Synchronization of Uncertain Doffing-Holmes Chaotic System Via Fractional Fuzzy Sliding Mode Control

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Abstract. This paper presents a fractional-order fuzzy sliding mode control (FOFSMC) scheme for synchronization of an uncertain chaotic system. A fractional-order fuzzy sliding mode controller (FOFSMC) can drive the error state trajectory of the master-slave system to converge to zero in finite time. The feasibility and effectiveness of the FOFSMC scheme are demonstrated via a numerical simulation. The numerical results demonstrate the ability of the FOFSMC scheme to synchronize the chaotic Daffing-Holmes systems. As a result, compared with conventional switching controllers, the proposed scheme has a lower implementation cost and can provide a reasonable tracking performance.

Introduction

The problem of synchronization is design a system (slave system), which follows the behavior of another one (master system). At first Pecora and Carroll demonstrated the synchronization of two identical chaotic systems under different initial conditions [1].

Nowadays, chaos and its synchronization has been found to be more attractive due to its potential applications in many fields of engineering and science such as in secure communications and signal processing [2], chemical reactions, power converters, biological systems, and information processing, etc.[3-5]. There is a close relationship between synchronization and control. From the viewpoint of control, the chaos synchronization problem entails controlling the dynamic behavior of a ‘‘slave’’ system by means of a control input, such that its oscillation, following a period of transition, mimics that of the ‘‘master’’ system. Synchronization of chaotic systems is a hard task because of their nonlinear behavior and sensitivity to the initial values.

The stability analysis of such a system could be mentioned as one of most important open areas of research in this field. Implementing the analysis methods like Lyapunov are difficult due to chaotic behavior of chaotic systems and its complex mathematical expression.Chaos has been developed and thoroughly studied over the past two decades. Over the last decades, Many methods and techniques have been developed, such as OGY method [6], feedback approach [7-8], adaptive method [9], time-delay feedback approach [10], and backstepping design technique [11], etc. However, most of the methods mentioned above synchronize two identical chaotic systems. In fact, in systems such as laser array, biological systems to cognitive processes, it is hardly the case that every component can be assumed to be identical. Consequently, in these years, more and more applications of chaos synchronization in secure communications make it much more important to synchronize two different chaotic systems [12-20]. Recently, the concept of integrating fuzzy logic control and sliding-mode control has become a popular research subject [21-24]. Fuzzy sliding-mode control can eliminate the chattering of sliding-mode control, and possess robustness in the presence of model uncertainty and disturbances. To suppress the chattering several techniques have been investigated in the literatures. In this paper, we design a fractional-order fuzzy sliding mode control (FOFSMC) scheme for synchronization of an uncertain chaotic system.

Basic definition and fundamentals of fractional calculus

The fractional-order differentiator can be denoted by a general fundamental operator ${}_aD_t^{\alpha}$ as a generalization of the differential and integral operators, which is defined as follows [25]

$$
{}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & , R(\alpha) > 0 \\ 1 & , R(\alpha) = 0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & , R(\alpha) < 0 \end{cases} \tag{1}
$$

Where α is the fractional order which can be a complex number, constant a is related to initial conditions. Three most commonly used definitions in Fractional calculus are Riemann-Liouville, Grunwald-Letnikov, and Caputo definitions The Riemann-Liouville definition [26] is given by:

$$ {}_{a}^{RL}D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \tag{2} $$

Where m is the integer satisfying $m-1<\alpha<m$. Let $D_t^{(\alpha)} f(t) = \frac{d^{\alpha} f(t)}{dt^{\alpha}}$ denotes the Riemann-Liouville fractional derivative of $f(t)$ of order α .

Fuzzy controller

Fuzzy Control (FC) has supplanted conventional technologies in many applications. FC is an alternative way to deal with the unknown process. However, the huge number of fuzzy rules for high-order systems makes the analysis complex. Therefore, much attention has been focused on the FSMC.

Fuzzy sliding mode control. In this section, in order to eliminate the chattering problem fuzzy sliding mode control methodology is proposed. The main advantage of this method is that the robust behavior of the system is guaranteed. The second advantage of the proposed scheme is that the performance of the system in the sense of removing chattering is improved in comparison with the same SMC technique without using FLC. A fuzzy sliding surface will be introduced to develop the control law equation (12). The corresponding fuzzy rule table is presented in Table 1.

Table 1. FSMC rule table.

S	NB	NM	NS	ZE	PS	PM	PB
K	PB	PM	PS	ZE	NS	NM	NB

Where S is the input variable of the fuzzy system and K is the output variable of the fuzzy system, and NB, NM, NS, ZE, PS, PM, PB are the linguistic terms of antecedent fuzzy set. They mean Negative Big, Negative Medium, Negative Small, Zero, Positive Medium Positive Small and Positive Big, respectively. The width of boundary layer Φ_k , influences the chattering magnitude of the control signal, whilst the gain K_k , will influence speed of synchronization. GA is used to search for a best fit for these parameters in (12). The synchronization error and the chattering of the controlled response are chosen as a performance index to select the parameters.

Performance criteria

For evaluating the performance of the controller, we can use the performance criteria as follows:

(a) Integral of the absolute value of the error (IAE)

$$ IAE = \int_0^{t_f} |e(t)| dt \tag{3} $$

(b) Integral of the square value (ISV) of the control input

$$ ISV = \int_0^{t_f} u^2(t) dt. \tag{4} $$

Both IAE is used as objective numerical measures of tracking performance for an entire error curve, where t_f represents the total running time. The criterion IAE will give an intermediate result. The criterion ISV shows the consumption of energy.

System description

In [27] the nominal dynamic equation of Duffing-Holmes chaotic system are described as:

$$ \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1 - \alpha x_2 - x_1^3 + \beta \cos(t). \end{cases} \tag{5} $$

The initial states of master and slave system are assumed to be $x_1(0)=0.2$, $x_2(0)=0.2$ and $y_1(0)=0.1$, $y_2(0)=-0.2$, respectively. To demonstrate the chaotic behaviour of dynamic (5), the phase portrait of $x_1(t)$ and $x_2(t)$ is shown in Fig 1. Parameters α and β are chosen 0.25 and 0.3, respectively.

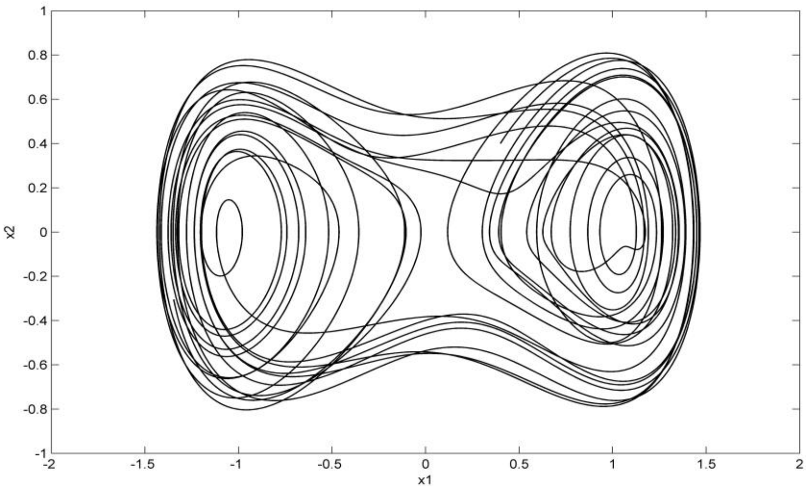

Fig. 1 Phase plane trajectory of Duffing-Holmes chaotic system, x1 versus x2.

Now suppose that let Duffing-Holmes slave system is perturbed by uncertainly and disturbance, which as follows:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_1 - \alpha y_2 - y_1^3 + \beta\cos(t) + \Delta f(Y,t) + d(t) + u(t). \end{cases} \tag{6}$$

Where $\Delta f(Y,t)$ and $d(t)$ are chosen $0.1\sin(t)\sqrt{y_1^2 + y_2^2}$ and $0.1\sin(t)$ respectively.

Synchronization of uncertain Doffing-Holmes chaotic system

Classical PID surface sliding mode controller. A typical sliding surface can be expressed as:

$$S(e(t)) = S_1(e(t)) = \lambda_1 e_1(t) + \int e_1(t)dt + e_2(t),\ \ \lambda_1 > 0 \tag{7}$$

Taking the time derivative from both sides of equation in (7), obtains:

$$\dot{S}(e(t)) = \lambda_1 \dot{e}_1(t) + e_1(t) + \dot{e}_2(t), \quad \lambda_1 > 0 \tag{8}$$

The input control signal can be defined as:

$$u(t) = \begin{bmatrix} -\lambda_1(x_2 - \dot{y}_1) - y_1 + \alpha y_2 + y_1^3 - \beta\cos(t) - 0.1\sin(t)\sqrt{y_1^2 + y_2^2} - \\ 0.1\sin(t) + \dot{y}_2 - x_1 + y_1 - K\,\mathrm{sgn}(S(t)) \end{bmatrix} \tag{9}$$

Where switching feedback control gain might be any positive number, and substitution of Eq. (9) into (8), results:

$$\dot{S}(t) = -K\,\mathrm{sgn}(S(t)) \tag{10}$$

A simulation result for this controller has been shown in Fig. 2 Chattering phenomena has occurred when the state hits the sliding surface Fig. 2(a)-(c). After reaching time the actual trajectory response x_1 is almost identical to the desired command y_1, the same results is noticed for x_2 and y_2.

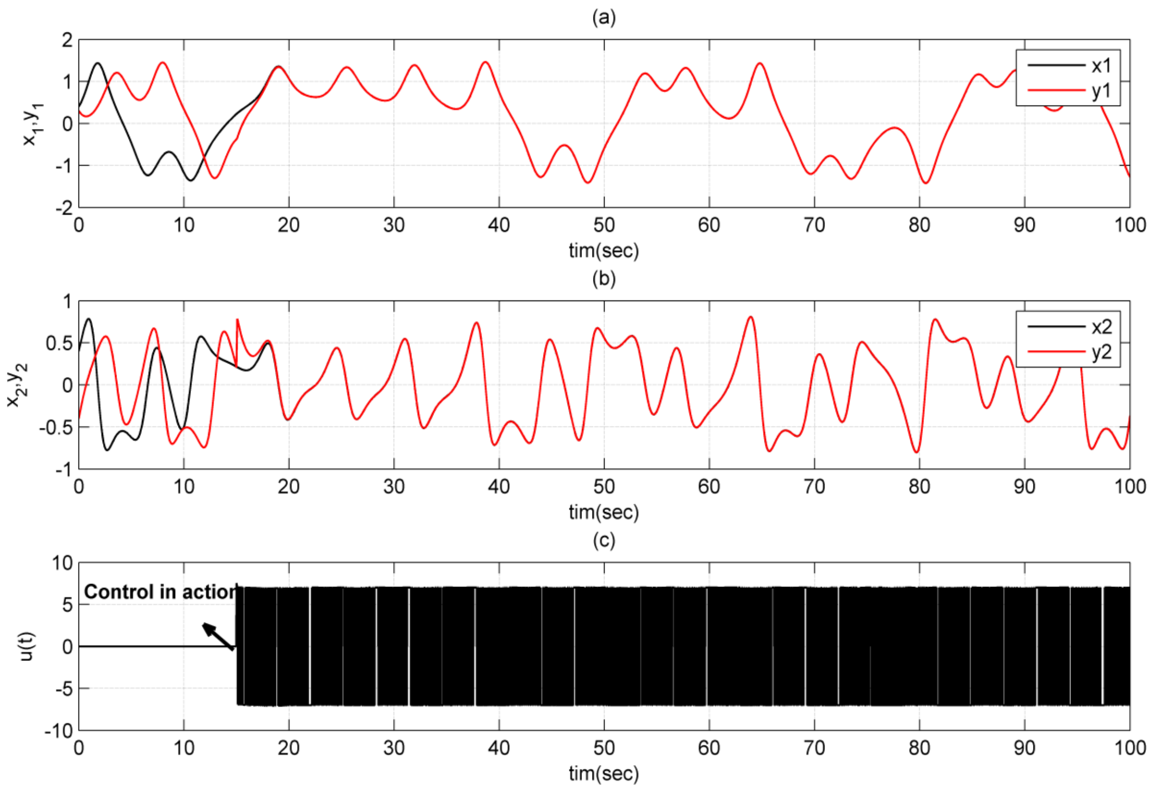

Fig. 2 PIDSMC Synchronization results of Duffing-Holmes system with signum function and different initial conditions, where $(x_1(0), x_2(0)) = (0.4, 0.4)$ and $(y_1(0), y_2(0)) = (0.3, -0.4)$. (a) The time waveform $x_1(t)$ and $y_1(t)$ versus time t, (b) The time waveform $x_2(t)$ and $y_2(t)$ versus time t, and (c) Variation of control action over time. Note that control u(t) is activated at $t = 15$ sec.

Fractional order PI$^\lambda$D$^\mu$ surface Sliding mode controller. the fractional PI$^\lambda$D$^\mu$ sliding surface can be expressed as:

$$S(e(t)) = S_1(e(t)) = \lambda_1 e_1(t) + D^{-\lambda} e_1(t) + D^{\mu} e_1(t), \;\; \lambda_1 > 0 \tag{11}$$

Taking the time derivative from both sides of (11), results:

$$\dot{S}_1(e(t)) = \lambda_1 \dot{e}_1(t) + D^{1-\lambda}\left(e_1(t)\right) + D^{1+\mu}\left(e_1(t)\right) \tag{12}$$

The input control signal can be defined as:

$$u(t) = \begin{bmatrix} D^{1-\mu}\left(-\lambda_1\left(x_2 - \dot{y}_1\right)\right) - D^{2-\lambda-\mu}\left(x_2 - y_1\right) - y_1 + \alpha y_2 + y_1^3 - \beta\cos(t) - 0.1\sin(t)\sqrt{y_1^2 + y_2^2} - \\ 0.1\sin(t) + \ddot{y}_1 - K\,\mathrm{sgn}(S(t)) \end{bmatrix} \tag{13}$$

Where switching feedback control gain might be any positive number, and substitution of Eq. (13) into (14), results:

$$\dot{S}(t) = -K\,\mathrm{sgn}(S(t)) \tag{14}$$

When the control law the control signals is chosen as (13), chattering phenomena will occur as soon as the state hits the sliding surface because of discontinuity in signum function. To reduce the chattering a saturation function is used instead of the signum function. Hence, the alternative control signal in (9) becomes as:

$$u(t) = \begin{bmatrix} -\lambda_1(x_2 - \dot{y}_1) - y_1 + \alpha y_2 + y_1^3 - \beta\cos(t) - 0.1\sin(t)\sqrt{y_1^2 + y_2^2} - \\ 0.1\sin(t) + \dot{y}_2 - x_1 + y_1 - K\,sat\left(S/\Phi\right) \end{bmatrix} \tag{15}$$

Fractional surface sliding mode control (15) will be

$$u(t)=\begin{bmatrix} D^{1-\mu}\left(-\lambda_1(x_2-\dot{y}_1)\right)-D^{2-\lambda-\mu}(x_2-y_1)-y_1+\alpha y_2+y_1^3- \\ \beta\cos(t)-0.1\sin(t)\sqrt{y_1^2+y_2^2}-0.1\sin(t)+\ddot{y}_1-K\,sat\left(S/\Phi\right) \end{bmatrix} \tag{16}$$

The simulation results of employing genetic based fuzzy $PI^{\lambda}D^{\mu}$ sliding mode control ($PI^{\lambda}D^{\mu}$FSMC) with +0.25 variations in system parameters the system responses are illustrated in Fig. 3. As shown in Fig. 3(a), and (b), the proposed controller ($PI^{\lambda}D^{\mu}$FSMC) provides a fast tracking capability in the various uncertainties in comparison with the response obtained by employing the PIDSMC, and one can find that the $PI^{\lambda}D^{\mu}$FSMC provides a smooth control action. The chattering of u(t) is shown minimized in Fig. 3(c).

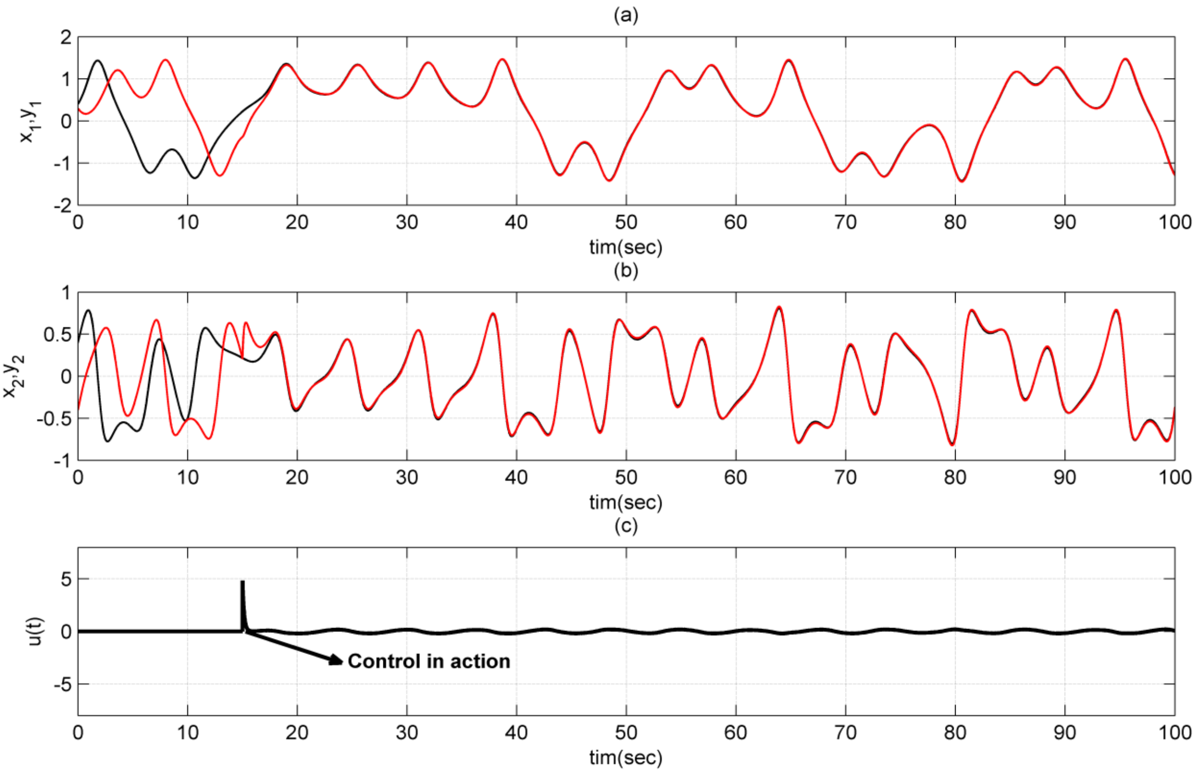

Fig. 3 $PI^{\lambda}D^{\mu}$FSMC Synchronization results of Duffing-Holmes system with saturation function and different initial conditions, where $(x_1(0),x_2(0))=(0.4,\,0.4)$ and $(y_1(0),y_2(0))=(0.3,-0.4)$. (a) The time waveform $x_1(t)$ and $y_1(t)$ versus time t, (b) The time waveform $x_2(t)$ and $y_2(t)$ versus time t, and (c) Variation of control action over time. Note that control u(t) is activated at $t=15$ sec.

From Fig. 6(a), and (b), it is observed that employing the $PI^{\lambda}D^{\mu}$FSMC can impressively improve the tracking performance and provides a faster tracking response with minimum reaching phase time in comparison with the conventional controller PIDSMC Fig. 4(a), and (b). In addition, all performance criteria (IAE, ITAE, and ISV) of employing the proposed $PI^{\lambda}D^{\mu}$FSMC are minimized comparing with these by employing the PIDSMC. The performance indices are tabulated in Table 2.

Table 2:Results of controller's performances

Controller	synchronization error	IAE	ISV
PIDSMC	e_1	2.9245	100.42
	e_2	2.5652	
$PI^{\lambda}D^{\mu}$FSMC	e_1	1.3979	10.36
	e_2	1.1648	

Conclusion

In this paper, a fractional fuzzy sliding mode control scheme for synchronization of chaotic and uncertain Duffing-Holmes system is addressed. The FOFSMC, incorporating the fractional-order controller (FOC), the SMC, and a fuzzy logic controller (FLC), is designed to retain the advantages of the SMC while reducing the chattering. The numerical results demonstrate the ability of the FOFSMC scheme to synchronize the chaotic systems. As a result, compared with conventional switching controllers, the proposed scheme has a lower implementation cost and can provide a reasonable tracking performance.

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