(IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 2, Issue 1,Jan-Feb 2012, pp.727-733

DESIGN OF INTEGRATED RELIABILITY REDUNDANCY SYSTEM WITH MULTIPLE CONSTRAINTS – INTEGER PROGRAMMING APPROACH

P.Venkateswarlu[1], B.Dattatraya Sarma[2], Khadarbabu .Sk[3], M.N.Srinivas[4], G.Sankaraiah[5] and C.Uma Shankar[6]

[1] Gokula Krishna College of Engineering, Sullurpeta,,A.P., India

[2] Sri Venkateswara Engineering College, Nellore, A.P., India

[3,4] School of Advanced Sciences, VIT University, Vellore,TN,India

[5] G.Pulla Reddy Engg College, Kurnool,A.P., India

[6] Rayalaseema University ,Kurnool A.P., India

ABSTRACT

In the present scenario of globalization and liberalization, it is imperative that Indian industries become fully conscious of the needs to produce reliable products meeting International standards. Reliability of a system can be maximized by attaching parallel components to each of the components in the system such that if one component fails, one of its parallel components comes into operation and the system does not fail until all parallel units fail. Integrated Reliability Model refers to the determination of the number of components, component Reliabilities, stage Reliabilities and the system Reliability wherein the problem considers both the unknowns that is the component Reliabilities and the number of components in each stage for the given cost constraint to maximize the System Reliability. In this work an attempt is made to develop an integrated reliability model for a series-parallel configuration subject to the cost, weight and volume constraints .The Lagrangian Multiplier method is used to determine the component reliabilities, stage reliabilities, number of components in each stage and the system reliability for a series-parallel configuration**.** Using this method, rounded values cannot be obtained, but number of components should be an integer. To get the integer value, Integer programming has been successfully used.

NOMENCLATURE:

R_s = System Reliability
R_j = Reliability of stage j, $0< R_j < 1$
r_j = Reliability of each component in stage j, $0< rj < 1$
x_j = Number of components in stage j
c_j = Cost coefficient of each component in stage j
w_j = Weight coefficient of each component in stage j
v_j = Volume coefficient of each component in stage j
C_o = Maximum allowable System cost
W_o = Maximum allowable System weight
V_o = Maximum allowable System volume
F = Lagrangian function
$r_{j,mi}$ = Minimum reliability in function
$r_{j,max}$ = Maximum reliability in function
f_j = Feasibility factor of cost function
g_j = Feasibility factor of weight function
h_j = Feasibility factor of volume function

1. INTRODUCTION:

It is a known fact that reliability program increases the initial cost of every device, instrument or system and also it is true that the reliability decreases when the complexity of the system increases [2]. In this type of complex situation, reliability of a product or service is best assured when it is designed by the design engineer and built in by production engineer, rather than conducting externally an experiment by a reliability engineer [1].

Once the product is accepted by the buyer and put into operation, either by itself or as a part of a larger assembly, the quality of performance would be judged by how long the product gives useful service; this is indicated by the word "RELIABILITY" [4].

The objective of the present work is to maximize the reliability of a system subject to cost constraint where the reliability of a system can be maximized in two different ways.

The Mathematical function $c_j = e^{[(1-f_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]}$ [7] considered to establish the proposed Integrated Reliability Model with Redundancy for Series-Parallel Configuration System with multiple Constraints and the analysis is demonstrated with the case study for the mathematical function [3,5]. The navel aspect this work is the mathematical function was applied for series system by Mettas[7], but author in this paper applied series-parallel configuration with multiple constraints.

2. MATHEMATICAL MODEL:

The objective function and the constraints of the model are

Maximize $$R_S = \prod_{j=1}^{n} [1-(1-r_j)^{x_j}]$$

Subjected to the constraints

$$\sum_{j=1}^{n} c_j . x_j \le C_0 \quad (1)$$

$$\sum_{j=1}^{n} w_j . x_j \le W_0 \quad (2)$$

$$\sum_{j=1}^{n} v_j . x_j \le V_0 \quad (3)$$

Integrated Reliability Model (IRM) is one where the component reliabilities (r_j) and the number of components (x_j) in each stage are treated as unknowns. In this type of situation the system reliability (R_s) has to be maximized for the given cost, weight and volume by determining the components reliabilities and the number of components required for each stage [9]..

For a given series – parallel system, the Integrated Reliability Model refers to the determination of the number of components (x_j), component reliabilities (r_j), stage reliabilities (R_j) and the system reliability (R_s) [6] . To determine these parameters, Lagrangian method is applied due to its simplicity and accuracy.

3. PROCEDURE FOR PROBLEM FORMULATION:

In most Reliability Optimization Problems, the decision variables are the number of redundancies that are integers, the component Reliabilities that are real numbers or a combination of both. In the method, which uses differentiation the decision variables have to be continuous. In this study, to formulate the Integrated Reliability Models under the specified mathematical function [8], considered that there are 'n' statistically independent stages in series with x_j statistically independent components in each stage

System Reliability for the given Cost function

$$R_s = \prod_{j=1}^{n} R_j \tag{3.1}$$

Cost coefficient of each component in stage j is derived from the following relationship between cost and reliability

$$c_j = e^{[(1-f_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]} \tag{3.2}$$

Since Cost constraint is linear in x_j

$$\sum_{j=1}^{n} c_j.x_j \le C_o \tag{3.3}$$

Substituting (3.2) in (3.3)

$$\sum_{j-1}^{n} e^{[(1-f_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]}.x_j - C_O \le 0 \tag{3.4}$$

$$\sum_{j-1}^{n} e^{[(1-g_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]}.x_j - W_O \le 0 \tag{3.5}$$

$$\sum_{j-1}^{n} e^{[(1-h_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]}.x_j - V_O \le 0 \tag{3.6}$$

Non - Negativity restriction $x_j \ge 0$

The transformed equations through the relation $x_j = \dfrac{\ln(1-R_j)}{\ln(1-r_j)}$ (3.7)

Are Maximize $R_s = \prod_{j=1}^{n} \left[1-(1-r_j)^{x_j}\right]$

(3.8)

Subject to the constraints

$$\sum_{j=1}^{n}\left[\left(e^{[(1-f_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]}\right).\frac{ln(1-R_j)}{ln(1-r_j)}\right] - C_O \le 0$$

(3.9)

$$\sum_{j=1}^{n}\left[\left(e^{[(1-g_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]}\right).\frac{ln(1-R_j)}{ln(1-r_j)}\right] - W_O \le 0$$

(3.10)

$$\sum_{j=1}^{n}\left[\left(e^{[(1-h_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]}\right).\frac{ln(1-R_j)}{ln(1-r_j)}\right] - V_O \le 0$$

(3.11)

Non - Negativity restriction $x_j \ge 0$

A Lagrangian function is formulated as

$$F = R_S + \lambda_1 \left[\left\{ \sum_{j=1}^{n} \left\{ e^{[(1-f_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]} . \frac{\ln(1-R_j)}{\ln(1-r_j)} \right\} \right\} - C_0 \right] +$$

$$\lambda_2 \left[\left\{ \sum_{j=1}^{n} \left\{ e^{[(1-g_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]} . \frac{\ln(1-R_j)}{\ln(1-r_j)} \right\} \right\} - W_0 \right] + \lambda_3 \left[\left\{ \sum_{j=1}^{n} \left\{ e^{[(1-h_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]} . \frac{\ln(1-R_j)}{\ln(1-r_j)} \right\} \right\} - V_0 \right]$$

(3.12)

The stationary point can be obtained by differentiating the Lagrangian function [10] with respect to $R_j, r_j, \lambda_1, \lambda_2 and \lambda_3$.

$$\frac{\partial F}{\partial r_j} = \lambda_1 \left[\sum_{j=1}^{n} e^{[(1-f_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]} . \ln(1-R_j)/\ln(1-r_j) . \left\{ (1-f_j).(r_{j,\max} - rj, \min)/(r_{j,\max} - r_j)^2 + 1/(\ln(1-r_j)(1-r_j)) \right\} \right]$$

$$-\lambda_2 \left[\left\{ \sum_{j=1}^{n} e^{[(1-g_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]} \right\} . [1/\ln(1-r_j)][1/(1-R_j)] \right] - \lambda_3 \left[\left\{ \sum_{j=1}^{n} e^{[(1-h_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]} \right\} . [1/\ln(1-r_j)][1/(1-R_j)] \right] = 0.$$

(3.13)

$$\frac{\partial F}{\partial \lambda_1} = \left[\left\{ \sum_{j=1}^{n} \left\{ e^{[(1-f_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]} . \frac{ln(1-R_j)}{ln(1-r_j)} \right\} \right\} - C_0 \right] = 0$$

(3.14)

$$\frac{\partial F}{\partial \lambda_2} = \left[\left\{ \sum_{j=1}^{n} \left\{ e^{[(1-g_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]} . \frac{ln(1-R_j)}{ln(1-r_j)} \right\} \right\} - W_0 \right] = 0$$

(3.15)

$$\frac{\partial F}{\partial \lambda_3} = \left[\left\{ \sum_{j=1}^{n} \left\{ e^{[(1-h_j)(r_j - r_{j,min})/(r_{j,max} - r_j)]} . \frac{ln(1-R_j)}{ln(1-r_j)} \right\} \right\} - V_0 \right] = 0$$

(3.16)

4. Case Problem

Consider the case of a system with three stages for which the component reliability is given by the equation: $c_j = e^{[(1-f_j)(r_j - r_{j,\min})/(r_{j,\max} - r_j)]}$.

To determine the optimum component reliability(r_j), stage reliability(R_j),number of components in each stage(x_j) and the system reliability(R_s) not to exceed system cost Rs.1000,weight of the system 1500 kg and volume of the system 2000 cm^3.

4.1 Constants: The necessary data for constants is shown in the Table 1 below.

Table 1 Constants

Stage	f_i	g_i	h_i	$r_{i,min}$	$r_{i,max}$
1	0.9	0.5	0.2	0.5	0.99
2	0.9	0.5	0.2	0.5	0.99
3	0.9	0.5	0.2	0.5	0.99

The component reliabilities, stage reliabilities, number of components in each stage and the system reliability are determined and are presented in the following tables.

4.2 Cost Constraint Details (Without X_j Rounding Off):
Reliability design relating to cost is shown in the following Table 2

Table 2 Cost Constraint Details (Without X_j Rounding Off)

Stage	r_j	R_j	x_j	c_{jX100}	$c_j.\ x_j$
01	0.701	0.9399	2.33	110.27	256.93
02	0.70	0.9518	2.52	110.00	277.2
03	0.80	0.9886	2.78	110.51	307.21
Total Cost					841.34

4.3 Weight Constraint Details (Without X_j Rounding Off):
The relevant results relating to weight are shown in the following Table 3

Table 3 Weight Constraint Details (Without X_j Rounding Off)

Stag	r_j	R_j	X_j	W_{jx100}	$W_j.\ X_j$
01	0.701	0.9399	2.33	163.08	379.97
02	0.70	0.9518	2.52	161.6	407.23
03	0.80	0.9886	2.78	164.8	458.14
Total Weight					1245.34

4.4 Volume Constraint Details (Without X_j Rounding Off):
The relevant results relating to volume are shown in the following Table 4

Table 4 Volume Constraint Details (Without X_j Rounding Off)

Stage	r_j	R_j	X_j	V_{jx100}	$V_j\ .X_j.$
01	0.701	0.9399	2.33	218.7	509.57
02	0.70	0.9518	2.52	215.5	543.06
03	0.80	0.9886	2.78	222.6	618.83
Total Volume					1671.46

System reliability $=R_s=0.8844$

5 Reliability Design with x_j Rounding Off:

The reliability design is reestablished by considering the values of x_j to be integers(by rounding off the value of x_j to the nearest integer) and the relevant results relating to cost, weight and volume are presented in the following table, further giving the information by calculating the variation due to cost, weight, volume and system reliability (before and after rounding off x_j.)

5.1 Reliability Design Relating to Cost with Rounding Off:
The relevant results relating to cost are shown in the following Table 5

Table 5 Reliability Design Relating to Cost with Rounding Off

Stage	r_j	R_j	x_j	c_{jx100}	$c_j.\ x_j$
01	0.701	0.9732	3	110.27	330.31
02	0.70	0.9730	3	110.0	330.00
03	0.80	0.9920	3	110.51	331.53
Total Cost					991.84

Variation in total cost = 18.82%

5.2 Reliability Design Relating to Weight with Rounding Off:

The relevant results relating to weight are shown in the following Table 6

Table 6 Reliability Design Relating to Weight with Rounding Off

Stage	r_j	R_j	X_j	W_{jx100}	$W_{j.}\ X_j$
01	0.701	0.9732	3	163.08	489.24
02	0.70	0.9730	3	161.6	484.8
03	0.80	0.9920	3	164.8	494.40
Total Weight					1468.44

Variation in total weight = 18.83%

5.3 Reliability Design Relating to Volume with Rounding Off:

The relevant results relating to volume are shown in the following Table 7

Table 7 Reliability Design Relating to Volume with Rounding Off

Stage	r_j	R_j	X_j	V_{jx100}	$V_j\ .X_{j.}$
01	0.701	0.9732	3	218.7	656.1
02	0.70	0.9730	3	215.5	646.5
03	0.80	0.9920	3	222.6	667.8
Total Volume					1970.4

Variation in total volume = 18.79%
System reliability = 0.9394
Variation In system reliability = 06.22%

6 Integer Programming solution:

The integer programming for the function under consideration is applied to establish and to optimize an integrated reliability model for redundant systems with multiple constraints.This approach is useful in optimizing the design with the values of x_j's to be integers, which are highly useful for practical implementation to real life problems.

6.1 Reliability Design Relating to Cost:

Reliability design relating to cost is shown in the following Table 8

Table 8 Reliability Design Relating to Cost

Stage	r_j	R_j	x_j	c_{jx100}	$c_{j.}\ x_j$
01	0.701	0.9732	3	110.27	330.31
02	0.70	0.9730	3	110.0	330.00
03	0.80	0.9600	2	110.51	221.02
Total Cost					881.33

Variation in total cost = 04.75%

6.2 Reliability Design Relating to Weight:

Reliability design relating to weight is shown in the following Table.9

Table 9 Reliability Design Relating to Weight

STAGE	r_j	R_j	X_j	W_{jx100}	$W_{j.}\ X_j$

01	0.701	0.9732	3	163.08	489.24
02	0.70	0.9730	3	161.6	484.8
03	0.80	0.9600	2	164.8	329.60
TOTAL WEIGHT					1303.64

Variation in total weight = 04.68%

6.3 Reliability Design Relating to Volume:

Reliability design relating to volume is shown in the following Table.10

Table 10 Reliability Design Relating to Volume

Stage	r_j	R_j	X_j	V_{jx100}	$V_j .X_{j.}$
01	0.701	0.9732	3	218.7	656.1
02	0.70	0.9730	3	215.5	646.5
03	0.80	0.9600	2	222.6	445.2
Total Volume					1747.8

System Reliability (R_s) = 0.9091
Variation in total volume = 04.57%
Variation in system reliability = 02.79%

7. CONCLUSIONS

An Integrated Reliability Model (IRM) for a series-parallel redundant system with multiple constraints for the Mathematical function is established while optimizing the system reliability. The IRM refers to determination of number of components (x_j), Component Reliabilities (r_j), Stage Reliability (R_j) and the System Reliability (R_s). These values are calculated using Lagrangian Multiplier Approach where the values are found to be real values which may be infeasible for practical application. To overcome this flaw the Integer programming method is applied to derive an integer solution by treating the inputs from the Lagrangian method. The model is demonstrated through a case problem which infers the conventional analogy that the system reliability increases as increase in cost of the system. When the value of number of components (x_j) is rounded off for want of integer solution, a customary increase in cost is found but the approach of rounding off the values leads to variation in cost and this is overcome by application of a scientific integer programming method where the values of (x_j) are derived in the form of integers. The proposed model is pretty useful for the reliability design engineer to produce quality and reliable goods particularly when the cost of the system is very low.

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