

SYSTEM IDENTIFICATION USING AN AFFINE COMBINATION OF TWO LMS ADAPTIVE FILTERS

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Abstract:

The design of many adaptive filters constrained with the tradeoff between convergence speed and steady-state mean-square error (MSE), i.e. Fast converging filter produce a large steady-state mean-square deviation (MSD) and slow converging filter produce the small mean square deviation. This limitation is usually independent of the type of adaptive algorithm, i.e., least mean-square (LMS), normalized least mean-square (NLMS), recursive least squares (RLS), or affine projection (AP). In this paper we proposed a new Affine combining algorithm which works with two fixed step size LMS algorithms and the overall response is sum of two filter outputs weighted-multipliers with scalars 'λ(n)' and '(1-λ(n))' which is shown in figure 1. The overall filter response is a function of scalar λ(n) & by estimating the optimized scalar mixing parameter λ_o(n) at every iteration 'n' such that it produce a small MSE. In order to meet the above mentioned requirement (i.e. producing small MSE) we proposed two schemes namely

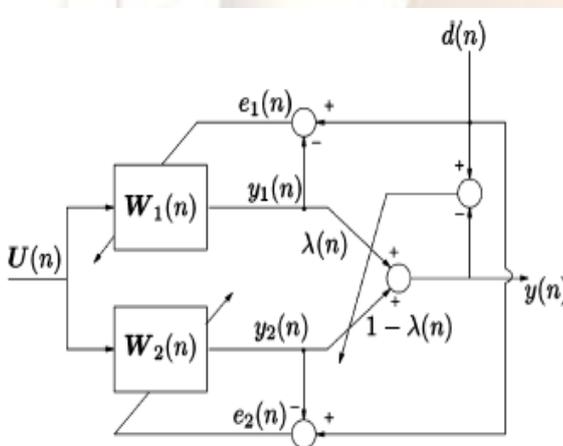


Fig 1: Affine combination of a two LMS Adaptive filters with mixing parameter λ(n)

Stochastic Gradient Approach and Error power based Schemes to update the λ(n) at every iteration. We can prove that this affine combination algorithm can effectively track the fluctuations in the statistics of non-stationary filter input. The attractive feature of this scheme is it can produce a very small MSE without sacrificing convergence speed. We can use this affine combination algorithm for the evaluation of system identification application. Finally we compare the performance of affine algorithm with standard LMS, Time varying LMS, and RLS algorithms.

1. Introduction

A Combination algorithm is proposed, which uses a convex combination of two fixed step-size adaptive filters, where adaptive filter w1 (n) uses a larger step size than adaptive

filter w2 (n). The key to this scheme is the selection of the scalar mixing parameter λ(n) for combining the two filter outputs. The convex combination performed as well as the best of its Components in the MSE sense. These results indicate that a combination of adaptive filters can lead to fast convergence rates and good steady-state performance, an attribute that is usually obtained only in variable step-size algorithms.

The achievable performance is studied for an affine combination of two LMS adaptive filters using the structure shown in Fig.1 with stationary signals. Here, the combination parameter λ(n) is not restricted to the range (0,1). Each adaptive filter is estimating the unknown channel impulse response using the same input data. Thus, w1(n) and w2(n) are statistically dependent estimates of the unknown channel. There exists a single combining parameter sequence λ(n) which minimizes the MSD. The parameter λ(n) does not necessarily lie within (0,1) for all n. Thus, the output y(n) in Fig.1 is an affine combination of the individual outputs y1(n) and y2(n).

The adaptive scheme is first studied from the view point of an optimal affine combiner. The value of λ(n) that minimizes the MSE for each n conditioned on the filter parameter at iteration 'n' is determined as a function of the unknown system response. This leads to an optimal affine sequence λ_o(n). The statistical properties of an optimal affine combiner are then studied. It is shown that λ_o(n) can be outside of the interval (0,1) for several iterations. Most importantly, λ_o(n) is usually negative in steady-state.

Finally, two realizable schemes for updating λ(n) are proposed. The first scheme is based on a stochastic gradient approximation to λ_o(n). The second scheme is based on the relative values of averaged estimates of the individual error powers. Both schemes are briefly studied, and their performances are compared to that of the optimal affine combiner.

1.1 Optimal Affine Combiner :

The system under investigation is shown in Fig1. Each filter uses the LMS adaptation rule but with different step sizes μ₁ and μ₂

$$w_i(n+1) = w_i(n) + \mu_i e_i(n) u(n) \quad (1)$$

$$e_i(n) = d(n) - w_i^T(n)u(n) : i=1, 2 \quad (2)$$

$$d(n) = e_0(n) w_0^T(n)u(n) \quad (3)$$

where $W_i(n)$, for $i=1, 2$ are the N-dimensional adaptive coefficient vectors, is assumed zero-mean, and statistically independent of any other signal in the system, and the input process is assumed wide-sense stationary. $U(n)$ is the input vector. It will be assumed, without loss, that $\mu_1 > \mu_2$, so that will, in general, $W_1(n)$ converges faster than $W_2(n)$. Also, $W_2(n)$ will converge to the lowest individual steady-state weight miss adjustment. The weight vectors $W_1(n)$ and $W_2(n)$ are coupled both deterministically and statistically through $U(n)$ and $e_0(n)$.

The outputs of the two filters are combined as $y(n) = \lambda(n)y_1(n) + [1 - \lambda(n)] y_2(n)$

Where $y_i(n) = w_i^T(n) U(n)$ for $i=1, 2$ and overall system error is

$$e(n) = d(n) - y(n) \quad (5)$$

Equation (5) can be re-written as

$$y(n) = \lambda(n)W_1^T(n)U(n) + [1 - \lambda(n)]W_2^T(n)U(n) \quad (6)$$

where $W_{12}(n) = W_1(n) - W_2(n)$ Equation 6 shows that $y(n)$ can be interpreted as a combination of $W_2(n)$ and a weighted version of the difference filter $W_{12}(n)$. It also shows that the combined adaptive filter has an equivalent weight vector given by

$$w_{eq}(n) = \lambda(n)w_{12}(n) + w_2(n) \quad (7)$$

Subtracting (1) for $i=2$ from (1) for yields a recursion for

$$\lambda_0(n) = \frac{[w_{12}^T(n)R_u w_{12}(n)]}{[w_{12}^T(n)R_u w_{12}(n)]} \quad (8)$$

Which is the expression for the optimum affine combiner, as a function of unknown system response.

2. Iterative Algorithms To Adjust Affine Mixing Combiner $\lambda(n)$:

2.1 Stochastic Gradient Approach:

Consider a stochastic gradient search to estimate the optimum instantaneous value of $\lambda(n)$. The stochastic gradient algorithm to update $\lambda(n)$ is

$$\lambda(n+1) = \lambda(n) + \mu_\lambda [d(n) - w_{12}^T(n)U(n)] \quad (9)$$

$$\text{where } w_{12}(n) = \lambda(n)w_1(n) + [1 - \lambda(n)]w_2(n) \quad (10)$$

Equation (9) is a linear first order stochastic time-varying recursion in the scalar parameter $\lambda(n)$. The stochastic behavior of this recursion has been analyzed .The

accuracy of the theoretical analysis and the performance of the proposed algorithm for adjusting $\lambda(n)$ are evaluated here. Appropriate values of μ_λ were chosen so that the algorithm was able to track the adaptation of $W_1(n)$ and $W_2(n)$.

Sufficiently small values of μ_λ were found so that stochastic approach was stable. However, these values were not large enough to track the adaptation of $W_1(n)$ and $W_2(n)$

Fig. 2 shows the behavior of the optimum behavior of $\lambda(n)$. It can be easily verified that the performance of the stochastic gradient algorithm for $\lambda(n)$ is very close to that of the optimum combiner .A good-to-excellent agreement between the theory and the simulations can be verified, especially convergence time and the steady-state behavior. Again, the fluctuations in the initial transient phase indicate the action of the stability control.

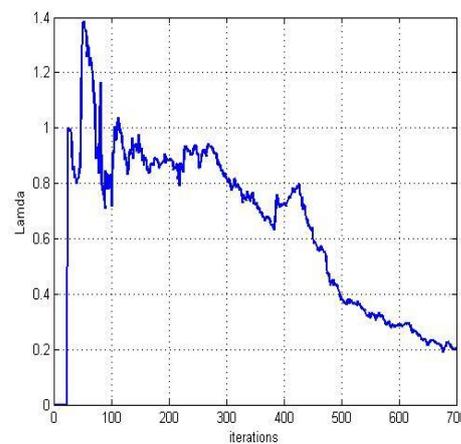


Fig: 2 values of $\lambda(n)$ in stochastic gradient scheme over the iterations.

2.2 Error Power Based Scheme :

A function of time averaged error powers could be a good candidate for an estimator of the optimum $\lambda(n)$ for each n . The individual adaptive error powers are good indicators of the contribution of each adaptive output to the quality of the present estimation of $d(n)$. These errors are readily available and does not need an estimate of the additive noise power.

Consider a uniform sliding time average of the instantaneous Errors

$$e_1^2(n) = 1/k \sum_{m=n-k+1}^n e_1^2(m) \quad (14)$$

$$e_2^2(n) = 1/k \sum_{m=n-k+1}^n e_2^2(m) \quad (15)$$

where 'k' is the averaging window length. Then, consider the instantaneous value of $\lambda(n)$ is defined as

$$\lambda(n) = 1 - k \left\{ \text{erf} \left(\frac{e_1^2(n)}{e_2^2(n)} \right) \right\} \quad (16)$$

$$\text{where } \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2/2} dt$$

Note that a stochastic analysis of the transient behavior is quite complicated for this algorithm because of the erf nonlinearity. Nevertheless, the theoretical analysis of

the optimum case provided useful insights for the design and evaluation of the algorithm.

performance compared to 0.0369 which is produced by standard LMS with the step size $\mu_1=0.05$.

3. System Identification Simulation Setup:

3.1 System Identification process-Description:

In the class of applications dealing with identification of system impulse response, an adaptive filter is used to provide a linear model that represents the best fit to an unknown plant

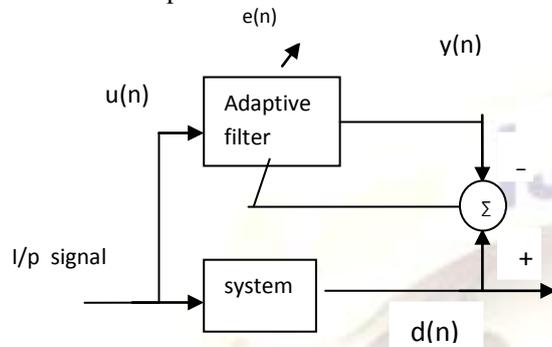


Fig.3 System Identification-Block diagram

Here, same input is given to both the adaptive filter and the plant. The output of the plant will serve as the desired signal for the adaptation process.

Simulation setup:

As discussed in 5.2, flow chart for the simulation, the input noisy signal and the desired signal and the filter parameters are same for all the four simulation procedures and they are characterized as follows,

(i) Input signal to the filter:

- Amplitude: 1 V
- Signal type: Normal distribution(optional)
- Variance: 1.060
- Samples: 1000
- Initial Phase: 0

(ii) Desired response (Actual signal with added noise):

- Noise Amplitude: 0.15 V
- Noise Type: Normal distribution

(iii) System to be identified:

- Number of samples: 25
- Adaptive filter Type: FIR
- Order: 25
- No. Of Iterations: 100-1000
- Step sizes: $\mu_1=0.05$, $\mu_2=0.01$

4. Results:

Using the above mentioned simulation setup we obtained the simulated result of identifying the system response shown in fig 4. The table 1 indicates the MSE values after the various no of iterations for various algorithms. The above two simulation graphs will shows the accuracy of system identification application using error power based scheme for 700 iterations. The system proposed to be identified has the 25 coefficients. This scheme is producing the Mean Square Error of 0.0236 which is better

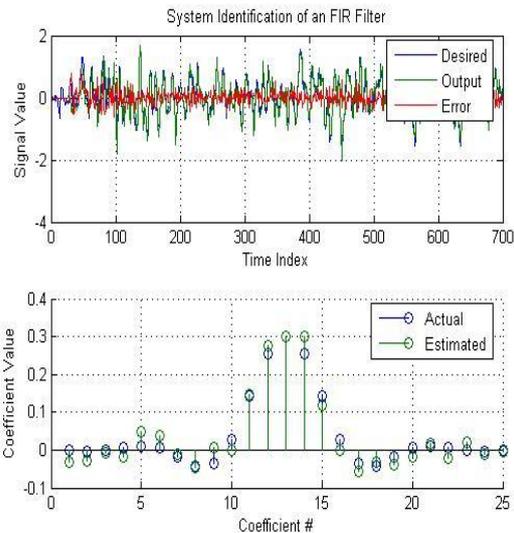


Fig4: System Identification using an Affine LMS of Error Power Based Scheme

No. of iterations	Affine Stochastic	LMS1	LMS2	Affine Error power scheme	TV-LMS	RLS
100	0.0455	0.0652	0.1283	0.0728	0.0639	0.0046
200	0.0413	0.0538	0.0860	0.0530	0.0479	0.0037
300	0.0280	0.0428	0.0602	0.0386	0.0372	0.0026
400	0.0246	0.0390	0.0490	0.0330	0.0332	0.0025
500	0.0213	0.0399	0.0420	0.0272	0.0305	0.0021
600	0.0211	0.0490	0.0368	0.0259	0.0323	0.0024
700	0.0187	0.0369	0.0326	0.0236	0.0281	0.0022

Table1: List of MSE's for (a) standard LMS $\mu_1=0.05$, $\mu_2=0.01$ (b) Time varying LMS of $c=2$, $\mu=0.02$ (c) RLS of $\lambda=0.95$ (d)Affine stochastic of $\mu_1=0.05$, $\mu_2=0.01$ (e) Affine Error power scheme of $\mu_1=0.05$, $\mu_2=0.01$ for system identification application(filter order=25)

Fig 5. shows that affine stochastic scheme is very close to the performance of RLS the filter coefficients, by applying the Combination of two LMS adaptivefilter-algorithm instead fo conventional LMS, DCT-LMS,TV-LMS,RLS. The attractive feature of affine scheme over the RLS is, the computational complexity is just in the order of $O(m)$,where

m is the filter order ,but in RLS it is in the order of $O(m^2)$. The Matrix singularity conditions also sometimes limiting the usage of RLS in practical applications.

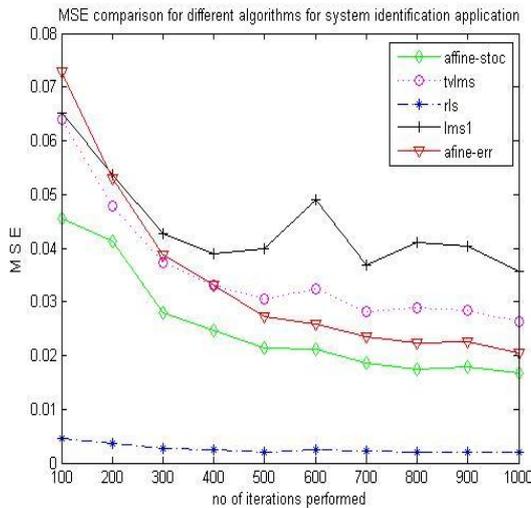


Fig 5:

Learning curve comparison in Affine stochastic LMS ,Time varying LMS,RLS, Standard LMS, Affine Error power based Schemes

5. Conclusion & Future Scope :

We observed that the Affine combination of two LMS adaptive filters is performing very close to the RLS and so much better than standard LMS while maintaining both the good rate of convergence and small MSE.

The sub band filtering has very good significance in the speech processing application, where we have to improve the fidelity of the speech signal in the lower sub bands of the speech which is to be processed. One more possible usage of this affine combination is while

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Author Biographies



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