Modelling and analysis of single expansion chamber using Response Surface Methodology

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Abstract
The present paper discusses development of a second order model for predicting sound transmission losses of the single expansion chamber (Muffler). Firstly, Transfer Matrix Method (TMM) is used to predict the sound transmission losses of expansion chamber. Then, accuracy of the TMM is validated against the experimental results. Finally, the second order model is developed using response surface method (RSM). Also, fractional factorial design (FFD) is carried out to investigate the effect of the expansion chamber dimensions on the sound transmission losses at desired pure tone 500 Hz.
Analysis of the RSM showed that Muffler diameter, Muffler length and Pipe diameter are significant parameters. In addition their square and interaction terms are enough to obtain the response variable (transmission losses). The study also found that the predictive model development by RSM close to those predict by TMM.

1-Introduction
The expansion chamber is one of the more significant tools which are used to attenuate the noise emitted from the exhaust system of the vehicle engine. The first attempted to modeling the simple expansion chamber was reported by Davies et al (1954). They used transmission line theory by assuming both continuity of pressure and volume velocity at discontinuities. Igarashi and his co. workers (Igarashi and Toyama 1958 , Miwa and Igarashi 1959 and Igarashi and Arai 1960) in a series of reports determined the transmission characteristic of expansion chamber and resonator. They used an electrical analogy to 4-pole parameters and determine the transmission losses of muffler. Then, in the early seventies, other design techniques gradually evolved for one dimensional analysis of muffler. Alfredson and Davies (1971) developed equation from an energy balance point of view of acoustic pressure in simple area expansion and determine the attenuation in such silencer. This is one of the first worker to consider mean flow in silencer [1,2].
The cascading property makes the 4-pole approach more convenient in modeling mechanical systems, because it allows formulating different models independently and combining them by simply multiplying their 4-pole matrices. Since, the 4-pole method formulates the system equation in two terminal variables, typically the pressure and volume flow in acoustics, it has been applied to systems composed of acoustic elements. Therefore, applications are mainly found in duct acoustics .
The performance of an acoustic muffler is measured in term of one of the following parameters, insertion losses, transmission losses and noise reduction. Each performance parameters have advantages and disadvantages. For instance, the transmission losses can be used to describe muffler in terms of the 4- pole parameter and does not involve the source characteristic. This facilitates the theoretical prediction of muffler performance. The noise reduction is dependent not only on the muffler properties but also on the exhaust system radiation impedance. Insertion losses depend on the coupling of the source and radiation elements with the muffler. Predict of the insertion losses become difficult because of the need to evaluate the source and radiation element characteristic[8,11].
From the previous data it was observed that the insertion losses are more useful to the user but it’s difficult to predict. Noise reduction does not give solely a specific description of the muffler as compared to the transmission losses. Transmission losses is easy to predict the acoustic transmission behavior of one element or more element [4,10].
As an important subject in the statistical design of experiment, the response surface methodology is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest in influenced by several variables and objective is to optimize this response [3].
The response surface methodology is practical, economical and relatively easy for use and it has been used by a lot of researchers. An experiment is a series of tests, called runs, in which changes are made in the input variable ($x_1, x_2,\ldots x_k$) to identify the reasons for changes in the output response (y). If all of these variables are assumed to be measurable, the response surface can be expressed as [3]:

$$Y=f\,(x_1, x_2,\ldots x_k). \qquad (1.1)$$

In this paper, the RSM is carried out to model and analysis the effect of the single expansion chamber dimensions on the sound transmission losses and comparison the results by TMM.

2- Mathematical models
For plan wave propagation in the rigid walled tube with sufficiently dimensions [tube length (L) and tube cross section (D)] filled with in viscous fluid, the small amplitude waves travel as plane waves. The acoustic pressure P and particle

velocity V at all points of a cross section are the same. By other word, the acoustic pressure P and particle velocity V any where in the tube element can be expressed as the sum of left and right traveling wave [1,4,9].
Consider the sound transmission in the tube of acoustic medium of finite length L as shown in Fig. (1), for the input side is at x=0 and output side is at x=L, the acoustic pressure and particle velocity can be related by:

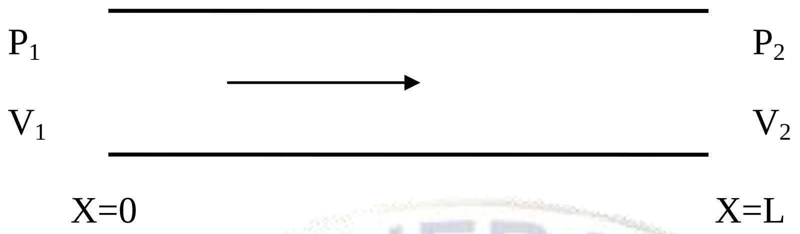

Figure (1)

$P_1=AP_2 + BV_2$ (2.1)
$V_1=CP_2 + DV_2$ (2.2)

Equations 1 and 2 can be put in a matrix form as following

$$\begin{bmatrix} P_1 \\ V_1 \end{bmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{bmatrix} P_2 \\ V_2 \end{bmatrix} \qquad (2.3)$$

Where, A, B, C and D are called 4-pole parameter and equation (2.3) called 4-pole parameter equation or the transfer matrix equation.
According to Munjal [ ], the 4- pole parameter are:

$A = \exp(-jM\, Km\, L)^* \cos(Km\, L)$ (2.4)
$B = \exp(-jM\, Km\, L)^* jz \sin(Km\, L)$ (2.5)
$C = \exp(-jM\, Km\, L)^* j(1/z) \sin(Km\, L)$ (2.6)
$D = \exp(-jM\, Km\, L)^* \cos(Km\, L)$ (2.7)

Where, $Km = k/(1- M2)$ is the convective wave number, M is the mean flow Mach number, $k = \omega/c$ is the acoustic wave number, ω is the angular frequency , c is the speed of sound ,$z = \rho c/S$ is acoustic impedance, S is the area of the pipe, ρ is the fluid density and j is the square root of (-1).
Equation (2.3) relates two input variables and two output variables, the particle velocity and acoustic pressure. In actual use, the simple expansion chamber consists of different elements (tubes) connecting together as shown in figure (2).

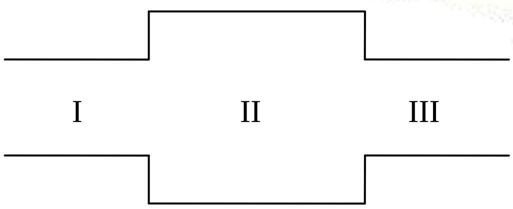

Figure 2.Outline of one expansion chamber

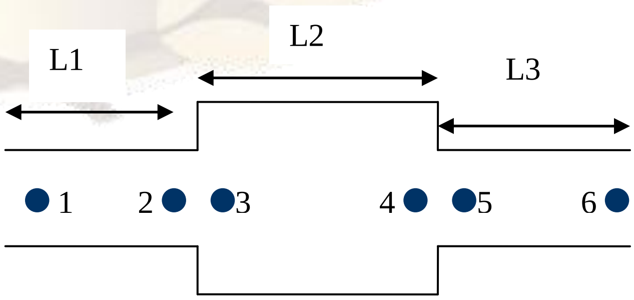

Figure 3. The acoustical nodes inside the acoustical field

To determine the sound transmission losses for the expansion chamber shown in figure (2) the expansion chamber is divided into three elements I, II, III. In case of applying the equation (2.3) for each element, the transfer matrix for each element can be expressed as the following.
As indicated in figure (3), on basis of the plane wave theory, the 4- pole matrix between point 1 and 2 for element I is

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = e^{-j\ M_1\ Km_1\ L_1} \begin{bmatrix} TI_{11} & TI_{12} \\ TI_{21} & TI_{22} \end{bmatrix} \begin{pmatrix} P_2 \\ U_2 \end{pmatrix} \qquad (2.8)$$

Where;
TI11= cos (Km1 L1) ; TI12= jZsin (Km1 L1) ;
TI21= j(1/Z)sin (Km1 L1) ; TI22 = cos (Km1 L1) ;
Similarly, the four pole matrix between point 3 and 4 for element II is

$$\begin{pmatrix} P_3 \\ U_3 \end{pmatrix} = e^{-j\ M_2\ Km_2\ L2} \begin{bmatrix} TII_{11} & TII_{12} \\ TII_{21} & TII_{22} \end{bmatrix} \begin{pmatrix} P_4 \\ U_4 \end{pmatrix} \qquad (2.9)$$

Where;
T II 11= cos (Km2 L2) ; T II 12= jZsin (Km2 L2) ;
T II 21= j(1/Z)sin (Km2 L2) ; T II 22 = cos (Km2 L2) ;

Similarly, the four pole matrix between point 5 and 6 for element III is

$$\begin{pmatrix} P_5 \\ U_5 \end{pmatrix} = e^{-j\ M_3\ Km_3\ L3} \begin{bmatrix} TIII_{11} & TIII_{12} \\ TIII_{21} & TIII_{22} \end{bmatrix} \begin{pmatrix} P_6 \\ U_6 \end{pmatrix} \qquad (2.10)$$

Where;
T III 11= cos (Km3 L3) ; T III 12= jZsin (Km3 L3) ;
T III 21= j(1/Z)sin (Km3 L3) ; T III 22 = cos (Km3 L3) ;

The total transfer matrix is assembled by multiplication of equations (2.8) , (2.9) and (2.10)

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = e^{-j\ [M_1 Km_1 L1\ +\ M_2\ Km_2\ L2\ +\ M_3 Km_3\ L3]} \left[T\mathrm{I}\right]\left[T\mathrm{II}\right]\left[T\ \mathrm{III}\right] \begin{pmatrix} P_6 \\ U_6 \end{pmatrix} \qquad (2.11)$$

Equation (2.11) can be rewrite as

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} P_6 \\ U_6 \end{pmatrix} \qquad (2.12)$$

Transmission losses can be calculated in term of 4- pole parameter as [4,7]

$$TL = 20\log\left[\frac{1+M_{in}}{1+M_{out}}\sqrt{\frac{Z_{in}}{4Z_{out}}}\left|T_{11} + \frac{T_{12}}{Z_{in}} + Z_{out}\,T_{21} + \frac{Z_{out}}{Z_{in}}T_{22}\right|\right] \qquad (2.13)$$

Where, M_{in} , M_{out} and Z_{in} , Z_{out} are the Mach number and acoustic impedance at in and out of the tubes.

3- Design of experiment

A commercial statistical analysis software "Minitab" has been employed for design of experimental. Response surface methodology is used to find a combination of factors which gives the optimal response. Response surface methodology is actually a collection of mathematical and statistical technique that is used for molding and analysis of problems in which a response of interest is influence by several variables and the objective is to optimize the response.

There are two methods of design of experiments which are based on response surface analysis, Central Composite Design (CCD) and Box-Bahnken Design (BBD). Both of these methodologies require a quadratic relationship between experimental factor and the responses [12].

In this study, A Box-Bahnken Design of experiments approach is chosen. The Box-Bahnken Design requires three factors and employs fewer data point than the Central Composite Design. Another important feature of the Box-Bahnken Design is that it has points at the vertices of the cube as defined by the ranges of factors [3]. Box-Bahnken Design also ensures that all the factors are never set their high level simultaneously.

The equation of the second order can be usually expressed as

$$Y = bo + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 + \sum_{i=1}^{k} \sum_{j=2}^{k} b_{ij} X_i X_j \qquad (3.1)$$

Where (Y) is the response variable, Xi are the input variables and β are the coefficients to be estimated. The idea β sound as long as a second order model can sufficiently estimate the design code.

From the equation (3.1), it can be noted that the second order equation includes terms square and interaction terms.

The above equation can be also rewrite in matrix notation.

$$Y = b_0 + X^t b + X^t B X \qquad (3.2)$$

Where

$$b_0 = (b_0),\ X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ \dots \\ x_k \end{bmatrix},\ \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \dots \\ \dots \\ \beta_k \end{bmatrix},\ B = \begin{bmatrix} \beta_{11} & \frac{\beta_{12}}{2} & \dots & \frac{\beta_{1k}}{2} \\ \frac{\beta_{12}}{2} & \beta_{22} & \dots & \frac{\beta_{2k}}{2} \\ \dots & \dots & \dots & \dots \\ \frac{\beta_{1k}}{2} & \frac{\beta_{2k}}{2} & \dots & \beta_{kk} \end{bmatrix} \qquad (3.3)$$

The second order model is flexible, because it can take a variety of functional form and approximates the response surface locally. Therefore, this model is usually a good estimation of true response surface.

4- Case study

To study and modeling of the effect of the expansion chamber dimensions, three kind of design parameters for example: expansion chamber length (L2), expansion chamber diameter (D2) and inlet pipe (D1) are chosen as the tuned variables. Table (2) illustrated the levels of variables and coding identifications used in this case [5]. Also a 500 Hz is chosen as the target frequency during the study. Table (3) shows the necessary order and combinations of the expansion chamber dimensions and results obtained using TMM.

Before using the design of experiment, an accuracy check of the mathematical model of simple expansion chamber shown in figures (4) is performed with experimental data from Kim et al (1990). The accuracy comparison between the theoretical transfer matrix method and experimental data for the model shows a good agreement. Therefore, the proposed mathematical model is acceptable [6].

code	1	0	-1
Muffler diameter (m)	0.1404	0.108	0.0756
Muffler length (m)	0.2704	0.208	0.1456
Pipe diameter (m)	0.04745	0.0365	0.02555

Table (2). Variables coding identifications

Run Order	Muffler diameter (m)	Muffler length (m)	Pipe diameter (m)	STL
1	0	0	0	12.4967
2	-1	-1	0	6.6791
3	0	0	0	12.4967
4	0	1	-1	15.0765
5	0	1	1	5.372
6	1	1	0	13.4876
7	-1	0	1	3.0811
8	-1	1	0	4.1794
9	1	0	1	12.4967
10	1	0	-1	23.1246
11	0	-1	-1	18.7898
12	0	-1	1	8.3604
13	1	-1	0	17.1654
14	-1	0	-1	12.4967

Table (3). Expansion chamber dimensions and results obtained using TMM.

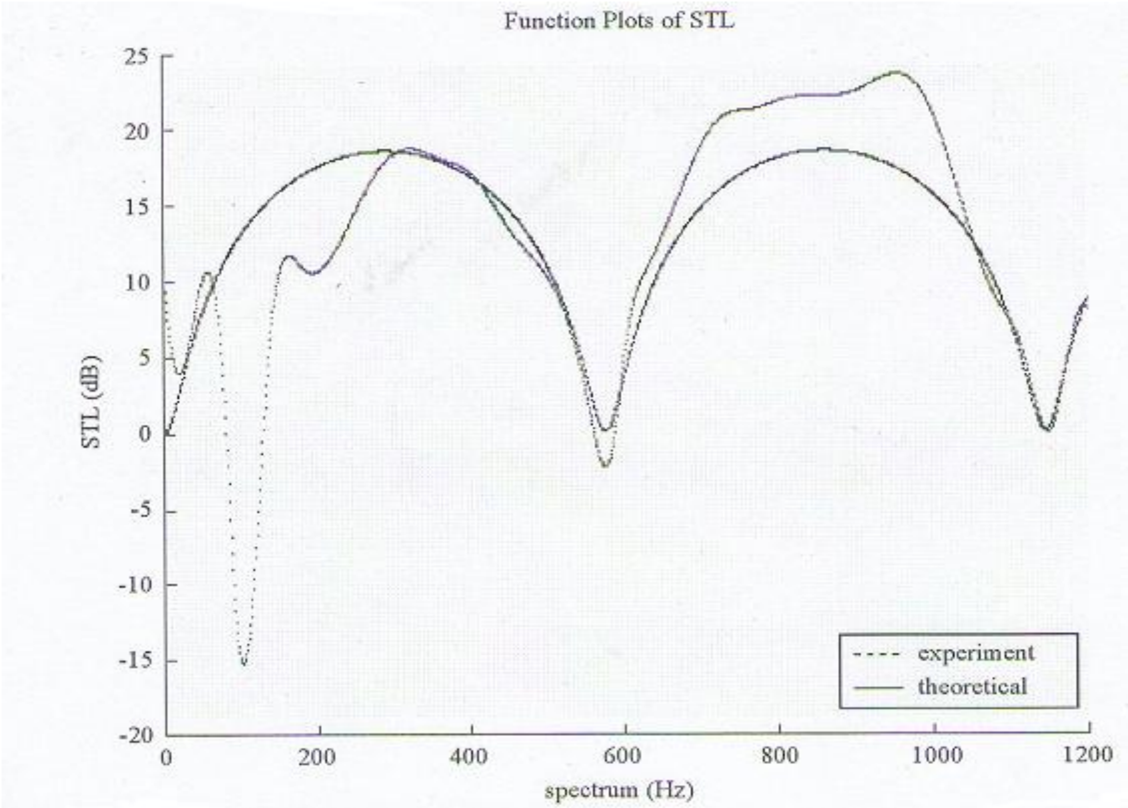

Figure 4. performance of single expansion chamber without mean flow
[D1=D3 = 0.0365 (m) , D2 = 0.15 (m) , L1=L3=0.1 (m)
and L2= 0.3 (m)][experimental data are from Kim et al (1990)].

5- Results and analysis

The analysis of second order model is usually done using computer software. Investigation of adequacy of second order model for sound transmission losses are presented at 500 Hz. Minitab software is applied to analysis all the three factors with their high and low level. The estimated regression coefficients for sound transmission losses are shown in table (4).

Term	Coeff	SE Coeff	T	P
Constant	12.4967	0.07342	170.217	0.000
A	4.9798	0.03671	135.658	0.000
B	-1.6099	0.03671	-43.857	0.000
C	-5.0222	0.03671	-136.813	0.000
AA	-0.6094	0.05804	-10.499	0.000
BB	-1.5095	0.05804	-26.007	0.000
CC	0.9124	0.05804	15.721	0.000
AB	-0.2945	0.05191	-5.673	0.005
AC	-0.3031	0.05191	-5.838	0.004
BC	0.1812	0.05191	-53.491	0.025

Table (4) Estimate regression coefficients for STL

The small p-value (less than 0.05) indicates that the corresponding model terms are sufficient. From the table (1), it is clear that the linear terms A, B and C and square terms A^2, B^2 and C^2 and interaction terms AB and AC are sufficient model terms to the response variable (STL).

The adequacy of the second order model is also verified using the analysis of variance (ANOVA). The ANOVA uses F – distribution method .This method employs the regression approach with adjusted mean sum of squares (Adj MS), the sum square (Seq SS) and the adjusted sum of squares after removing insignificant terms from the model (Adj SS). At a level of confidence of 95%, the model is checked for adequacy and results are presented in table (5)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	9	434.839	434.839	48.315	4481.97	0.00
Linear	3	420.955	420.955	140.298	13014.75	0.00
Square	3	13.098	13.098	4.366	405	0.00
Interaction	3	0.846	0.846	0.282	26.15	0.004
Residual Error	4	0.043	0.043	0.011		
Lack-of-Fit	3	0.043	0.043	0.014		
Pure Error	1	0.000	0.000	0.000		
Total	13	434.882				

Table (5) , Analysis of Variance for STL

The model is adequate as the p= values of the lack of fit are not significant. But the small p – value for the interaction and the square terms implies that the model could fit and it is adequate.
Figure (5) shows the results verified by the pareto chart, which displays the interactions in terms of the significance. It is clear that the terms A and C are most important factors affecting the sound transmission losses followed by the term B then the interaction between the different factors.

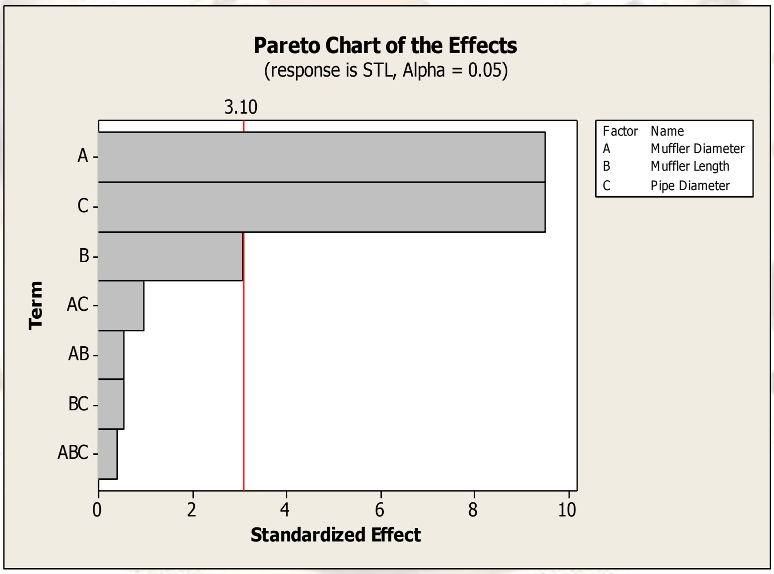

Figure (5) Pareto Charts; Significant Interactions at a 0.05 Significance level

The second order equation for predicting the sound transmission losses is expressed as:

$$Y=[(-0.0262444)+(340.561)*A+(141.520)*B-(977.071)*C-(580.478)*A^2-(387.662)*B^2+(7609.83)*C^2-(145.678)*AB-(854.262)*AC+(265.228)*BC]$$

6-Model check

Table (6) shows the sound transmission losses values predicated by transfer matrix method and the respective values predicated by the response surface method. It is clear that the predicated values by response surface method are good agreement with the predicated by transfer matrix method. This indicates that the obtained model is useful to predicted values of sound transmission losses as shown in figure (6).

Test runs	Muffler diameter (m)	Muffler length (m)	Pipe diameter (m)	STL by TMM (dB)	STL by RSM (dB)	Deviation (%)
1	0.0756	0.1456	0.02555	12.5263	12.6895	-1.30286
2	0.0756	0.1768	0.031025	9.8209	9.6885	1.348145
3	0.0918	0.1456	0.02555	15.772	15.9944	-1.41009
4	0.0918	0.1768	0.031025	12.9172	12.9306	-0.10374
5	0.0918	0.208	0.0365	9.8545	9.7968	0.585519
6	0.1088	0.2392	0.0365	11.4333	11.4453	-0.10496
7	0.1242	0.208	0.041975	12.4755	12.4967	-0.16993
8	0.1404	0.2704	0.04745	9.2216	9.2113	0.111694
9	0.0756	0.1768	0.031025	9.8209	9.6885	1.348145
10	0.1088	0.1768	0.02555	19.0832	19.1684	-0.44647
11	0.0918	0.2704	0.041975	4.7658	4.8522	-1.81292
12	0.1404	0.1768	0.02555	23.7702	23.581	0.795955
13	0.1088	0.208	0.031025	15.3622	15.3807	-0.12043
14	0.1242	0.1768	0.0365	15.3355	15.3103	0.164325
15	0.1242	0.2392	0.031025	16.348	16.4221	-0.45327
16	0.1404	0.2392	0.02555	21.6846	21.8773	-0.88865
17	0.1404	0.208	0.041975	14.4326	14.5735	-0.97626
18	0.1088	0.208	0.0365	12.6193	12.6209	-0.01268
19	0.1242	0.208	0.0365	14.8342	14.866	-0.21437
20	0.1404	0.1456	0.02555	23.681	23.3277	1.491913
21	0.1404	0.2704	0.02555	19.5097	19.5631	-0.27371
22	0.1242	0.2392	0.04745	9.4076	9.3271	0.855691
23	0.1088	0.1768	0.04745	8.8427	8.6977	1.639771
24	0.1404	0.208	0.04745	12.4543	12.4967	-0.34044

Table (6) comparison of the STL between the TMM and RSM

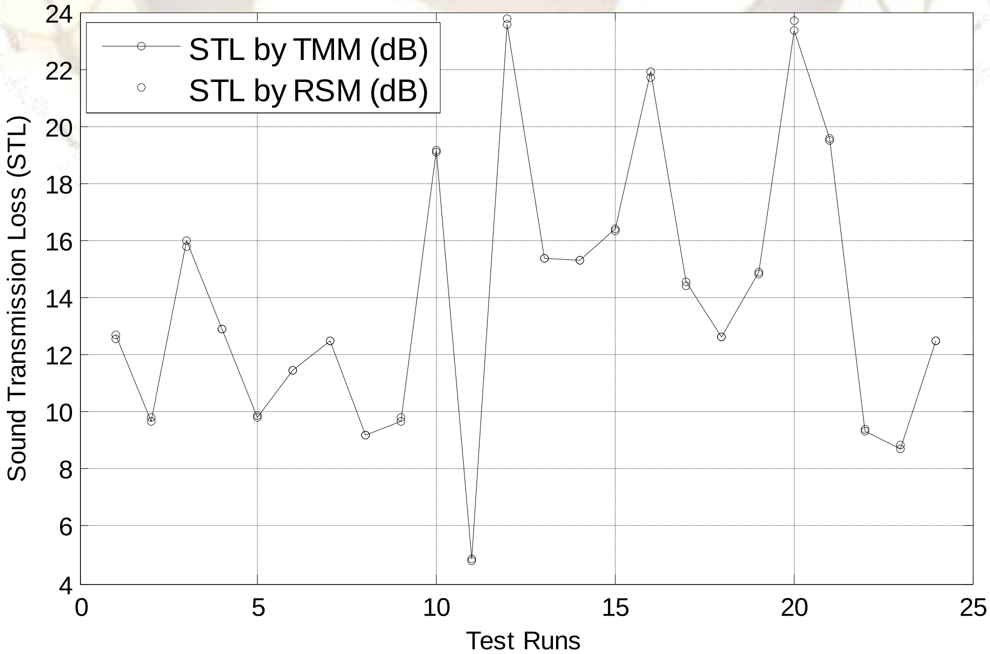

Figure (6) prediction sound transmission losses by TMM and RSM

7- Conclusion

In the present research, the second order model based on response surface method is developed to predict the sound transmission losses for single expansion chamber at certain frequency. The obtained results by RSM are compared with those predict by transfer matrix method. Comparison between two methods showed that the deviations are found to be within + 1.6397 % and -1.8129 %. It shows good agreement and also the adequacy of the developed model in predict.

In conclusion, the developed model is sufficient to predict the sound transmission losses for single expansion chamber without the need for complex calculations of the mathematical model.

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