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# Satellites: Linear Oscillation of the system (Equilibrium for small eccentricity)

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**ABSTRACT**
This paper deals with the linear oscillation of the system about the positions of equilibrium for small eccentricity. We will try to find the condition of equilibrium position.

**Key words:** Eccentricity; Oscillation; Equilibrium

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## I. INTRODUCTION

The effect of Earth's oblateness and magnetic force on the motion of a system of two artificial satellites connected by light, flexible and in extensible string. The physical properties of the celestial bodies is generally faced with two types of problems namely gravity gradient stabilization and altitude stabilization of the satellites. Gravity gradient stabilization means that the portion carrying the instrument in the satellites is always pointed towards the surface of the earth. This resulted in the formulation of the problem of the passive altitude stabilization of the satellites in the orbit. We come to know that,

$$\rho = \frac{1}{1+e\cos v}$$

Where
$e =$ Eccentricity of the orbit
$v =$ True anomaly of the centre of mass

By determining the motion of the other satellite, we apply the identity as

$$m_1\overrightarrow{\rho_1} + m_2\overrightarrow{\rho_2} = 0$$

Where

$\overrightarrow{\rho_1}, \overrightarrow{\rho_2} =$ Radius vector of $m_1, m_2$

There are three types of motions are given by
(i) Free motion
(ii) Constrained motion
(iii) Evolutional motion
(Combination of free and constrained motion)

Mathematical Approach
In the case of constrained motion
We apply

$$x^2 + y^2 = \frac{1}{\rho^2} \quad \ldots\ldots\ldots \quad (1)$$

We transform the polar form by replacing

$$\left.\begin{aligned} x &= (1+\cos v)\cos\psi \\ y &= (1+e\cos v)\sin\psi \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (2)$$

Diff [H]- (2) w.r. to v we obtain

$$x^1 = -\frac{\psi^1 \sin\psi}{\rho} - e\cos\psi.\sin v$$

$$x^{11} = -\frac{\psi^{11}\sin\psi}{\rho} - \frac{\psi^{1^2}\cos\psi}{\rho} + 2e\psi^1\sin\psi\sin v - e\cos\psi.\sin v$$

**II**$_y$ $$y^1 = \frac{\psi^1\cos\psi}{\rho} - e\sin\psi.\sin v$$

$$y^{11} = \frac{\psi^{11}\cos\psi}{\rho} - \frac{\psi^{1^2}\sin\psi}{\rho} - 2e\psi^1\cos\psi\sin v - e\sin\psi.\cos v$$

$$\left.\begin{aligned} &\rho^1 = \rho^2 e\sin v \rho = \frac{1}{1+e\cos v} \\ &i.e. e\sin v = \frac{\rho^1}{\rho^2} \end{aligned}\right\} \quad \ldots\ldots\ldots (3)$$

We have the system; when centre of mass moves along keplerian elliptical orbit in Nechville's co-ordinate then

$$\left.\begin{aligned} x^{11}-2y^{1}-3x\rho &= \lambda \propto^{x}-\frac{4A_{0x}}{\rho}-\frac{A\cos i}{\rho} \\ y^{11}+2x^{1} &= \lambda \propto^{y}+\frac{A_{0y}}{\rho}-\frac{A\cos ie^{1}}{\rho^{2}} \end{aligned}\right\} \qquad \text{.........} (4)$$

Replacing the value of $\rho, \rho^{1}$ *and* $x, y$ in (4) we have

$$-\lambda \propto \frac{1}{\rho}\cos\psi-\frac{4A_0\cos\psi}{\rho^2}-\frac{A\cos i}{\rho} \qquad \text{.........} (5)$$

$$\frac{\psi^{11}\cos\psi}{\rho}-\frac{\psi^{2}\sin\psi}{\rho}-2e\psi^{2}\cos\psi\sin v-e\sin\psi\cos v$$

$$-\frac{2\psi^{1}\sin\psi}{\rho}-2e\cos\psi\sin v=\lambda \propto \frac{1}{\rho\sin\psi}+\frac{A_0\sin\psi}{\rho^2}$$

$$-A\cos i.e\sin v \qquad \text{.........} (6)$$

Multiplying (5) by $\sin\psi$ and (6) by $\cos\psi$ and the subtracting first from the second

$$(1+e\cos v)\psi^{11}-2e\psi^{1}\sin v-2e\sin v+3\sin\psi\cos\psi$$

$$=5A_0(1+e\cos v)^2\sin\psi\cos\psi+A\cos i[(1+e\cos v)\sin\psi-e\sin v\cos\psi] \qquad \text{.........} (7)$$

Again (5) is multiply by $\cos\psi$ and (6) by $\sin\psi$ and adding we get

$$A_0(1+e\cos v)^2(4\cos^2\psi-\sin^2\psi)+\frac{A\cos i}{\rho}(\cos\psi+e\rho\sin v.\sin\psi)-\frac{\lambda\propto}{\rho} \qquad \text{.........} (8)$$

This equation determines undetermined Lagrange's multiplier.
The motion will be constrained as long as
$\lambda(t) >,0$ i.e $\lambda \propto (t) >,0$
It means the particle will start moving with in the circle of variable radius

$$x^2+y^2=\frac{1}{\rho^2}$$

Now, the equation of motion of the system is given by
$5A_0(1+e\cos v)^2\sin\psi\cos\psi+A\cos i[(1+e\cos v)\sin\psi-e\sin v.\cos\psi]$
……… (9)

This is a second order differential equation with periodic term from equation (9) eccentricity is very small that implies $e=0$ and there exists stable positions of equilibrium for equatorial orbit $(i=0)$ given by

(i) $\varphi_0=0$ , $\sin\psi_0=\dfrac{A}{(3-5A_0)}$

(ii) $\varphi_0=0$ , $\psi_0=0$

We focus on first case as the oscillation of the system about the stable position of equilibrium.

$\varphi_0=0$ ; $\sin\psi_0=\dfrac{A}{3-5A_0}$

e = to be taken as a small parameter

Replacing

$\psi=\psi_0+\partial$

$\psi^1=\partial^1$

$\psi^{11}=\partial^{11}$

$\sin\psi=\sin(\psi_0+\partial)=\sin\psi_0.\cos\delta+\cos\psi_0\sin\delta$

$$=\frac{A}{3-5A_0}+\delta\sqrt{1-\frac{A^2}{(3-5A_0)^2}} \qquad \text{.........} (10)$$

$\text{II}_y$

$\cos\psi=\cos(\psi_0+\delta)=\cos\psi_0\cos\delta-\sin\psi_0.\sin\delta$

$$=\sqrt{1-\frac{A^2}{(3-5A_0)^2}}-\delta\frac{A}{(3-5A_0)} \qquad \text{.........} (11)$$

There fore we observe that linear string the equation of motion w.r. to $\delta$ and $\delta^1$ in case of equatorial orbit $(i=0)$ we have

$$(1+e\cos v)\delta^{11}-2e\delta^{1}\sin v-2e\sin v+3\delta\left[1-\frac{2A^2}{(3-5A_0)^2}\right]+\frac{3A}{(3-5A_0)}\sqrt{1-\frac{A^2}{(3-5A_0)^2}} \qquad \text{......} (12)$$

$$=2e\sin v-\frac{3A}{(3-5A_0)}\sqrt{1-\frac{A^2}{(3-5A_0)^2}}$$

$$-Ae\sin v\left[\sqrt{1-\frac{A^2}{(3-5A_0)^2}}-\delta\frac{A}{(3-5A_0)}\right] \qquad \text{.........} (13)$$

Suppose

$$\delta=\frac{z}{1+e\cos\rho}=ze \qquad \text{.........} (14)$$

$$z=\delta(1+e\cos v)$$

$$z^1=(1+e\cos v)\delta^1-e\delta\sin v$$

$$z^{11}=(1+e\cos v)\delta^{11}-2e\delta^1\sin v-e\delta\cos v$$

$$z^{11}=e\frac{z}{(1+e\cos v)}\cos v=(1+e\cos v)\delta^{11}-2e\delta^1\sin v$$

$$z^{11}+ez.\rho\cos v=(1+e\cos v)\delta^{11}-2e\delta^1\sin v \qquad \text{.........} (15)$$

$$z^{11} + ze\cos v\left|1 - e\cos v + e^2\cos^2 v\right| + 3z\left|1 - e\cos v + e^2\cos^2 v\right|$$
$$\left[1 - \frac{2A^2}{(3-5A_0)^2}\right] - 5A_0 z(1 + e\cos v)\left\{1 - \frac{2A^2}{(3-5A_0)^2}\right\}$$
$$-5A_0\left(1 + 2e\cos v + e^2\cos^2 v\right)\frac{A}{3-5A_0}\sqrt{1 - \frac{A^2}{(3-5A_0)^2}}$$
$$-\frac{A^2}{(3-5A_0)}\left(1 - e\cos v + e^2\cos^2 v\right) - zA\sqrt{1 - \frac{A^2}{(3-5A_0)^2}}$$
$$= 2e\sin v - \frac{3A}{(3-5A_0)}\sqrt{1 - \frac{A^2}{(3-5A_0)^2} - Ae\sin v\sqrt{1 - \frac{A^2}{(3-5A_0)^2}}}$$
$$+\frac{A^2 e\sin vz}{(3-5A_0)}\left(1 - e\cos v + e^2\cos^2 v\right)$$
$$z^{11} + z\left[(3-5A_0) - \frac{2(3-5A_0).A^2}{(3-5A_0)^2}\right]$$
$$= e\left[2\sin v - A\sqrt{1 - \frac{A^2}{(3-5A_0)^2}}.\sin v + z\sin v.\frac{A^2}{(3-5A_0)}\right.$$
$$+z\cos v\left\{2 + 5A_0 - \frac{2(3+5A_0)A^2}{(3-5A_0)^2} + 10A_0\cos v.\right.$$
$$\left.\frac{A}{(3-5A_0)}\sqrt{1 - \frac{A^2}{(3-5A_0)^2} - \frac{A^2}{(3-5A_0)}.\cos v}\right]$$
$$+e^2\left[-\frac{z\sin 2vA^2}{2(3-5A_0)} + z\cos^2 v\left\{-2 + 6.\frac{A^2}{(3-5A_0)^2}\right\}\right.$$
$$\left.+5A_0.\cos^2 v.\frac{A}{3-5A_0}\sqrt{1 - \frac{A^2}{(3-5A_0)} + \frac{A^2\cos^2 v}{3-5A_0}}\right] + \qquad \text{.........} (16)$$

Suppose

$$(3-5A_0) - \frac{2A^2}{(3-5A_0)} = n_1^2 (say)$$
$$(3-5A_0)^2 - 2A^2 = n_1^2(3-5A_0)$$
$$2A^2 = (3-5A_0)\left(3 - 5A_0 - n_1^2\right)$$
$$A^2 = \tfrac{1}{2}(3-5A_0)\left(3 - 5A_0 - n_1^2\right)$$
$$A = \sqrt{\frac{1}{2}(3-5A_0)\left(3 - 5A_0 - n_1^2\right)}$$

Here

$$A\sqrt{1 - \frac{A^2}{(3-5A_0)^2}} = \sqrt{\frac{(3-5A_0)\left(3-5A_0-n_1^2\right)}{2}} \times$$
$$\sqrt{1 - \frac{(3-5A_0)\left(3-5A_0-n_1^2\right)}{2} \Big/ (3-5A_0)^2}$$
$$= 0$$

The equation (16) reduce to

$$z^{11} + n_1^2 z = e\left[2\sin v - \tfrac{1}{2}\sin v\sqrt{\left(3-5A_0+n_1^2\right)\left(3-5A_0-n_1^2\right)}\right.$$
$$+\frac{1}{2}z\sin v\left(3-5A_0-n_1^2\right) + z\cos v\left\{2 + 5A_0 - \frac{(3+5A_0)\left(3-5A_0-n_1^2\right)}{(3-5A_0)}\right\}$$
$$\left.+\frac{10A_0\cos v}{2(3-5A_0)}\sqrt{\left(3-5A_0+n_1^2\right)\left(3-5A_0-n_1^2\right)} - \frac{\cos v\left(3-5A_0-n_1^2\right)}{2}\right]$$
$$+e^2\left[-\frac{z\sin 2v\left(3-5A_0-n_1^2\right)}{4} + z\cos^2 v\left\{-2 + \frac{3\left(5A_0-n_1^2\right)}{(3-5A_0)}\right\}\right.$$
$$\left.+\frac{5A_0\cos^2\sqrt{\left(3-5A_0+n_1^2\right)\left(3-5A_0-n_1^2\right)}}{2(3-5A_0)} + \frac{\cos^2 v\left(3-5A_0-n_1^2\right)}{2}\right] \qquad \text{.........} (18)$$

Where,

$$n_1^2 = (3-5A_0) - \frac{2A^2}{3-5A_0}$$

## II. CONCLUSION:

We obtained that the linear oscillation of the system about the position of equilibrium.

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