FERROTOROIDIC PROPERTY FOR FERROIC SPECIES

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Abstract:
The Physical property tensors invariant under the grey groups and their subgroups constitutes all domains that arise in a phase transition. In this paper, the Ferrotoroidic moment domain pairs are calculated using coset decomposition for all the 324 feerrotoroidic ferroic species, taking 32 grey groups as prototypic point groups.

Key Words: Ferrotoroidic, toroidal moment, domain pairs

Introduction: A ferroic crystal contains one or more domains but of the different spatial orientation. A Ferroic crystals arises in a ferroic phase transition from a phase of higher symmetry to a phase of lower symmetry here grey group $G1^1$ is the prototypic point group and H is a Ferroic phase of a lower symmetry. Aizu has given all possible 773 species of the Ferroic crystals in phase transitions. This method was extended by D.B.Litvin[5], he has calculated twin laws of domain pairs for Ferrotoroidic ferroic species of eleven types of physical property tensors. In this paper the ferrotorodic domain pairs are calculated using coset decomposition for all the 324 Ferrotoroidic ferroic species, taking the 32 grey groups as prototypic point groups.

Domain states may be distinguished by the values of components of certain Spontaneous macroscopic tensorial properties. Ferrotoroidic type with toroidal moment "aV" for all 324 Ferroic species are calculated, taking grey group as the prototypic point group. Here "V" denotes a Polar vector, and "a" denotes zero-rank tensors that change sign under time inversion.

The transition to the multiferroic state, which involves spontaneous magnetization, polarization and toroidal moment, gives rise to spontaneous toroidic effects, these effects produce specific contributions to the spontaneous polarization and magnetization under applied magnetic or electric fields. which provide indirect indications of the existence and role of the toroidal moment in multiferroic materials.

A magnetic toroidal moment represents a vector-like electromagnetic multipole moment which breaks both space and time reversal Symmetries simultaneously. It can be represented by a current flowing through a solenoid bent into a torus, or alternatively, by a ring-like arrangement of magnetic dipoles. The toroidal moment has been proposed as the primary order parameter for the low-temperature phase transition from a ferroelectric into a simultaneously ferroelectric and weakly ferromagnetic, i.e, multiferroic phase as in case of boracites. This suggests that ferrotoroidicity is a fundamental form of ferroic order, equivalent to ferromagnetism, ferroelectricity, and ferroelasticity. In the following the case of magnetic toroidal moments are discussed and the term "Toroidal moment" is always used in the sense of"Magnetic toroidal moment".

Crystals $LiCoPO_4$ under time reversal and space inversion exhibit ferrotoroidic and antiferromagnetic domains, the ferrotiroidic domains in $LiCoPO_4$ using nonlinear optical techniques. $LiCoPO_4$ Crystallizes in the Olivine

Structure with the orthorhombic space group Pnma, and originally it was believed that the magnetic moments of the four Co ions in the unit cell are anti ferromagnetically aligned along the orthorhombic direction.

G be one of the 32 Crystallographic Point group and 1^1 is a group consisting of identity and time inversion R_2. The direct product of G and 1^1, Which is designated by $G1^1$ is known as grey group and 32 point groups in which R_2 does not occur explicitly nor in combination with Symmetry operations are known as ordinary point groups. The 58 groups in which R_2 occurs implicitly are known as magnetic variants of the 32 ordinary point groups. These 32 ordinary and 58 magnetic variants are known as magnetic point groups. Shubnikov (1951) discussed point groups are 122, where the ordinary point groups are 32, the grey groups are 32 and the magnetic point groups are 58.

coset decomposition of a group with respect to one of its subgroup has wide application in solid state physics, let H be a subgroup of G then the left (right) cosets, i.e.,

$$G = a_1H + a_2H + \text{---------} + a_i H + \text{--------} \text{ where } a_i \in G,\ i = 1, 2, 3, \text{-----}$$

This is coset decomposition of G with respect to H. coset decomposition has been applied in the analysis of ferroic crystals using coset decomposition of point groups and space groups (Aizu, [1]) . Here the numbers of distinct domains of ferroic crystals are equal to the number of distinct right or left coset and the coset representatives are F-operations that change one orientation state to another.

Representative domain pairs:

If G be the prototypic point group of the crystal and H be the ferroic point group of one of the domains, a ferroic phase transition is represented by $G1^1$ FH. The n domain states S_i, i=1, 2 - - - - n of a ferroic Crystal arise in a phase transition between the parent phase of symmetry G to a lower symmetry phase of subgroup symmetry H, then

$$G = H + g_2 H + g_3 H + \text{- - - - - - - - -} + g_n H \text{ Where } g_i,\ i = 1, 2, \text{- - - -}\ n \text{ are elements of G.}$$

If, $S_i = g^*S_j$, $S_j = g^*s_i$; then (Si, S_j) are domain pairs.

The pairs of domain states can be divided into subsets of ordered domain pairs, where all pairs of domain states are related by elements of the group G.

Ferrotoroidic Domain pairs for the Ferroic Species: 2221^1 F2^1

Aizu has introduced a classification of ferroic phase transitions into 773 species each characterized by a pair of point groups G and H .This method was extended by D.B.Litvin[5], he has calculated twin laws of domain pairs for Ferrotoroidic ferroic species of eleven types of physical property tensors. In this paper domain pairs all the Ferrotoroidic ferroic species are which exhibit ferroic moment are calculated and this is illustrated for the ferroic phase transition from 2(x) 2(y) $2(z)1^1$ to $2(z)^1$. Where 2221^1 are a prototypic point group and $2(z)^1$ is ferroic point group. The coset decomposition of $2_x2_y2_z1^1$ with respect to $2(z)^1$ is given by

$$\mathbf{G = H + C_{2y}H + R_2H + R_2C_{2Y}H}$$

and consequently there are four coset elements and the number of distinct domain pairs

(S_i, S_j i,j =1,2…4) are two.

Table 1: Domain pairs for ferroic species 2 (x)2(y)2(z)1'F2(z)'

S No	Prototypic point group, G	Ferroic Point group, H	Domain Pairs
1	2221^1	2	(T_1, T_2, O) $(-T_1, -T_2, O)$; $(-T_1, T_2, O)$ $(T_1, -T_2, O)$

Table 2 gives the list of all the domain pair representatives of toroidic physical property tensors. In the below mentioned table 2 the 2nd column represents Prototypic point group "G", 3rd column represents the Ferroic point group "H" and 4th column represents the Domain pairs.

Table 2

S No	Prototypic Point Group G	Ferroic Point Group H	Domain Pairs
1.	1^1	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$
2.	$\bar{1}1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$
3.	$\bar{1}1^1$	$\bar{1}^1$	$(T_1T_2,T_3)(-T_1,-T_2,-T_3)$
4.	21^1	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$
5.	21^1	2	$(0,0,T_3)(0,0,-T_3)$
6.	21^1	2^1	$(T_1,T_2,0)(-T_1,-T_2,0)$
7.	$m1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(T_1,T_2,-T_3)$ $(-T_1,-T_2,T_3)$
8.	$m1^1$	m	$(T_1,T_2,0)(-T_1,-T_2,0)$
9.	$m1^1$	m^1	$(0,0,T_3)(0,0,-T_3)$
10.	$2/m1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$
11.	$2/m1^1$	m^1	$(0,0,T_3)(0,0,-T_3)$
12.	$2/m1^1$	$2^1/m$	$(T_1,T_2,0)(-T_1,-T_2,0)$
13	2221^1	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(T_1,-T_2,-T_3)$ $(-T_1,T_2,T_3)$; $(-T_1,T_2,-T_3)$ $(T_1,-T_2,T_3)$; $(-T_1,-T_2,T_3)$;$(T_1,T_2,-T_3)$
14.	2221^1	2^1	$(T_1,T_2,0)(-T_1,-T_2,0)$; $(-T_1,T_2,0)(T_1,-T_2,0)$
15.	2221^1	2^12^12	$(0,0,T_3)(0,0,-T_3)$
16.	$mm21^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$; $(-T_1,T_2,T_3)$ $(T_1,-T_2,-T_3)$; $(T_1,-T_2,T_3)$;$(-T_1,T_2,-T_3)$

17.	$mm21^1$	m	$(0,T_2,T_3)(0,-T_2,-T_3); (0,-T_2,T_3)(0,T_2,-T_3)$
18.	$mmm1^1$	mm2	$(0,0,T_3)(0,0,-T_3)$
19.	$mmm1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(T_1,-T_2,-T_3)(-T_1,T_2,T_3); (-T_1,T_2,-T_3)(T_1,-T_2,T_3);(-T_1,T_2,T_3)(T_1,-T_2,-T_3); (T_1,-T_2,T_3)(-T_1,T_2,-T_3); (T_1,T_2,-T_3)(-T_1,-T_2,T_3)$
20.	$mmm1^1$	$2/m^1$	$(0,0,T_3)(0,0,-T_3);$
21.	$mmm1^1$	mm2	$(0,0,T_3)(0,0,-T_3);$
22.	41^1	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_2,T_1,T_3)(T_2,-T_1,-T_3); (T_2,-T_1,T_3)(-T_2,T_1,-T_3); (-T_1,-T_2,T_3)(T_1,T_2,-T_3)$
23.	41^1	2^1	$(T_1,T_2,0)(-T_1,-T_2,0); (-T_2,T_1,0)(T_2,-T_1,0)$
24.	41^1	4	$(0,0,T_3)(0,0,-T_3)$
25.	$\bar{4}1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3); (-T_2,T_1,T_3)(T_2,-T_1,-T_3); (T_2,-T_1,T_3)(-T_2,T_1,-T_3); (-T_1,-T_2,T_3)(T_1,T_2,-T_3)$
26.	$\bar{4}1^1$	2^1	$(T_1,T_2,0)(-T_1,-T_2,0); (-T_2,T_1,0)(T_2,-T_1,0)$
27.	$\bar{4}1^1$	$\bar{4}^1$	$(0,0,T_3)(0,0,-T_3)$
28.	$4/m1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_2,T_1,T_3)(T_2,-T_1,-T_3); (T_2,-T_1,T_3)(-T_2,T_1,-T_3); (-T_1,-T_2,T_3)(T_1,T_2,-T_3); (T_2,-T_1,-T_3)(-T_2,T_1,T_3); (-T_2,T_1,-T_3)(T_2,-T_1,T_3); (T_1,T_2,-T_3)(-T_1,-T_2,T_3)$
29.	$4/m1^1$	$\bar{1}^1$	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_2,T_1,T_3)(T_2,-T_1,-T_3); (-T_1,-T_2,T_3)(T_1,T_2,-T_3);$
30.	$4/m1^1$	2	$(0,0,T_3)(0,0,-T_3);$
31.	$4/m1^1$	m	$(T_1,T_2,0)(-T_1,-T_2,0); (-T_2,T_1,0)(T_2,-T_1,0)$
32.	$4/m1^1$	$4/m^1$	$(0,0,T_3)(0,0,-T_3);$
33.	4221^1	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_2,T_1,T_3)(T_2,-T_1,-T_3); (T_2,-T_1,T_3)(-T_2,T_1,-T_3); (-T_1,-T_2,T_3)(T_1,T_2,-T_3); (T_1,-T_2,-T_3)(-T_1,T_2,T_3);(-T_1,T_2,-T_3)(T_1,-T_2,T_3); (T_2,T_1,-T_3)(-T_2,-T_1,T_3); (-T_2,-T_1,-T_3)(T_2,T_1,T_3)$
34	4221^1	2^1(S)	$(0,T_2,T_3)(0,-T_2,-T_3); (-T_2,0,T_3)(T_2,0,-T_3); (T_2,0,T_3)(-T_2,0,-T_3); (0,-T_2,T_3)(0,T_2,-T_3);$
35.	4221^1	4	$(0,0,T_3)(0,0,-T_3)$
36.	4221^1	42^12^1	$(0,0,T_3)(0,0,-T_3)$
37.	$4mm1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_2,T_1,T_3)(T_2,-T_1,-T_3); (T_2,-T_1,T_3)(-T_2,T_1,-T_3); (-T_1,-T_2,T_3)(T_1,T_2,-T_3);(-T_1,T_2,T_3)(T_1,-T_2,-T_3); (T_1,-T_2,T_3)(-T_1,T_2,-T_3); (-T_2,-T_1,T_3)(T_2,T_1,-T_3); (T_2,T_1,T_3)(-T_2,-T_1,-T_3)$
38.	$4mm1^1$	m	$(0,T_2,T_3)(0,-T_2,-T_3); (-T_2,0,T_3)(T_2,0,-T_3); (T_2,0,T_3)(-T_2,0,-T_3); (0,-T_2,T_3)(0,T_2,-T_3);$
39.	$4mmm1^1$	mm2	$(0,0,T_3)(0,0,-T_3)$
40.	$4mm1^1$	4	$(0,0,T_3)(0,0,-T_3)$

41.	$4mm1^1$	$4mm$	$(0,0,T_3)(0,0,-T_3)$
42.	$\bar{4}2m1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_2,T_1,-T_3)$ $(T_2,-T_1,T_3)$; $(T_2,-T_1,-T_3)$ $(-T_2,T_1,T_3)$; $(-T_1,-T_2,T_3)$ $(T_1,T_2,-T_3)$;$(T_1,-T_2,-T_3)$ $(-T_1,T_2,T_3)$; $(-T_1,T_2,-T_3)(T_1,-T_2,T_3)$; $(-T_2,-T_1,T_3)$ $(T_2,T_1,-T_3)$; (T_2,T_1,T_3) $(-T_2,-T_1,-T_3)$
43.	$\bar{4}2m1^1$	$2^1(P)$	$(T_1,T_2,0)(-T_1,-T_2,0)$; $(-T_2,T_1,0)(T_2,-T_1,0)$; $(T_1,-T_2,0)$ $(-T_1, T_2,0)$ $(-T_2,-T_1,0)$;$(T_2,T_1,0)$
44.	$\bar{4}2m1^1$	$2^12^12(p)$	$(0,0,T_3)(0,0,-T_3)$
45.	$\bar{4}2m1^1$	$mm2$	$(0,0,T_3)(0,0,-T_3)$
46.	$4/mmm1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_2,T_1,T_3)$ $(T_2,-T_1,-T_3)$; $(T_2,-T_1,T_3)$ $(-T_2,T_1,-T_3)$; $(-T_1,-T_2,T_3)(T_1,T_2,-T_3)$;$(T_1,-T_2,-T_3)(-T_1,T_2,T_3)$; $(-T_1,T_2,-T_3)(T_1,-T_2,T_3)$;$(T_2,T_1,-T_3)(-T_2,-T_1,T_3)$;$(-T_2,-T_1,-T_3)(T_2,T_1,T_3)$; $(T_2,-T_1,-T_3)(-T_2,T_1,T_3)$; $(T_1,T_2,-T_3)(-T_1,-T_2,T_3)$; $(-T_1,T_2,T_3)(T_1,-T_2,-T_3)$;$(T_1,-T_2,T_3)(-T_1,T_2,-T_3)$;$(-T_2,-T_1,T_3)(T_2,T_1,-T_3)$; $(T_2,T_1,T_3)(-T_2,-T_1,-T_3)$
47.	$4/mmm1^1$	$2(p)$	$(0,0,T_3)(0,0,-T_3)$
48.	$4/mmm1^1$	$m^1(s)$	$(T_1,0,0)(-T_1,0,0)$;$(0,T_1,0)$ $(0,-T_1,0)$
49.	$4/mmm1^1$	$2^12^12(P)$	$(0,0,T_3)(0,0,-T_3)$
50.	$4/mmm1^1$	4	$(0,0,T_3)(0,0,-T_3)$
51.	$4/mmm1^1$	$4/m^1mm$	$(0,0,T_3)(0,0,-T_3)$
52.	31^1	3	$(0,0,T_3)(0,0,-T_3)$
53.	$\bar{3}\,1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_2,T_1-T_2,T_3)$ $(T_2,-T_1+T_2,-T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(T_1-T_2,T_1,-T_3)$; $(T_2,-T_1+T_2,-T_3)(-T_2,T_1-T_2,T_3)$; $(T_1-T_2,T_1,-T_3)(-T_1+T_2,-T1,T_3)$
54.	$\bar{3}1^1$	3	$(0,0,T_3)(0,0,-T_3)$
55.	$\bar{3}1^1$	$\bar{3}^1$	$(0,0,T_3)(0,0,-T_3)$
56.	321^1	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_2,T_1-T_2,T_3)$ $(T_2,-T_1+T_2,-T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(T_1-T_2,T_1,-T_3)$; $(-T_1+T_2, T_2 ,-T_3)(T_1-T_2,-T_2,T_3)$; $(T_1,T_1-T_2,-T_3)$ $(-T_1,-T_1+T_2,T_3)$; $(-T_2,-T_1,-T_3)$ (T_2,T_1, T_3)
57.	321^1	32^1	$(0,0,T_3)(0,0,-T_3)$
58.	$3m1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(-T_2,T_1-T_2,T_3)$ $(T_2,-T_1+T_2,-T_3)$; $(-T_1+T_2,-T_1,T_3)$ $(T_1-T_2,T_1,-T_3)$; $(T_1-T_2,-T_2,T_3)(-T_1+T_2,T_2,-T_3)$;$(T_2\ T_1,T_3)(-T_2\ ,-T_1,-T_3)$; $(-T_1,-T_1\ +T_2,T_3)(T_1,T_1\ -T_2\ ,-T_3)$
59.	$3m1^1$	3	$(0,0,T_3)(0,0,-T_3)$
60.	$3m1^1$	$3m$	$(0,0,T_3)(0,0,-T_3)$

61.	$\bar{3}m1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_2,T_1-T_2,T_3)\ (T_2,-T_1+T_2,-T_3);$ $(-T_1+T_2,-T_1,T_3)\ (T_1-T_2,T_1,-T_3);\ (-T_1+T_2,T_2,-T_3)(T_1-T_2,-T_2,T_3);$ $(T_1,\ T_1-T_2,-T_3)\ (-T_1,-T_1+T_2,-T_3);\ (-T_2,-T_1,-T_3)(T_2,T_1,T_3);$ $(-T_1,-T_2,-T_3)(T_1,T_2,T_3);(T_2,-T_1+T_2,-T_3)\ (-T_2,T_1-T_2,T_3);$ $(T_1-T_2,T_1,-T_3)\ (-T_1+T_2,-T_1,T_3);(T_1-T_2,-T_2,T_3)(-T_1+T_2,T_2,-T_3);$ $(-T_1,-T_1+T_2,T_3)(T_1,\ T_1-T_2,-T_3);(T_2,T_1,T_3)(-T_2,-T_1,-T_3)$
62.	$\bar{3}m1^1$	m	$(0,0,T_3)(0,0,-T_3)$
63.	$\bar{3}m1^1$	3	$(0,0,T_3)(0,0,-T_3)$
64.	$\bar{3}m1^1$	32^1	$(0,0,T_3)(0,0,-T_3)$
65.	$\bar{3}m1^1$	3m	$(0,0,T_3)(0,0,-T_3)$
66.	61^1	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(T_1-T_2,\ -T_1,T_3)\ (-T_1+T_2,\ -T_1,-T_3);$ $(T_2,-T_1+T_2,T_3)\ (-T_2,T_1,-T_2,-T_3)\ ;\ (-T_2,T_1,-T_2,T_3);\ (T_2,-T_1+T_2,-T_3)$ $(-T_1+T_2,\ -T_1,T_3)(T_1-T_2,\ ,T_1,-T_3);(-T_1,-T_2,T_3);(T_1T_2,\ -T_3)$
67.	61^1	2	$(0,0,T_3)(0,0,-T_3)$
68.	61^1	6	$(0,0,T_3)(0,0,-T_3)$
69.	$\bar{6}1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_1+T_2,-T_1,-T_3)\ (T_1-T_2,T_1,T_3)\ ;$ $(-T_2,T_1-T_2,-T_3)(T_2\ ,\ -T_1+T_2,T_3);(-T_2T_1-T_2,\ T_3)(\ T_2,-T_1+T_2,\ -T_3);$ $(-T_1+T_2,-T_1,T_3)(T_1-T_2,T_1,-T_3);(T_1,T_2,-T_3);(-T_1,-T_2,T_3)$
70.	$\bar{6}1^1$	m	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(-T_1+T_2,-T_1\ ,-T_3)\ (T_1-T_2,T_1,T_3);$ $(-T_2,T_1-T_2,-T_3)\ (T_2\ ,-T_1+T_2,T_3)$
71.	$\bar{6}1^1$	$\bar{6}^1$	$(0,0,T_3)(0,0,-T_3)$
72.	$6/m1^1$	1	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3);(T_1-T_2,T_1,T_3)\ (-T_1+T_2,\ -T_1,-T_3);$ $(-T_2,\ -T_1+T_2,T_3)\ (T_2,T_1-T_2,-T_3);\ (T_2,-T_1+T_2,-T_3)(-T_2,T_1-T_2,T_3);$ $(T_1-T_2,T_2,-T_3)(-T_1+T_2,-T_1,T_3);(T_1-T_2,T_1,-T_3)(-T_1+T_2,-T_1,T_3);$ $(-T_1,-T_2,T_3)(T_1,T_2,-T_3)$
73.	$6/m1^1$	6	$(0,0,T_3)(0,0,-T_3)$
74.	$6/m1^1$	$6/m^1$	$(0,0,T_3)(0,0,-T_3)$
75.	6221^1	3	$(0,0,T_3)(0,0,-T_3)$
76.	6221^1	6	$(0,0,T_3)(0,0,-T_3)$
77.	6221^1	62^12^1	$(0,0,T_3)(0,0,-T_3)$
78.	$6mm1^1$	2	$(0,0,T_3)(0,0,-T_3)$
79.	$6mm1^1$	mm2	$(0,0,T_3)(0,0,-T_3)$
80.	$6mm1^1$	3	$(0,0,T_3)(0,0,-T_3)$

81.	$6mm1^1$	6	$(0,0,T_3)(0,0,-T_3)$
82.	$6mm1^1$	6mm	$(0,0,T_3)(0,0,-T_3)$
83.	$\bar{6}m21^1$	3	$(0,0,T_3)(0,0,-T_3)$
84.	$\bar{6}m21^1$	3m	$(0,0,T_3)(0,0,-T_3)$
85.	$\bar{6}m21^1$	$\bar{6}^1$	$(0,0,T_3)(0,0,-T_3)$
86.	$\bar{6}m21^1$	$\bar{6}^1m2^1$	$(0,0,T_3)(0,0,-T_3)$
87.	$6/mmm1^1$	2(p)	$(0,0,T_3)(0,0,-T_3)$
88.	$6/mmm1^1$	m(p)	$(T_1,T_2,0)(-T_1,-T_2,0)$; $(T1-T_2,T_1,0)(-T1+T_2,-T_1,0)$; $(T_2,-T_1+T_2,0)$ $(-T_2, T_1- T_2, 0)$;$(-T_1+T_2, T_2, 0)$;$(T_1-T_2,-T_2,0)$; $(T_1,T_1-T_2,0)(-T_1,-T_1+T_2,0)$;$(-T_2,-T_1,0)$;$(T_2,T_1,0)$
89.	$6/mmm1^1$	6	$(0,0,T_3)(0,0,-T_3)$
90.	$6/mmm1^1$	$6/m^1$	$(0,0,T_3)(0,0,-T_3)$
91.	$6/mmm1^1$	$6/m^1 mm$	$(0,0,T_3)(0,0,-T_3)$
92.	231^1	2	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,T_3,0)(0,-T_3,0)$
93.	231^1	2^12^12	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,T_3,0)(0,-T_3,0)$
94.	$m31^1$	$\bar{1}^1$	$(T_1,T_2,T_3)(-T_1,-T_2,-T_3)$;$(T_1,-T_2,-T_3)(-T_1,T_2,T_3)$; $(-T_1, T_2,-T_3)(T_1,-T_2, T_3)$;$(-T_1,-T_2, T_3)$ $(T_1, T_2,-T_3)$; $(T_3,T_1,T_2,)(-T_3,-T_1,-T_2)$;$(-T_3,T_1,-T_2,)(T_3, -T_1,T_2)$; $(- T_3,-T_1, T_2,)(T_3, T_1,-T_2)$;$(T_3, -T_1, -T_2)$; $(-T_3,T_1, T_2,)$ $(T_2,T_3,T_1)(-T_2,-T_3,-T_1)$;$(T_2,-T_3,-T_1)$ $(-T_2,T_3,T_1)$;$(-T_2,T_3,-T_1)(T_2,-T_3,T_1)$; $(-T_2,-T_3,T_1)(T_2,T_3,-T_1)$
95.	$m31^1$	2^12^12	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,T_3,0)(0,-T_3,0)$; $(0,0,-T_3)(0,0,T_3)$;$(0,-T_3,0)(0,T_3,0)$; $(-T_3,0,0)$ $(T_3,0,0)$
96.	$m31^1$	mmm^1	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,-T_3,0)(0,T_3,0)$;
97.	4321^1	2(p)	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,T_3,0)(0,-T_3,0)$;
98.	4321^1	$2^1(p)$	$(T_1,T_2,0)(-T_1,-T_2,0)$; $(T_1,-T_2 ,0)(-T_1,T_2,0)$; $(0, T_1,T_2)$ $(0, T_1,-T_2)$; $(T_2,T_1,0)$;$(-T_2,-T_1,0)$; $(0,-T_2,T_1)(0,T_2,-T_1)$; $(-T_1,0,T_2)(T_1,0,-T_2)$; $(-T_1,0,-T_2)(T_1, 0, T_2)$; $(T_2,0,T_1,)$ $(-T_2,0,-T_1)$; $(T2,0,-T1,)$ $(-T_2,0,T_1)$; $(T_2, -T_1,0)(-T_2,-T_1,0)$
99.	4321^1	$2^12^12(pp)$	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,T_3,0)(0,-T_3,0)$;
100.	4321^1	4	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,T_3,0)(0,-T_3,0)$;
101.	$\bar{4}3m1^1$	2	$(0,0,T_3)(0,0,-T_3)$;$(T_3,0,0)(-T_3,0,0)$;$(0,T_3,0)(0,-T_3,0)$;

102.	$\bar{4}3m1^1$	2^12^12	$(0,0,T_3)(0,0,-T_3);(T_3,0,0)(-T_3,0,0);(0,T_3,0)(0,-T_3,0);$
103.	$\bar{4}3m1^1$	$\bar{4}^1$	$(0,0,T_3)(0,0,-T_3);(T_3,0,0)(-T_3,0,0);(0,T_3,0)(0,-T_3,0);$
104.	$\bar{4}3m1^1$	$\bar{4}^12^1m$	$(0,0,T_3)(0,0,-T_3);(T_3,0,0)(-T_3,0,0);(0,T_3,0)(0,-T_3,0);$
105.	$m3m1^1$	$2^1(s)$	$(0,0,T_3)(0,0,-T_3);(T_3,0,0)(-T_3,0,0);(0,T_3,0)(0,-T_3,0)$
106.	$m3m1^1$	m(s)	$(0,0,T_3)(0,0,-T_3);(T_3,0,0)(-T_3,0,0);(0,0,T_3)(0,0,-T_3)$
107.	$m3m1^1$	$2^1/m(p)$	$(T_1,T_2,0)(-T_1,-T_2,0); (T_1,-T_2, 0)(-T_1,T_2,0); (0, T_1,T_2,) (0, -T_1,-T_2,);$ $(0, T_1 ,-T_2)(0,-T_1 , T_2); (T_2,0,T_1,) (-T_2,0,-T_1) ; (T_2,0,-T_1,) (-T_2,0,T_1);$ $(T_2, T_1,0)(-T_2,-T_1,0); (0,-T_2,T_1)(0,T_2,-T_1);(-T_1,0,T_2)(T_1,0,-T_2);$ $(0,-T_2,-T_1)(0,T_2,T_1); (-T_1,0,-T_2)(T_1,0,T_2); (T_2, -T_1,0)(-T_2,T_1,0)$
108.	$m3m1^1$	4	$(0,0,T_3)(0,0,-T_3);(T_3,0,0)(-T_3,0,0);(0,T_3,0)(0,-T_3,0)$
109	$m3m1^1$	4mm	$(0,0,T_3)(0,0,-T_3);(T_3,0,0)(-T_3,0,0);(0,T_3,0)(0,-T_3,0)$
110.	$m3m1^1$	3	$(\frac{T_1+T_2+T_3}{3} ,0,0), (-\frac{T_1+T_2+T_3}{3} ,0,0);$ $(0,\frac{T_1+T_2+T_3}{3} ,0), (0,-\frac{T_1+T_2+T_3}{3} ,0);$ $(0,0,\frac{T_1+T_2+T_3}{3}), (0,0-\frac{T_1+T_2+T_3}{3})$

Similarly, Ferrotoroidic domain pairs for all 324 Ferrotoroidic Ferroic species are calculated using the above procedure, out of which 110 domain pairs were shown in table 2 and the rest of domain pairs are available with the authors.

3. Conclusions:

D.B. Litvin has calculated twin laws of domain pairs for Ferrotoroidic ferroic species of eleven types of physical property tensors. In this paper the ferrotorodic domain pairs are calculated using coset decomposition for all the 324 Ferrotoroidic Ferroic species, taking 32 grey groups as prototypic point groups.

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