MHD Flow with Viscous Dissipation and Chemical Reaction over a Stretching Porous Plate in Porous Medium

P.K. Singh* **and Jai Singh****
Department of Mathematics University of Allahabad, Allahabad-211002

Abstract:
This paper investigates the MHD boundary layer flow with viscous dissipation and chemical reaction over a stretching porous plate in porous medium. The governing boundary layer equations and boundary conditions are simplified by using similarity transformations and the resulting equations are solved numerically. The effects of various flow parameters, in the form of dimensionless quantities, on the flow field are discussed and presented graphically.
Key words: Viscous dissipation, Chemical reaction, porous medium.

Introduction:

Two dimensional boundary layer flows due to stretching of any porous surface are important types of flows occurring in several engineering processes. Such processes are witnessed in the manufacturing of the materials using the method of extraction, in glass-fiber and paper production, cooling of metallic sheets etc.. In these processes, generally the final product of desired characteristic is obtained by controlling the rate of cooling in the process of stretching.

The viscous dissipative heat effects on the steady free convection and on combined free and forced convection flows have been extensively studied by Ostrach[1-5]. V.M.Soundalgekar [6] studied the viscous dissipation effect on unsteady free convection flow past an infinite, vertical porous plate with constant suction. Viscous dissipation effects on the unsteady free convection flow of an elastico-viscous fluid past an infinite vertical plate with constant suction have been studied by V.M.Soundalgekar and G.A.Desai[7].The problem of Dissipation effects on MHD nonlinear flow and heat transfer past a porous surface with prescribed heat flux have been studied by S.P. Anjali Devi and B. Ganga [8]. Abo-Eldahab and El Aziz [9] studied the effect of viscous dissipation and Joule heating on MHD free convection flow past a semi-infinite vertical flate plate in the presence of the combined effect of Hall in which he considered power- law variation of the wall temperature. The viscous and Joules dissipation and internal heat generation was taken into account in the energy equation. Combined effect of conduction and viscous dissipation on magneto hydrodynamics free convection flow along a vertical flat plate were discussed Abdullah et al.[10]. The effect of viscous and joules dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in porous medium was studied by Anjai Devi and B. Ganga [11]. Viscous dissipation effects on nonlinear MHD flow in a porous medium over a stretching porous surface have been studied by S.P. Anjali Devi and B.Ganga[12].An analysis of thermal boundary layer in an electrically conducing fluid over a linearly stretching sheet in the presence of a constant suction transverse magnetic field with suction or blowing at the sheet have been studied by Chaim[13]. The effect of the viscous dissipation term along with temperature dependence heat source/ sink on momentum, heat and mass transfer in a visco-elastic fluid flow over an accelerating surface were studied by Sonth et al.[14].

It is observed that the flow over a stretching plate has been studied under various physical conditions. It appear to us that perhaps no attempt has been made to analyse the simultaneous effects of viscous dissipation and chemical reaction effects on flow past a stretching porous surface in a porous medium under the influence of magnetic field, therefore, we have investigated this aspects in this paper.

Nomenclature

u – Velocity in X-direction

v- Velocity in Y-direction

a- dimensional constant

m = power law exponent

ν- Kinematic viscosity

c_p- Specific heat at constant presure

Sc- Schmidt number
Pr- Prandtl Number
ρ= density of porous medium
T_w - Temperature of the fluid near the palte
T_∞ - Temperature of the fluid away from the palte
C_w - Species concentartion near the palte
C_∞ - Species concentartion away from the palte
β – Stretching parameter
M - Magnetic field parameter

Mathematical Formulation:

We consider two dimensional, steady, MHD laminar boundary layer flow with heat and mass transfer of a viscous, incompressible and electrically conducting fluid past a stretching porous plate embedded in a porous medium. A uniform magnetic field of strength B_0 transverse to the plate is applied. x- axis is taken along the plate and y-axis is normal to it. Consider a plate, say ,a polymer sheet, emerging out of a slit at $x = 0, y = 0$ and subsequently being stretched, as in polymer extrusion process. Let us assume that the velocity at a point in the plate is proportional to the power of its distance from the slit and boundary layer approximations are applicable. Let u and v be velocity components in x and y directions respectively. The chemical reactions of first order with rate constant k_1 are taking place in the flow and it is assumed that the induced magnetic field and the heat generated due to chemical reaction are negligible .Under the Boussinesq approximation, the governing boundary layer flow equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{K}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x} + \mathrm{v}\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1 C \tag{4}$$

The boundary conditions are

$$\left.\begin{array}{l} u = a\,x^m,\ \mathrm{v} = \mathrm{v}_w(x),\quad T = T_w(x) = T_\infty + T_0\,x^n \\ C = C_w(x) = C_\infty + C_0 x^n \quad \text{at}\ \ y = 0 \\ u = 0, T = T_\infty, C = C_\infty\ \ at\ \ y \to \infty \end{array}\right\} \tag{5}$$

We introduce the following similarity transformations

$$\psi(x, y) = \left[\frac{2\nu x U(x)}{(1+m)}\right]^{\frac{1}{2}} f(\eta)$$

$$\eta = \left[\frac{(1+m)U(x)}{2\nu x}\right]^{\frac{1}{2}} y \tag{6}$$

$$v_w(x) = \lambda\sqrt{\frac{\nu a(1+m)}{2}}\,x^{(m-1)/2}$$

Where $\lambda > 0$ for suction at the plate and ψ is the stream function.
The velocity components are given by

$$u = \frac{\partial \psi}{\partial y}, \mathrm{v} = -\frac{\partial \psi}{\partial x} \qquad (7)$$

It can be easily verified that the continuity equation (1) is identically satisfied and introducing non-dimensional form of temperature and concentration as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty} \qquad (8)$$

Now equations (2) to (4) becomes

$$f''' + ff'' - \left(\delta^{-1} + M^2\right) f' - \beta f'^2 = 0 \qquad (9)$$

$$\theta'' + \Pr f\theta' - 2\beta \Pr f'\theta + Ec \Pr f''^2 = 0 \qquad (10)$$

$$\phi'' + Scf\phi' - Sc\, m\phi f' - \gamma Sc\phi = 0 \qquad (11)$$

With boundary conditions

$$\left.\begin{array}{lll} f(0) = \lambda, & f'(0) = 1 & f'(\infty) = 0 \\ \theta(0) = 1, & \theta(\infty) = 0 & \\ \phi(0) = 1, & \phi(\infty) = 0 & \end{array}\right\} \qquad (12)$$

where

$\beta = \dfrac{2m}{m+1}$ (Stretching parameter)

$\Pr = \dfrac{\mu C_p}{\nu}$ (Prandtl number)

$Ec = \dfrac{a^2}{C_p T_0}$ (Eckert number)

$Sc = \dfrac{\nu}{D}$ (Schmidt number)

$\delta = \dfrac{K\,a}{\nu}$ (Permeability parameter)

$\gamma = \dfrac{x^2 k_1}{\nu}$ (Chemical reaction parameter)

$M^2 = \nu \dfrac{2\sigma B_0^2}{\rho a\left(1+m\right)}$ (Magnetic parameter)

We solve equation (9),(10) and (11) with boundary conditions (12) using BVP4C method. Here, while solving the equations, have taken $m = 1$, which corresponds to the plate velocity varying linearly with the distance.

Results and Discussion:

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphs.

Fig.1. Represents the dimensionless temperature profiles for different values of Prandtl number with constant chemical reaction and permeability parameter. It is clear that the temperature of the fluid decrease as Prandtl number increases.

Fig.2. depicts the dimensionless temperature profiles for different values of Eckert number with constant chemical reaction and permeability parameter. It is observed that Eckert number has no significant effect on the temperature of the fluid.

The dimensionless velocity profiles for different values of suction parameter λ with constant chemical reaction and uniform magnetic field are presented in Fig.3. It is observed that the fluid velocity decreases as suction parameter increases

Fig.4 demonstrates the dimensionless concentration distribution for different values of Schmidt number with constant chemical reaction and uniform magnetic field. It is seen that the concentration profile decreases as we increase the Schmidt number.

The dimensionless concentration profiles for different values of chemical reaction parameter with uniform magnetic field and constant permeability parameter are depicted in Fig.5. It is observed that the concentration of the fluid decreases with increase in the chemical reaction parameter.

The dimensionless velocity profiles for different values of permeability parameter with uniform magnetic field and constant suction are depicted in Fig. 6. It is observed that the velocity of the fluid decreases as permeability parameter increases.

Fig.7 demonstrates that the dimensionless velocity profiles for different values of magnetic parameter with constant chemical reaction and permeability parameter. It is seen that, velocity decreases with increase in magnetic parameter.

Fig.8 demonstrates the dimensionless temperature profiles for different values of magnetic parameter with constant chemical reaction and permeability parameter. It is observed that temperature increases with increase in the magnetic parameter.

Concentration of the fluid decreases with increases of magnetic parameter and this is observed from Fig.9.

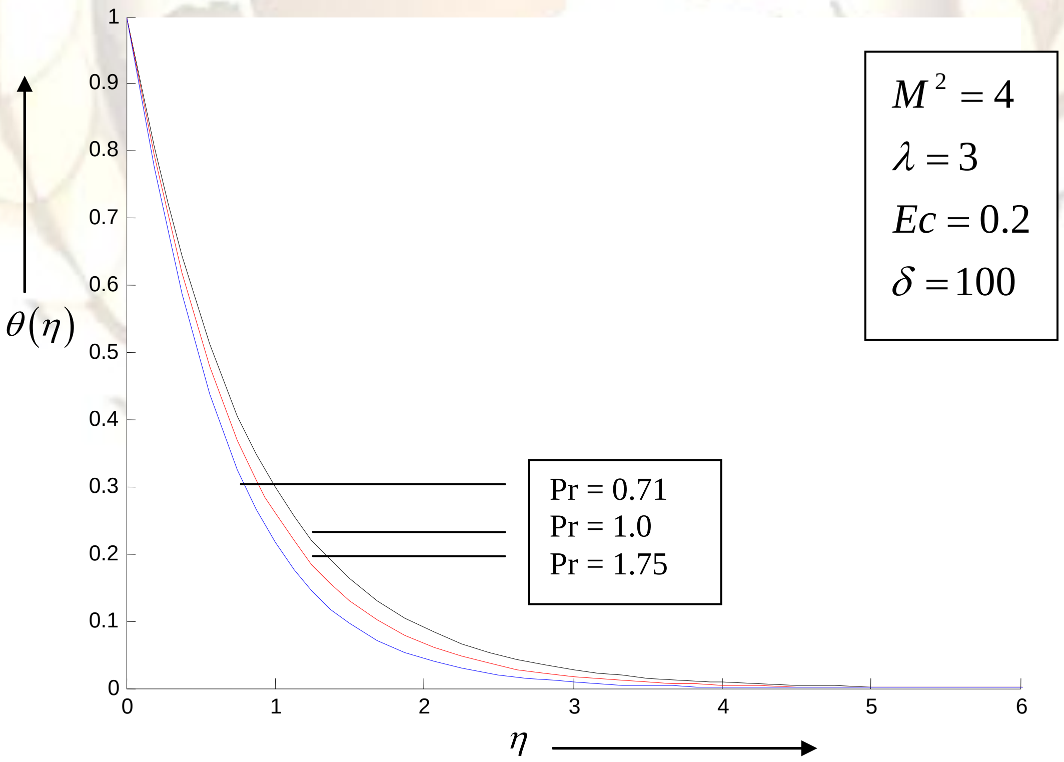

Fig.1. Effect of Pr over temperature distribution.

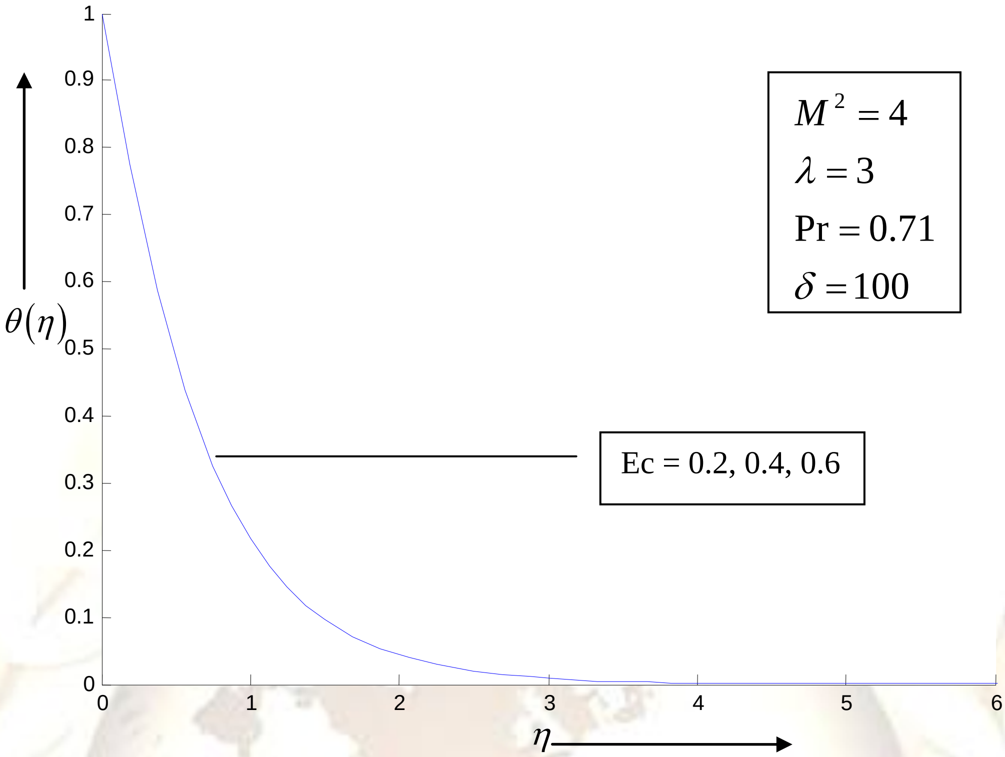

Fig.2 Effect of Ec over temperature distribution.

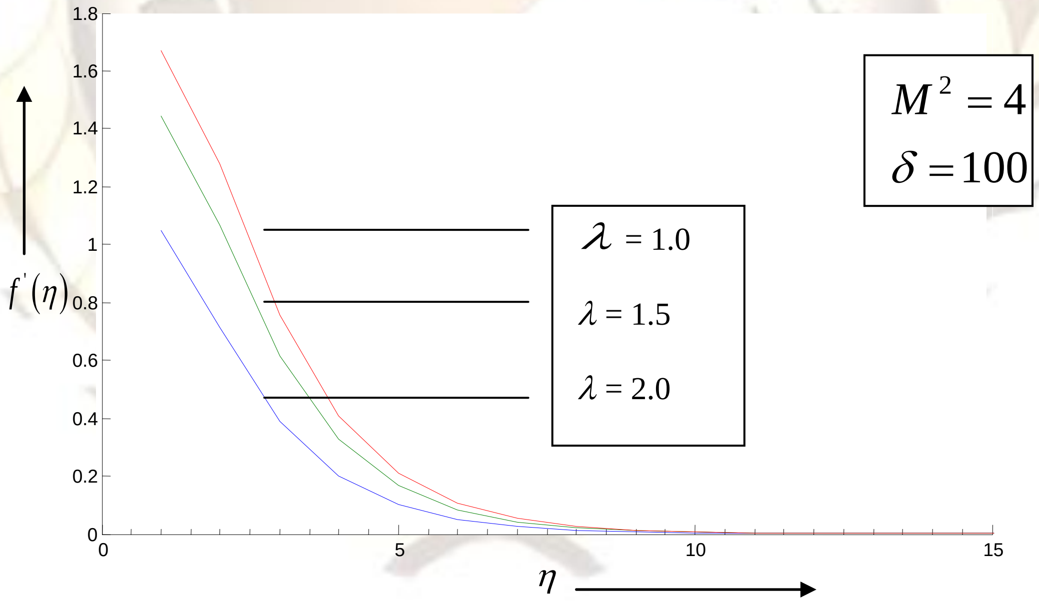

Fig.3 Non-dimensional velocity profiles for different values of λ .

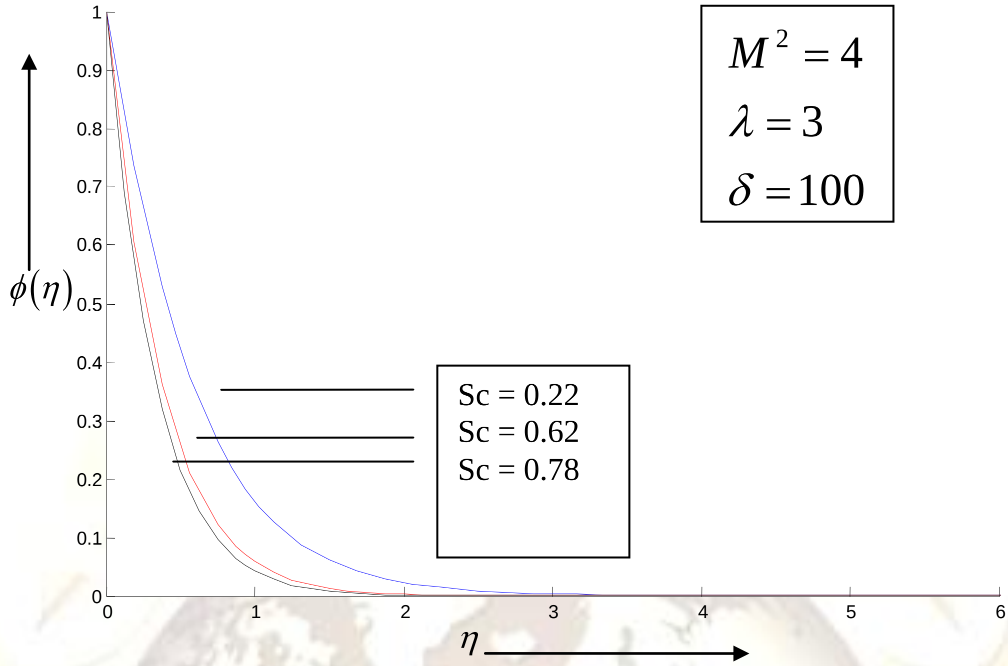

Fig.4 Dimensionless concentration profiles for different values of Sc.

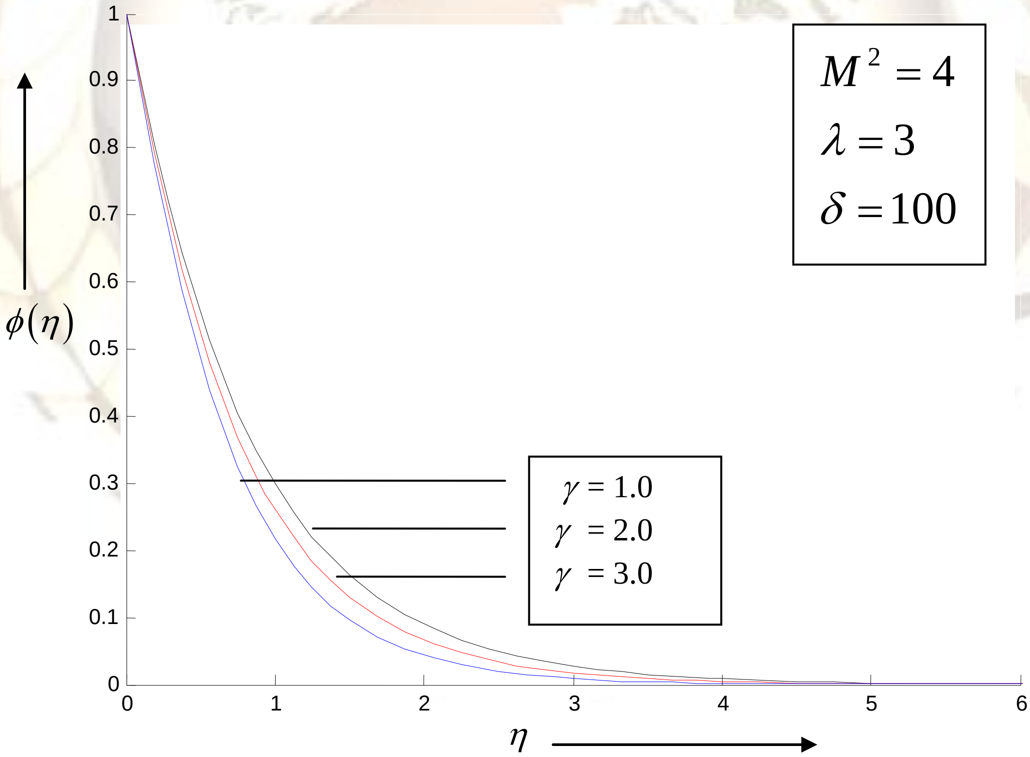

Fig.5 Chemical reaction effect over concentration profiles.

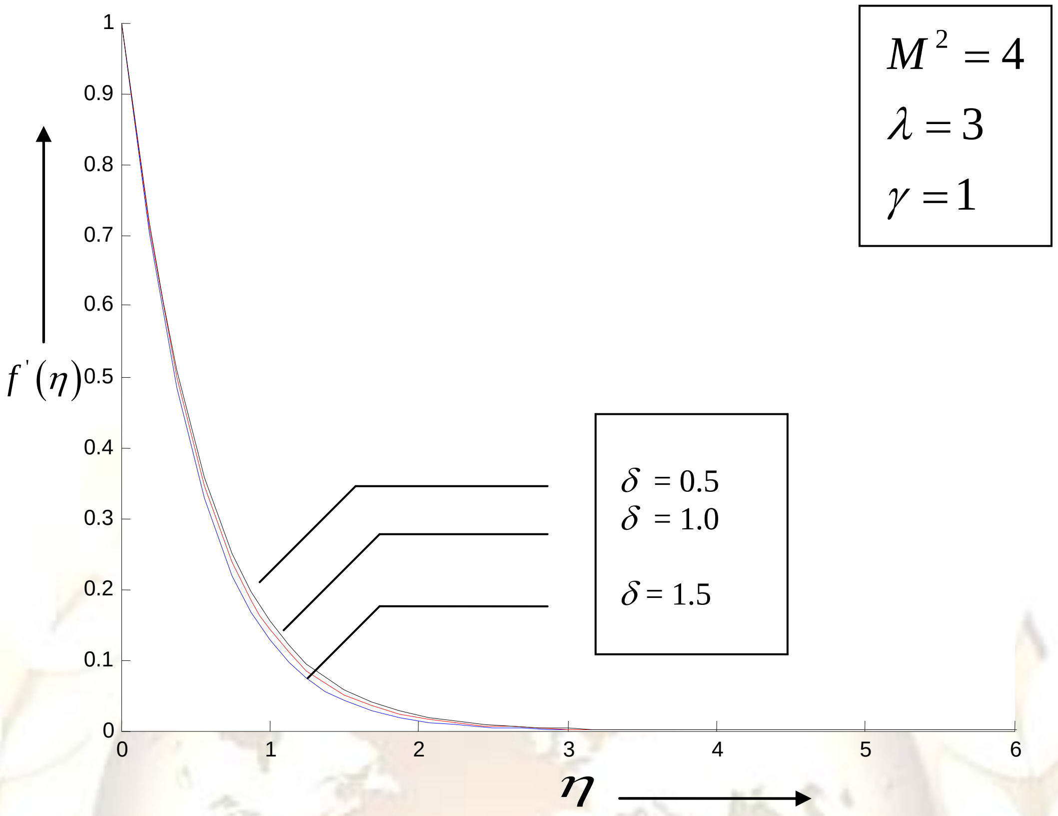

Fig.6 Velocity profiles for different values of permeability parameter.

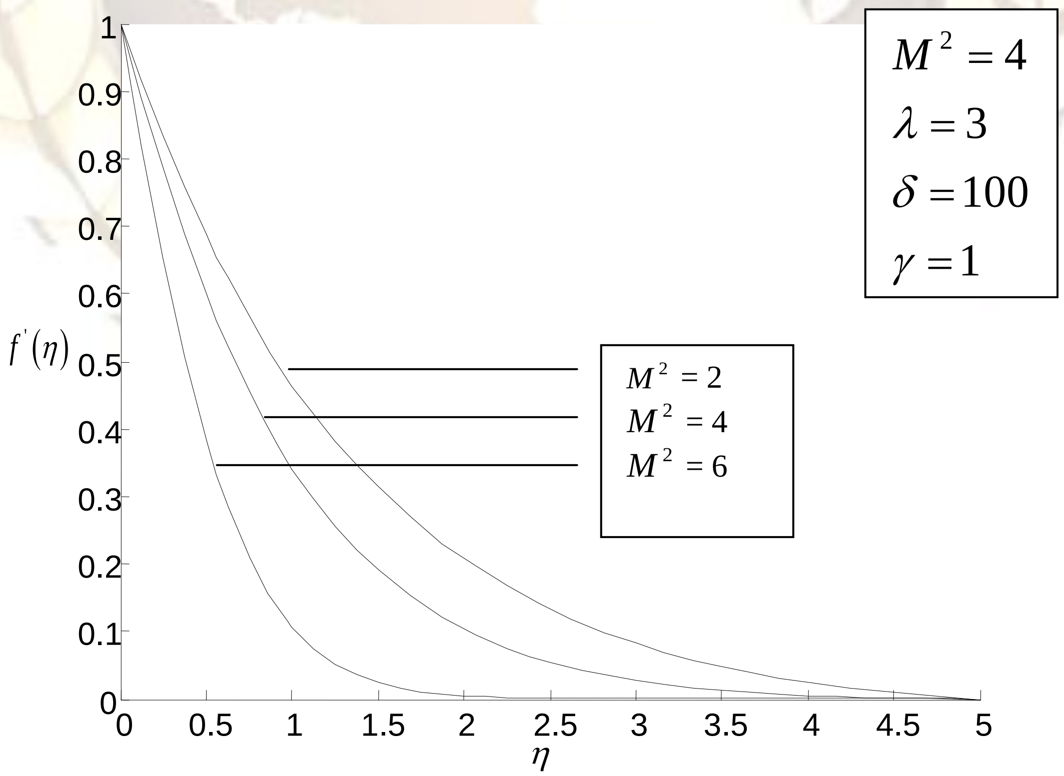

Fig.7. Non- dimensional velocity profiles for different values of M^2

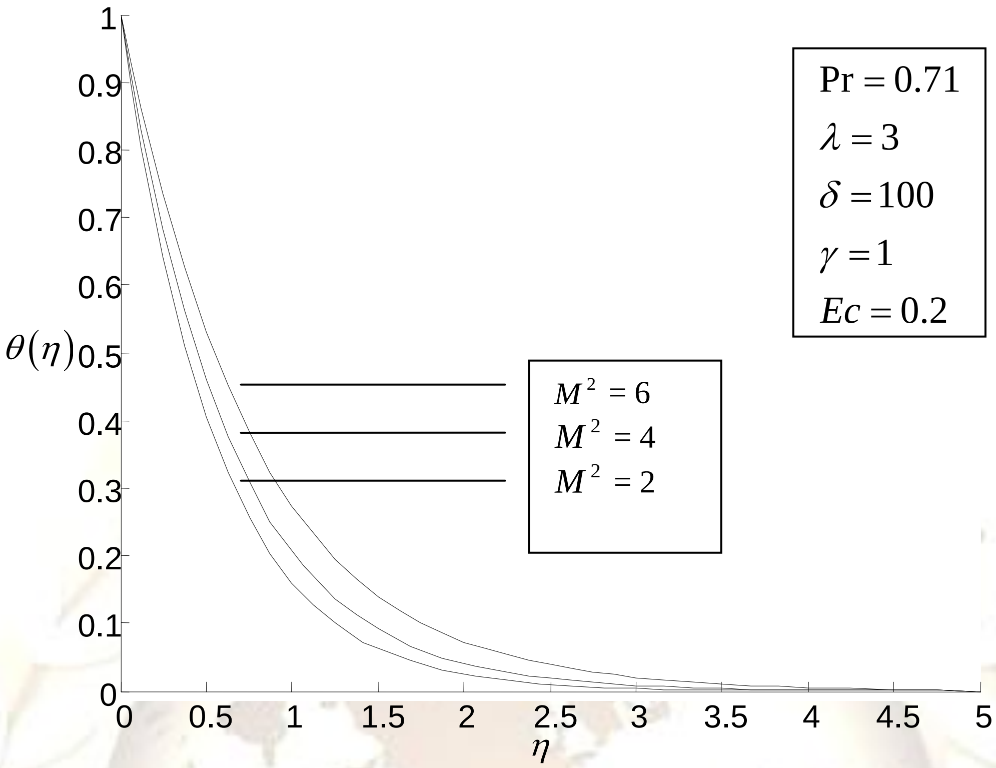

Fig. 8. Temperature distribution for various values of M^2.

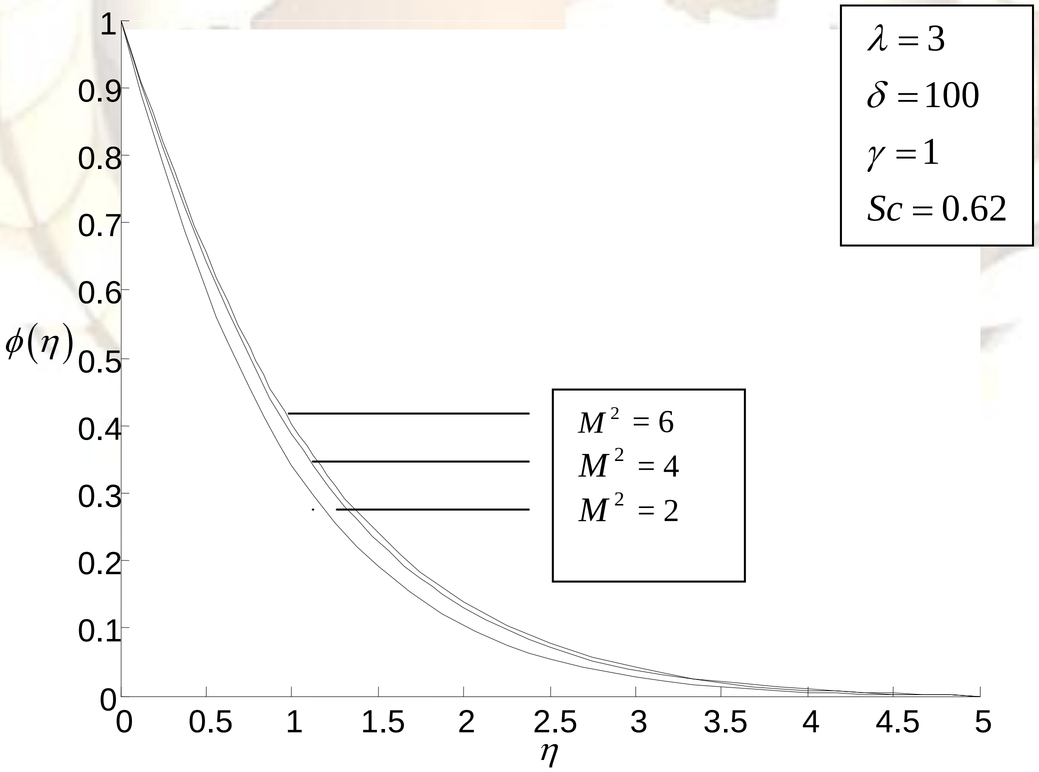

Fig.9. Dimensionless concentration distribution for different values of M^2

Conclusions:

- It is observed that increase in Prandtl number is to decrease the temperature.
- The effect of Eckert number over the temperature is insignificant.
- An increase in Schmidt number results in lowering the concentration distribution.
- Concentration decreases with increase in chemical reaction parameter.
- Velocity of the fluid increases when permeability decreases.
- The effect of magnetic parameter is to decrease the velocity whereas it has increasing effect on both temperature and concentration.

References:

1. S.Ostrach, New aspects of natural convection heat transfer, *Trans. Am .Soc. Mech .Engrs* 75, 1953, 1287-1290.
2. S. Ostrach, and Albers L.U., On pairs of solution of a class of internal viscous flow problems with body forces , NACATN 4273.
3. S. Ostrach, Unstable convection in vertical channels with heating from below, including effects of heat source and frictional heating, NACA TN 3458, 1955
4. S. Ostrach, Laminar natural convection flow and heat transfer of fluid with and without heat source in channels with constant wall temperatures, NACA TN 2863, 1952.
5. S. Ostrach, Combined natural and forced convection laminar flow and heat transfer of fluid with and without heat source in channels with linearly varying wall temperature, NACA,TN 3441, 1954.
6. V.M. Soundalgekar. , Viscous dissipation effect on unsteady free convection flow past an infinite, vertical porous plate with variable suction, *Int J. Heat and Mass Transfer*, 17,1974,85-92.
7. V.M. Soundalgekar and G.A Desai., Viscous dissipation effects on the unsteady free convective flow of an elastico-viscous fluid past an infinite vertical plate with constant suction, *Indian J. pure Appl. Math.*, 10 (11), 1979, 1397-1404.
8. S.P. Anajali Devi and B.Ganga, Dissipation effect on MHD flow and heat transfer past a porous surface with prescribed heat flux, *Journal of Applied Fluid Mechanics*, 3, 2010,1-6.
9. E.M. Abo-Eldahab ,El Aziz M.A.El Aziz, Viscous dissipation and Joules heating effects on MHD-free convection from a vertical plate with power –law variation in surface temperature in the presence of Hall and iso-slips currents, *Appl. Model*., 29 , 2005, 579-595.
10. Abdullah-Al-Mamun, Nur Hosain Md. Ariful Azim and Md. Abdul Maleque, Combined effect of conduction and viscous dissipation on Magneto hydrodynamics free convection flow along a vertical flat plate, *J.Naval Architecture and Mrine Engg.*, 4 , 2007, 87-98.
11. S.P. Anajali Devi and B. Ganga, Effects of Viscous and Joules dissipation on MHD flow,haet and mass transfer past a stretching porous surface embedded in a porous medium., *Nonlinear Analysis: Modelling and Control* , 14(3), 2009, 303-314.
12. Anajali Devi S.P. and Ganga B., Viscous dissipation effects on nonlinear MHD flow in a porous medium over a stretching porous surface, *Int.J.of Appl. Math. and Mech.*, 5(7), 2009, 45-59.
13. Chaim T.C., Magnetohydrodynamics heat transfer over a non-isothermal stretching sheet, *Acta Mech*.,122, 1977, 169-179.
14. R. M. Sonth, S. K. Khan, M.S. Abel, K.V.Prasad, Heat and mass transfer in a visco-elastic fluid over an accelerating surface with heat source /sink and viscous dissipation, *Heat Mass Transfer*, 38, 2002, 213-220.