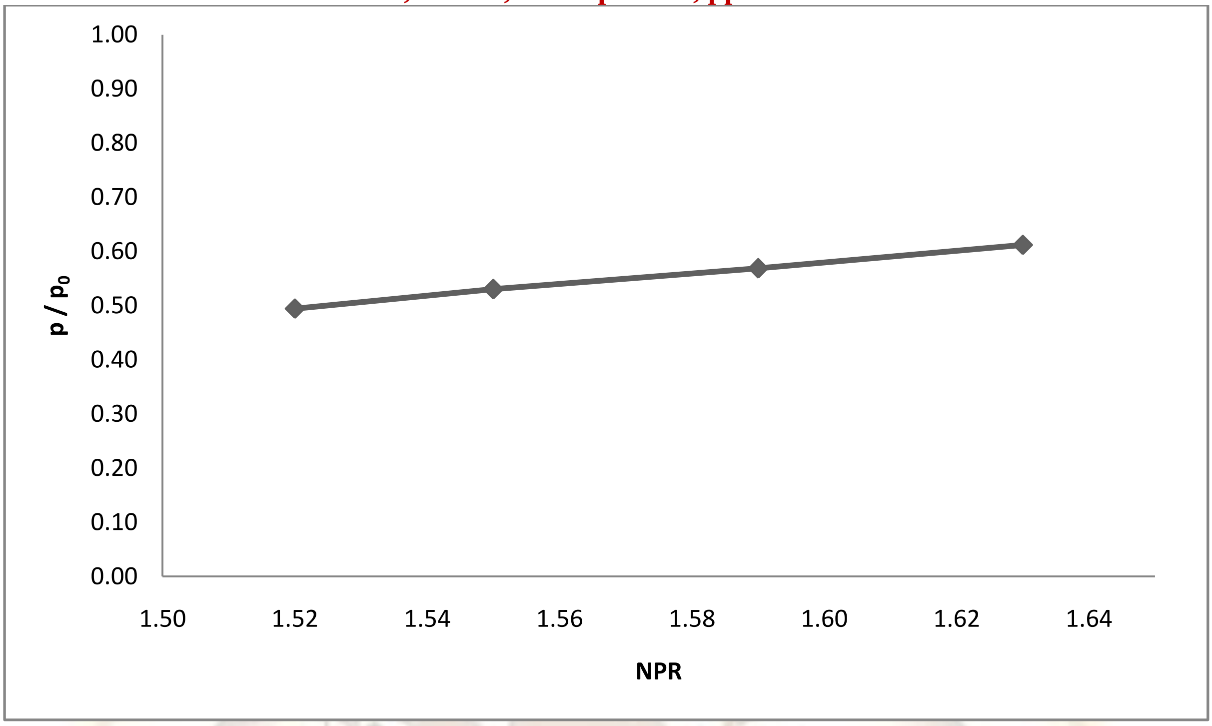


Fig.8 Exit pressure ratio for different NPR values

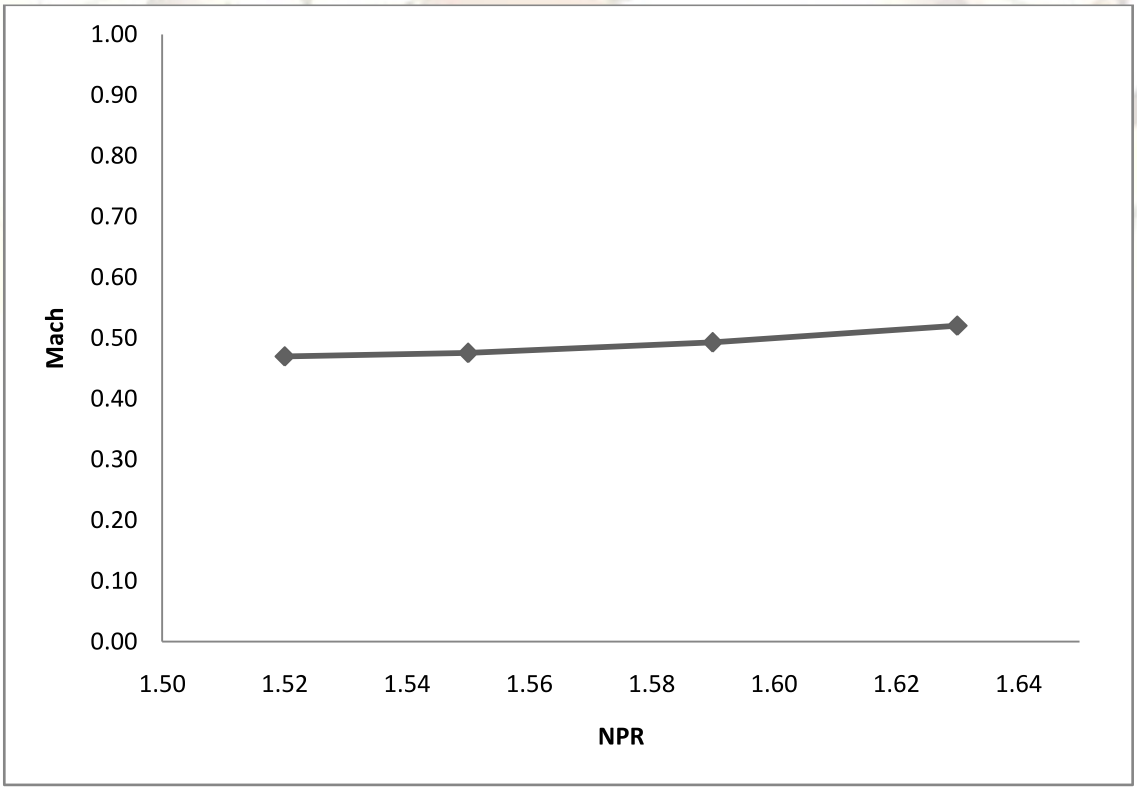


Fig.9 Exit Mach number for different NPR values

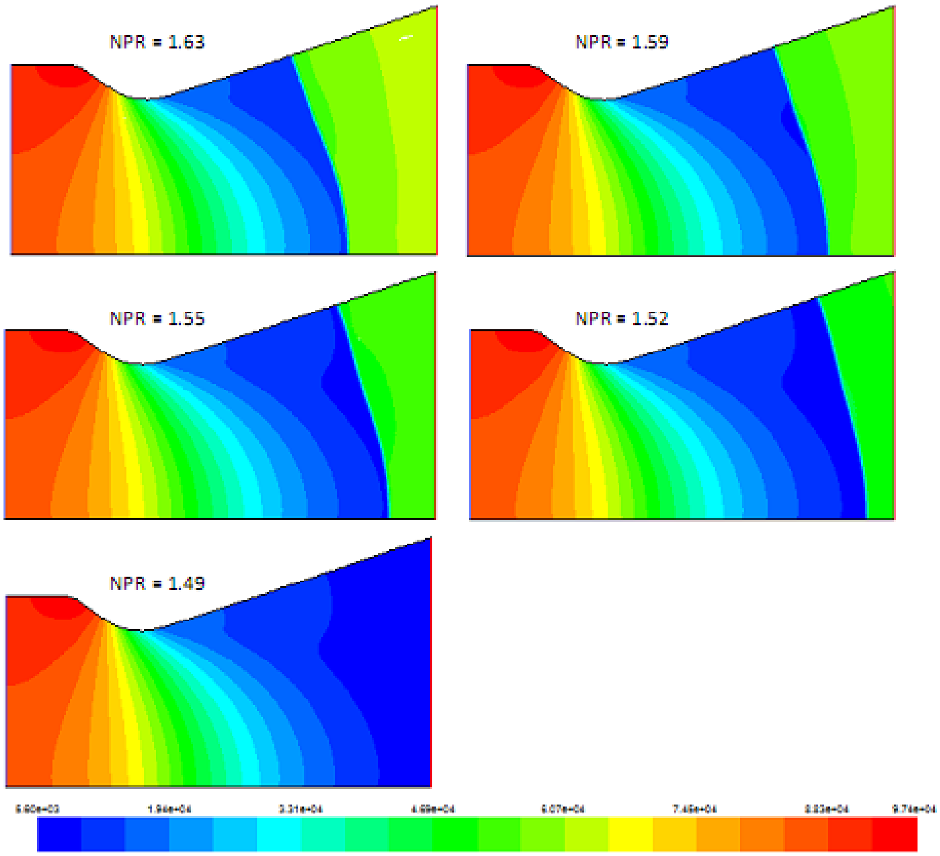


Fig.10 Static pressure contour for different NPR values

## Conclusion

Formation and movement towards the nozzle throat of the shockwave is computationally predicted. The results indicate increase in the static pressure, density and the static temperature across the shock. Mach number decrease across the shock. Deviation in the results from the analytical ones is due to the one dimensional approximation of the analytical approximation.

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