Study of Stability of a Highway Fill Application of the Analytical and the Finite Elements Methods

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ABSTRACT

The traditional tools used to treat the slopes stability problem are based on simple static approaches like calculation in limit equilibrium by the slices method. These approaches, though practicable, are not rigorous as they don't take into account the seismic action on the structures. Recent progresses made in data computing and numerical calculation fields (finite differences method, finite elements method) lead to better analysis the problem of slopes stability under seismic excitations. Indeed, they take into account all the important parameters, usually neglected when using simplified approaches, as initial state of subsoil sedimentary formations, water presence, presence of bad quality soil on surface, nonlinear behavior of the ground, and influence of possible devices of reinforcements...

The method of reduction of resistance properties known as "c-φ reduction", more than allowing the calculation of the slopes safety factor by the Finite elements method (FEM), has several advantages, such as the ability to predict stresses and strains at failure of any reinforcement elements as piles, anchoring or geotextiles. The application of these various approaches on a highway fill located in a zone known with seismic risk allows to understand the real behavior of the slope formations and to optimize its reinforcement.

Keywords **- Finite elements, Reinforcement, Safety factor, Seismicity, Slope stability.**

1. INTRODUCTION

The traditional techniques, founded on the limit equilibrium concept, are the most analysis methods commonly used for calculations of slopes stability, because of their simplicity and the reduced number of parameters they require, as slope geometry, topography, site geology, geotechnical parameters and hydrogeologic conditions. However, the methods founded on calculations with finite elements (FEM) have several advantages: i- modeling slopes with a high degree realism (complex geometry, loading sequences, presence of reinforcement device, water action, complex laws of geomaterials behavior,…), ii- better visualizing strains of the geological formations in place. The application of these various methods on an embankment permits more than their comparison to highlight all previously mentioned elements. Various calculations carried out illustrate perfectly the advantages of modeling the behavior by the finite elements method.

2. Methodology

2.1. Limit equilibrium methods

The limit equilibrium methods are the most used techniques for stability studies. These methods consist in cutting out the soil and the subsoil in fine slices so that their base can be considered as a straight line, then to write the equilibrium equations of the forces and/or moments. According to the assumptions made on the efforts between the slices and also the considered equilibrium equations, various techniques have been proposed (Table 1). They give in the majority cases quite similar results. The differences between the values of safety factor F obtained with the various methods are in general lower than 6% [1].
The principal methods based on the limit equilibrium are those of Fellenius [2] and Bishop [3]. The equilibrium of slice "i" on the horizontal slope portion potentially under rupture (Fig. 1) is written as follows:

$$dH_i - \sigma_i \tan\alpha_i\, dx + \tau_i\, dx = 0 \qquad (1)$$

Where
- H_i is the horizontal component of the force acting between two slices;
- σ_i and τ_i are normal and tangential stresses at the potential failure surface (at the slice i);
- α_i is the angle formed by the base of the slice i with the horizontal.

The equilibrium of slice i on the vertical is written:

$$dV_i - \gamma_i\, h_i\, dx + \sigma_i\, dx + \tau_i \tan\alpha_i\, dx = 0 \qquad (2)$$

Where
- V_i is the vertical component of the force acting between two slices;
- γ_i is the weight unit of the slice i.

Table 1: Main methods of limit equilibrium [1]

In the method of Fellenius [2], we assume that dH_i and dV_i

Methods	Advantages and Disadvantages
Slope Stability Charts (Janbu, 1968, Duncan, 1987)	- Satisfactory results in many cases - Speed
Ordinary Method off Slices (Fellenius, 1927)	- Only circular surfaces - Satisfy the moments equilibrium - Neither horizontal nor vertical forces equilibrium are satisfied
Bishop' S Modified Method (Bishop, 1955)	- Only circular surfaces - Satisfy the moments equilibrium - Satisfy only the equilibrium of the vertical forces
Force Equilibrium Method (Lowe & Karafiath, 1960, US Army Corps of Engineers, 1970)	- Any geometry of the failure surface - Does not satisfy the moments equilibrium - Satisfy the equilibrium of vertical and horizontal forces
Janbu' S Generalized Procedure of Slices (Janbu, 1968)	- All geometries of failure surfaces - Satisfy all equilibrium conditions - Numerical instability more frequent than with other methods
Morgenstern & Price' S Method (Morgenstern & Price' S Method, 1965)	- All geometries of failure surfaces - Satisfy all equilibrium conditions
Spencer' S Method (Spencer, 1967) - All geometries of failure surfaces	- Satisfy all equilibrium conditions

are nil, which implies that the normal stresses are estimated by:

$$\sigma_i = \gamma\, h_i \cos^2 \alpha_i \qquad (3)$$

Using the global definition of the safety factor, we obtain the equation (4).

$$F_{Fel} = \frac{\sum_{i=1}^{n}(C' + (\gamma h_i \cos^2 \alpha_i - u_i)\tan\varphi')\frac{1}{\cos \alpha_i}}{\sum_{i=1}^{n} \gamma h_i \sin \alpha_i} \qquad (4)$$

In the method of Bishop [3], we assume that $dV_i = 0$. Thus, considering the global definition of the safety factor, we obtain a relation of type FBish = f (FBish) (5). The safety factor is determined using an iterative procedure or the fixed point method.

$$F_{Bish} = \frac{\sum_{i=1}^{n}\left(C' + \left(\gamma h_i - \left(\frac{C'}{F_{Bish}} + \sigma' \frac{\tan\varphi'}{F_{Bish}}\right)\tan\alpha_i - u_i\right)\tan\varphi'\right)\frac{1}{\cos\alpha_i}}{\sum_{i=1}^{n}\gamma h_i \sin\alpha_i} \qquad (5)$$

The general procedure in all these methods can be summarized as follow:

1- hypothesis of the existence of at least one sliding surface;
2- Static analysis of normal and tangential stresses on the sliding surfaces;
3- Calculating the safety factor F, defined as the ratio of shear stress on the effective shear stress along the failure surface considered;
4- Determination of the critical failure surface which gives the least safety factor F, among the whole analyzed surfaces.

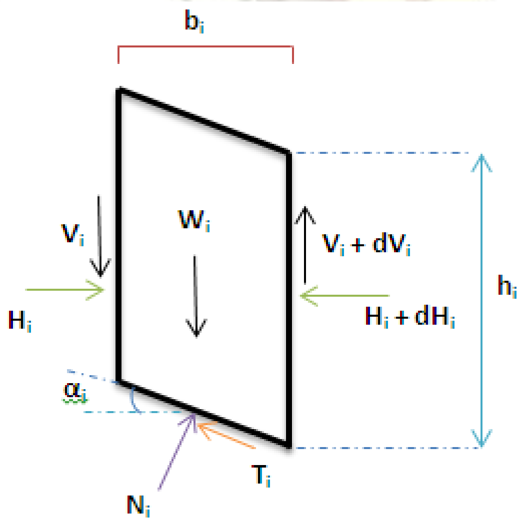

Figure 1: Circular failure according to Bishop and Fellenius methods

2.2. Pseudo static approach calculation

The principle of the pseudo-static approach consists in modeling the seismic solicitation by an equivalent acceleration which takes into account the probable reaction of the inclined massif. The pseudo-statics efforts are represented by two coefficients K_h and $\pm K_v$ called seismic coefficients, for characterizing the horizontal component directed downstream and the vertical component descending or ascending of the forces P applied to the inclined massif.

The safety factor F, calculated by the method of Bishop, is given by the following expression:

$$F = \frac{\sum \frac{c' b + (W - ub)\tan\varphi'}{\frac{1 + \tan\alpha \ \tan\varphi'}{F}\cos\alpha}}{\sum W\left[\sin\alpha - \frac{Y_G - Y}{R}\right] + k_v} \qquad (6)$$

Or

$$F = \frac{\sum \frac{c' + (\gamma h - u)\tan\varphi'}{\frac{1 + \tan\alpha \ \tan\varphi'}{F_{Fellenius}}\cos\alpha}}{\sum \gamma h\left[\sin\alpha + k_h\left(\cos\alpha - \frac{Y_G - Y}{R}\right) + k_v\right]} \qquad (7)$$

where:

- X_G and Y_G: coordinates of the center of gravity of the slice considered;
- α : inclination of the slice i with respect to the vertical crossing the center of the circle;
- c' and φ ': shear parameters of the soil slice;
- u: pore water pressure in the slice;
- K_h and K_v: seismic acceleration coefficients;
- R: radius of the slip circle.

2.3. Finite element methods

The different limit equilibrium methods are based on the arbitrary choice of a set of slip surfaces and to define the one that gives the minimal safety factor value. But lately, we observe an intensive use of numerical methods of analysis giving access to the constraints and deformations within the rock formations constituting the subsoil. To achieve this, it is necessary to know the behavior law of the formation considered; and then, the volume of this formation is divided into simple geometric elements, each element being subjected to the action of neighboring elements. The calculation will consist in determining the stress fields and displacement compatible with the mechanics equations and the constitutive law adopted [4, 5]. Thus, we will use the method of reduction of the properties of resistance of the rock formations, also known as method of “c-φ reduction” [6, 7]. The finite element method allows calculating stress and strain state in a rock mass, subjected to its self weight and taking into account the constitutive law adopted. In our calculations, a model with internal friction without work hardening (perfect elastoplastic Model: Mohr-Coulomb) is used, which corresponds to the basic assumptions of analytical methods.

In the Phi-C reduction approach, the strength parameters tanφ and C of the soil are successively reduced until failure of the structure occurs. The strength of interfaces characteristics, if used, is reduced in the same way. The strength of structural objects like plates and anchors is not influenced by Phi-c reduction. The total multiplier ΣMsf is used to define the value of the soil strength parameters at a given stage in the analysis:

$$\sum M_{sf} = \frac{\tan\varphi_{input}}{\tan\varphi_{reduced}} = \frac{c_{input}}{c_{reduced}} \qquad (8)$$

Where the strength parameters with subscript ‘input’ refer to the properties entered in the material sets and parameters with the subscript “reduced” refer to the reduced values used in the analysis. The total safety coefficient *F* is:

$$F = \frac{available\ strength}{strength\ at\ failure} \qquad (9)$$

= value of ΣMsf at failure

3. Case study: highway fills

The study area is in the Fès-Taza corridor which constitutes the narrowest part of the South-Rifain furrow (Morroco). The Lower Jurassic carbonate formations constituting the Middle Atlas Causse plunge to the north by stepped accident under Miocene marls. The Miocene consists of marly series with 400 to 500m thick, resting in discordance on Palaeozoic and Jurassic. Sandy-gritty and marly sandstone levels are found in this marly serie, especially along Fez-Taza corridor (Fig.2).

Miocene deposits, mainly represented by the blue argillaceous marls, are not very permeable except of intercalated sandy and sandstone horizons. Groundwater in the Miocene deposits is unconfined in the southern part of Fes-Taza corridor, where the deposits thickness is low and sandy levels outcrop.

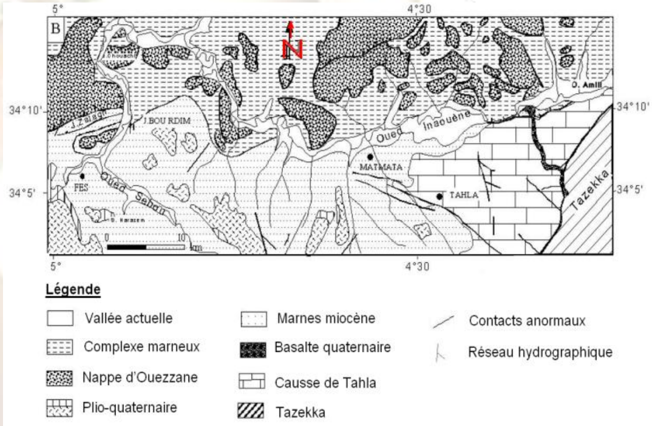

Figure 2: Geological map of the study area [8]

The case study relates to an embankment that is part of the highway section Ras-Tabouda Tahala (Province of Taza).

The geological formations are predominantly argilo-marly, of age going from Triassic to Oligocene and form a little accentuated relief and deeply notched by wadis and ravines (Fig. 3). The studied area is in a zone at the seismic risk (zone III; [9]) with horizontal coefficient of seismic acceleration equal to 0.16g. The piezometric level of the study area fluctuates in a seasonal way. A piezometric survey conducted on October 12th, 2010, revealed the existence of a water level between 1.30m and 4m of depth.

The calculations are carried out on the profiles of ground considered to be representative (Fig. 4). The mechanical characteristics adopted in this study of stability are obtained from laboratory and field tests (Table 2).

Calculations are carried out according to several scenarios by combining the various situations of presence or absence of water and taking into account or not of the seismic loading using two softwares "Plaxis"[10] and "GeoSlope" [11]. The first is based on limit equilibrium methods whereas the second is a finite elements code. For our study we adopted a plane strain model with 15 nodes and 692 elements (Fig. 4).

Table 2: Mechanical properties of formations

Formation	γ	γsat	K	ν	E	C	φ	ψ
	(kN/m3)		(m/s)		(kPa)	(kPa)	(°)	(°)
Fill	22	24	1.E-5	0.3	8000	0	33	3
yellow marl	18	20	1.E-7	0.3	8000	1	19	0
gray marl	18	20	1.E-9	0.3	30000	15	20	0

γ: Unit weight γsat: saturated unit weight K:coefficient of permeability
ν: Poisson's ratio E: Young modulus C: cohesion
φ: natural friction angle ψ: dilatancy

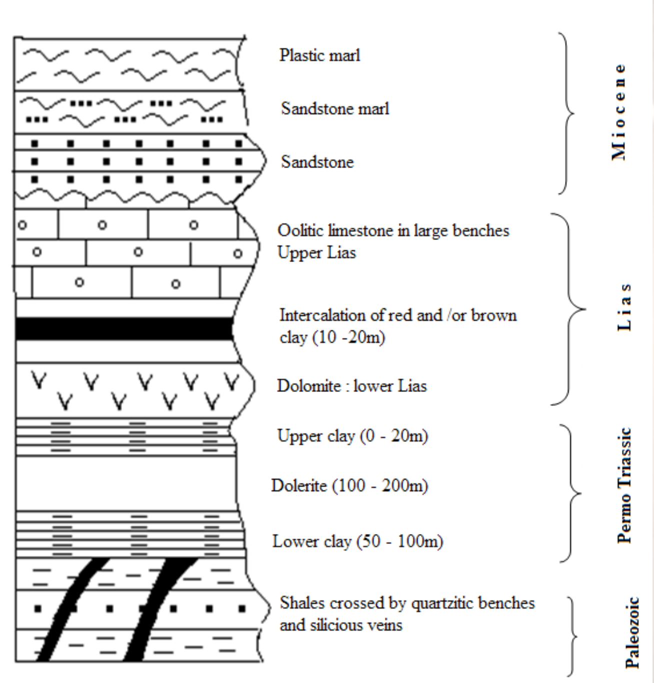

Figure 3: Stratigraphic column of the sedimentary formations that constitue the slope

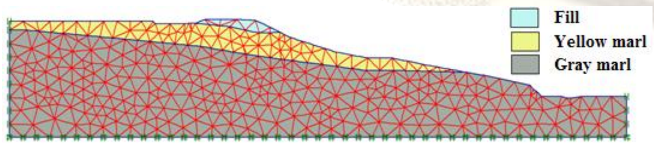

Figure 4: Geometry and mesh of the model studied

4. Results and discussions

4.1. Dry state

In the case of the highway embankment, the C - φ reduction method according to the Mohr Coulomb criterion, underestimates the safety factor value about 0.6% compared to the value obtained by the method of Fellenius which is most secure of the analytical methods and 5% compared to Bishop method (Fig.5 and Table 3).

Table 3: Values of the safety factors in a dry state

	Analytical methods				FEM
	Fellenius	Simplified Bishop	Janbu	Morgenstern-Price	Phi-C reduction
F	0.928	0.972	0.940	0.972	0.922

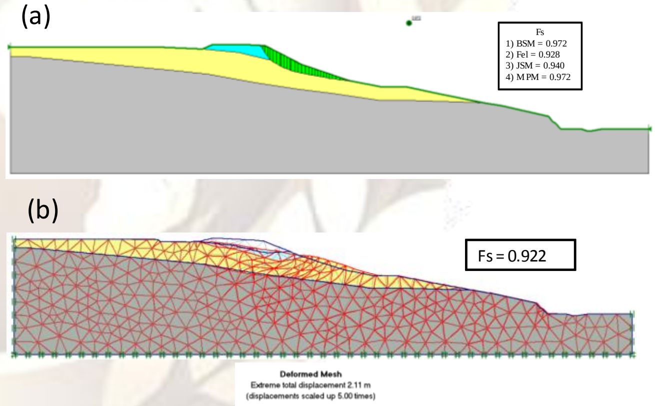

Figure 5. Safety factors calculated in a dry state; a- Geoslope and b- Plaxis.

4.2. Presence of water

The previous calculations were performed assuming that the water pressures are uniformly nil in the slope. Taking into account water effects can be done in different ways according to the methods of calculating used. The presence of ground water destabilizes the slope and shows a decrease in safety factor (Table 4). Thus, the c-φ reduction method according to Mohr Coulomb criterion, underestimates the safety factor value about 5% compared to the value obtained by Fellenius method and 15% compared to Bishop method (Fig.6 and Table 4).

Table 4: Values of the safety factors with presence of water

	Analytical methods				FEM
	Fellenius	Simplified Bishop	Janbu	Morgenstern-Price	Phi-C reduction
F	0.836	0.937	0.857	0.935	0.790

4.3. Dynamic conditions

The majority of the stability studies are done by static analysis. However, in a seismic region, the earthquake is the most damaging factor which causes slope instability. Therefore, it is also necessary to analyze the stability under dynamic conditions. The c-φ reduction method according to the Mohr Coulomb criterion, overestimates the safety factor value about 22% compared to the value obtained by Fellenius method and 15% compared to Bishop method (Fig.7 and Table 5).

Table 5: Values of the safety factors under dynamic conditions

	Analytical methods				FEM
	Fellenius	Simplified Bishop	Janbu	Morgenstern-Price	Phi-C reduction
F	0.650	0.686	0.659	0.679	0.793

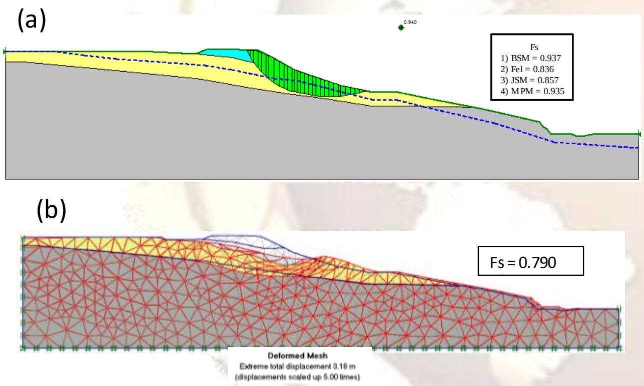

Figure 6. Safety factors calculated with presence of water: a- Geoslope and b- Plaxis.

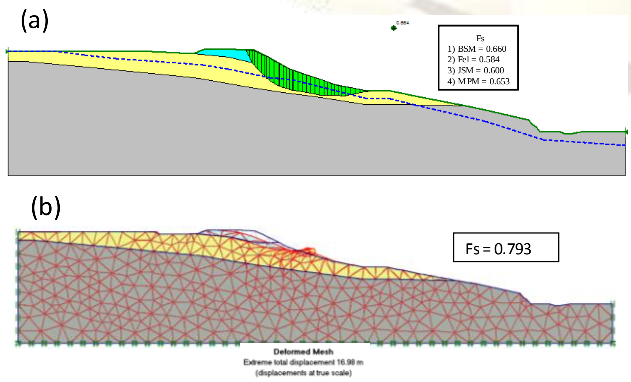

Figure 7. Safety factors calculated under dynamic conditions and presence of water: a- Geoslope and b- Plaxis.

4.4. Reinforcement

The improvement of the performance of the bad quality subsoil, by incorporating vertical inclusions, constitutes a suitable solution as well in term of reducing settlement as of increase in the bearing capacity. This technique is particulary applied for road or railway embankments built on soft soil layer [12, 13, 14]. According to preceding calculations, the slope is extremely unstable, and requires reinforcement's means. The choice of reinforcement method varies with the characteristics and the state of each site: soil type, drainage conditions, overloads and economic costs.

Considering the bad geotechnical characteristics of the marl (rain precipitations can generate crackings), we propose the installation of piles with the following characteristics:

- reinforced concrete piles with a diameter a = 1.2 m and length L = 12 m, centered on the interface yellow marl / gray marl (presumed failure surface);
- The piles have a shearing force of Pp=300KN;
- The use of mesh 5m x 5m (ie D1 = 5m, so D2 =3.8 m) arranged on two platforms with three rows of piles.

The finite element method, overestimates the safety factor value about 6% compared to Bishop method (Fig.8 and Table 6).

Table 6: Values of the safety factors after reinforcement under dynamic conditions

	Analytical methods			FEM
	Simplified Bishop	Janbu	Morgenstern-Price	Phi-C reduction
Fs	1.335	1.335	1.332	1.422

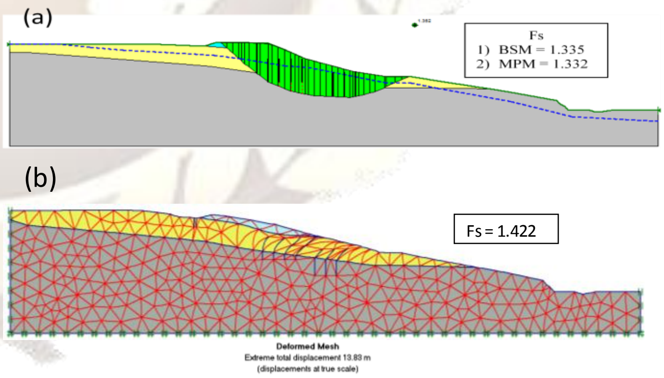

Figure 8. Safety factors calculated with reinforcement under dynamic conditions and presence of water: a- Geoslope and b- Plaxis.

5. Discussion

The safety factor obtained using the c-φ reduction method according to the Mohr Coulomb criterion remains comparable to those obtained by the analytical methods (with or without pore water pressures). The noted difference is due to the fact that for the analytical methods, the safety factors are supposed to be constant along the sliding surface. However, under seismic excitation, the reduction of the safety factor by the analytical methods is about 30% whereas it does not exceed 14% in the case of calculation by finite elements method. The results obtained by finite elements method show that the solution of reinforcement selected ensures the stability of the slope, whereas with the same reinforcement, the analytical methods show that it is still unstable, so FEM optimizes the reinforcement. Moreover the finite element methods that gives access to the stresses and strains within the subsoil, offer the possibility to use a detailed calculations in the form of curves: displacements (Fig. 9), the curve representing the evolution of the safety factor versus displacements (Fig. 10), localization of strains (Fig. 11) and the plastic zones (Fig. 12).

Taking into account of the constitutive law in finite element codes helps identify the state of stress and strain in different parts of the subsoil. Figure 9 which shows total displacements highlights the limit between the zone where there is no displacement and the zones where displacements occur (non null values). The circular shape of that limit points out the slip surface adopted by the analytical methods (Fig. 9). The displacements are important at the slope and the greatest value is at the slope's toe (Fig. 10). The identification of the failure curve in Plaxis is based on the localization of deformations on the slope (Fig. 11). We find again the circular shape of the slip surfaces. The figure 12 shows the concentration of the plastic points inside of this same limit.

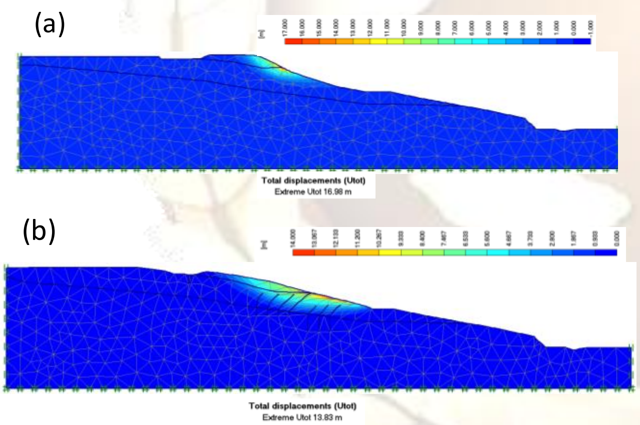

Figure 9: The total displacements obtained by Plaxis under dynamic conditions with presence of water (a- without reinforcement, b- with reinforcement)

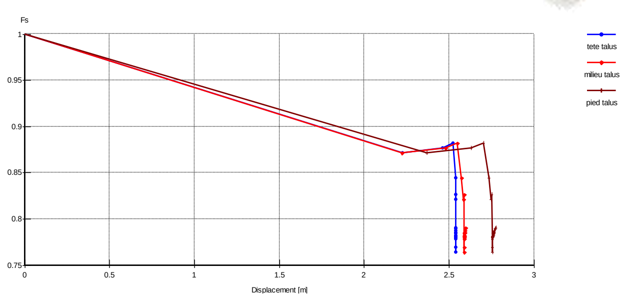

Figure 10: Evolution of the safety factor versus displacements

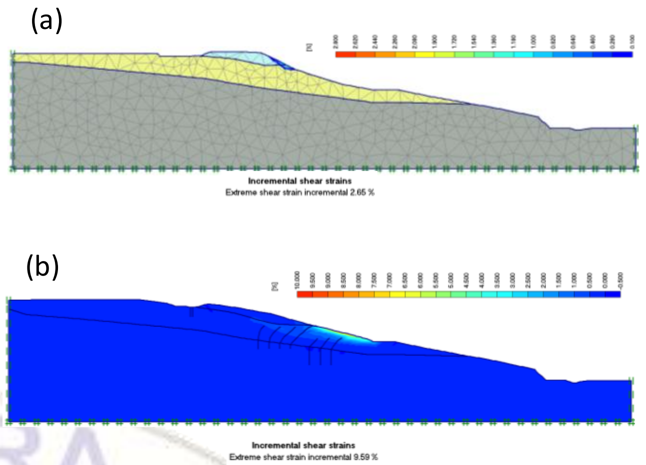

Figure 11: Localization of the deformations by Plaxis (a- without reinforcement, b- with reinforcement)

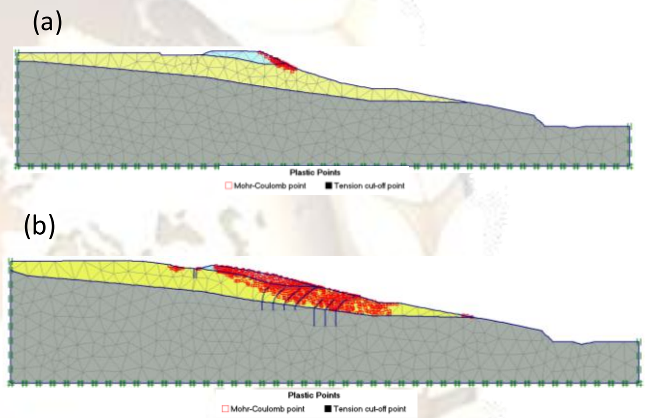

Figure 12: Localization of the plasticized zones (a- without reinforcement, b- with reinforcement)

6. Conclusion

The case study relates to an embankment that is part of the highway section Ras-Tabouda Tahala (Province of Taza). Miocene blue marl poses problems for any construction in the corridor south-Rif (Gharb and Fez Saiss), such as highway and others. This work will bring, to policy makers, solutions and approaches to prevent major civil engineering structure landslides and differential settlement.

The stability study of this highway fill allowed comparing the calculation results of the safety factor and defining the equilibrium of the slope over the limit equilibrium, using different methods: limit equilibrium methods and finite elements method. The law of stress-strain behavior that fails to methods of limit equilibrium is integrated into the finite element methods. The safety factors obtained by the finite element method and conventional methods, both in dry and saturated state do not show any significant difference. On the other hand under the dynamic conditions, the two approaches give different values of the safety factor, which influences the choice of reinforcement. In fact the results of the analysis of the slope stability after reinforcement show that the finite element modeling optimizes comfort. The identification of the failure curve obtained by finite elements method, based on the localization of deformation along the slope takes a circular shape which is mostly adopted by analytical methods.

The determination of the safety factor is insufficient to identify issues of slope stability. The various calculations performed illustrate perfectly the benefits that can be gained from modeling the behavior by the finite elements method; i- calculation of displacements obtained by finite element method to estimate the real settlement and optimize the reinforcement, ii- prediction of the failure mode, iii- use the results of field tests to better approximate the real behavior of structures.

REFERENCES

[1] Duncan JM, Buchignani AL, De Wet M (1987) An engineering manual for slope stability studies. Virginia Tech, Blacksburg.

[2] Fellenius W (1927) Erdstatische Berechnungen mit Reibung und Kohasion. Ernst, Berlin.

[3] Bishop AW (1955) The use of the slip circle in the stability analysis of slopes. Géotechnique, London, 5, 1, 7-17.

[4] Zienkiewicz OC, Taylor RL (2000) The finite element method solid mechanics: volume 2, 459 p.

[5] Dhatt G, Touzot G, (1981) une présentation de la méthode des éléments finis, 543 pages

[6] San, K.C., and Matsui, T .(1992) : "Application of Finite Element Method System to Reinforced Soil", Proc. Int . Symp. on Earth Reinforcement Practice, Kyusu, pp.403- 408

[7] Ugai, K. (1990), Availability of shear strength reduction method in stability analysis, Tsuchi-to-kiso, 38- 1:67-72, JSSMFE

[8] Suter G.1980. Carte géologique du Maroc au 1:500000: Notes et Mémoires Serv. géol. Maroc, 245.

[9] Badrane S. (2007)- contribution à l'étude de l'Aléa sismique au Maroc. Doctorat en sciences, 137 pages, Ecole Mohammadia d'Ingénieurs, Univ. Mohamed V, Rabat-Agdal.

[10] PLAXIS (2004). Finite Element Code for Soil and Rock Analyses . PLAXIS-2D Version 8, Reference Manual, Edited by Brinkgreve, et al., DUT, the Netherlands. www.plaxis.nl

[11] SLOPE/W (2002). Stability Analysis. Users Guide Version 5, GeoSlope Office, Canada. www.geoslope.com.

[12] WOOD H. J. *The design and construction of pile-supported embankments for the A63 Selby Bypass*. In: NEWSON T. A. Ed. Proc. of the BGA Int. Conf. on Foundations : Innovations, Observations, Design and Practice, Dundee, Ecosse, 2-5 septembre 2003. Londres : Thomas Telford, 2003, pp 941-950.

[13] ZANZIGER H., GARTUNG E. *Performance of a geogrid reinforced railway embankment on piles*. In: DELMAS S., GOURC J.P. Eds. Proc. of the 7th Int. Conf. on Geosynthetics, 22-27 septembre 2002, Nice. Lisse, Pays-Bas : Swets & Zeitlinger, 2002, pp 381-386.

[14] QUIGLEY P., O'MALLEY J., RODGERS M. *Performance of a trial embakment constructed on soft compressible estuarine deposits at Shannon, Ireland*. In: VERMEER P. A., SCHWEIGER H., KARSTUNEN M., CUDNY M. Eds. Proc. of the Int. Workshop on

[15] Geotechnics of Soft Soils - Theory and Practice, 17-19 septembre 2003, Noordwijkerhout, Pays-bas. Essen : Verlag Glückauf, 2003, pp 619-624.

[16] Akhssas A., Bahi L., Lahmili A., Ouadif L. (2011) Study of the stability of rock slope. Canadian Journal on Scientific and Industrial Research Vol. 2, No. 2, 35-39.

[17] Duncan JM (1996) State of the art: limit equilibrium and finite-element analysis of slopes. Journal of Geotechnical Engineering, 122, 577–596.

[18] Fredlund, D. G., and Krahn, J. 1977. "Comparison of Slope Stability Methods of Analysis," *Canadian GeotechnicaI Journal*, Vol 14, No. 3, pp 429-439.

[19] Zheng H, Liu DF, Li CG (2005) Slope stability analysis based on elastoplastic finite element method. Int. J. Numer. Meth. Eng., 64, 1871–1888.

[20] Zheng YR, Zhao SY, Kong WX, Deng CJ (2005) Geotechnical engineering limit analysis using finite element method. Rock Soil Mech., 26, 1, 163–168.

[21] Zou JZ, Williams DJ (1995) Search for critical slip surface based on finite element method. Canad . Geotech. J., 32, 2, 233–246.