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Navier-Stokes fractional equations and streams gauging

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ABSTRACT
Navier-Stokes fractional equations can describe different movement states and explain regime changes and manifest them in Blasius exponents. In particular, in this paper, alpha-viscosity is related to eddy-viscosity and a generalization of the frictional stress is formulated in terms of the movement regime and the spatial occupancy index $\boldsymbol{\beta}$. A methodology based on the wave analysis of the hydrograph is also presented, a process in which the wave velocity is determined, it is adjusted with the water profile slope due the regime change, and the real discharge is obtained, which is determined from its quotient with the measured discharge, while the latter is described according to Manning's model. This methodology was used in the estimation of the hydrograph for Carrizal River, Tabasco, Mexico. Authors concluded that Blasius exponent with values between $\mathbf{0} < \boldsymbol{\theta} < 2$ improves the representation of the traditional model determined by means of the water depth - discharge curve. Results obtained suggest a revision of water depth - discharge curves ($\boldsymbol{h} - \boldsymbol{Q}$) of hydrometric stations in order to obtain values closer to reality.
Keywords: Navier-Stokes fractional equations; streams gauging; wave celerity; water depth - discharge curves; flow regime; turbulence

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I. INTRODUCTION

In references [1-2] a fractional version of Navier-Stokes equations were presented. These are papers in which it is fundamentally stated that viscous stresses produce a momentum dispersive flow, which is described by means of a fractional Darcy's law; and that the divergence of the dispersive flow coincides with the temporal change of momentum, according to Newton's law.

On the other hand, Navier-Stokes fractional equations are simplified taking them to their form of boundary layer equations, also of fractional character. This form is achieved considering its relatively thin thickness, which implies that the main velocity is established in the downstream direction, with a large vertical velocity gradient compared to the longitudinal, which leads the velocity to satisfy the non-slip condition at the channel bed; and, on the contrary, with light pressure gradients in the vertical transverse direction compared with the strong ones in the longitudinal direction, [3].

Downstream velocity and vertical transverse velocity potentials are introduced. Momentum equation is expressed in terms of the downstream velocity potential. The frictional stress is produced by calculating the fractional derivative of the potential function, and it is obtained that the stress exhibits the power form of the inverse of the indexed Reynolds number and dependent on the spatial occupation index; in addition, it decreases with the horizontal distance to a power dependent on the spatial occupancy degree. The proportionality constant contains a viscosity power and the curvature value of the sub potential function at the origin.

Frictional force is obtained integrating the friction stress; it is expressed in a dimensionless form by introducing the drag coefficient. It follows that this coefficient is also represented as a power of the indexed Reynolds number, power determined by the spatial occupation index through an increasing function, so that the greater the spatial occupation index, the greater the exponent of the indexed Reynolds number, which is achieved with the laminar movement; on the contrary, the greater the turbulence, the lower the spatial occupation index and the lower the exponent of the indexed Reynolds number. Stress expression is reintroduced in the friction force calculation and it is observed that it can be described as a fractional derivative through *'Hadamard functionals'* whose order depends on the spatial occupation index.

An alternative expression for drag is obtained, of widely used in hydraulics, such as the Darcy-Weisbach friction factor, which is also represented as a power dependent on the spatial occupation index as an additional parameter. Then, the hydraulic slope and the shear velocity can also be found; and, finally, the dimensionless velocity as the ratio between average velocity and shear velocity.

To gauge a stream in a cross section consists in determine the volume of water that passes through it per unit of time. There are several ways to gauge a stream depending on river characteristics to be measured, as well as available equipment [4].

But in addition, to gauge a stream and determine the volume of water that passes through it, in the unit of time, is a daily task in hundreds of hydrometric points in many places of the country, and from these results depends diverse engineering applications, so it is hoped that its value could be as close as possible to reality. Most of river gauging stations have water depths - discharges curves $(h - Q\ curves)$ and often discharge is determined by measuring the level, elevation or height of water, and then estimating it by means of the curve, which is done under the hypothesis of a permanent and uniform flow; but this reduces Navier-Stokes fractional equations to Bernoulli's theorem, a traditional way of approaching the problem where an abstraction of various movement states is doing, walls roughness presence, and the involuntary presence of sediments.

Traditionally, the hydrograph wave analysis, which passes through the gauging section, is done by determining the celerity value and it's relation with the channel slope and with $(Q(h))$ curve data, instantaneous discharge for any event is determined, with the discharge ratio and its relation with the slope ratio, being the exponent equal to $1/2$,following Manning model [5].

This methodology was applied by [6] for Carrizal River, Tabasco, Mexico. Since then this methodology has not been revised, but with the presentation of [1-2] of Navier-Stokes fractional equations, it is shown that the exponent is not exactly a constant but varies with movement states, so the hydrograph analysis of data obtained from the curve must be reworked in order to determine an instantaneous discharge, taking into account several exponents of slopes ratio.

Therefore, the purpose of this research paper is to present a proposal for the relationship between alpha-viscosity and eddy-viscosity, update the traditional model by means of the Navier-Stokes fractional equations, and with that, determine hydrographs with real discharges originated by hourly elevations records.

II. MATERIALS AND METHODS

Navier-Stokes fractional equations: Fluid movement is described from the Eurelian point of view considering a volume of fluid limited by a boundary surface; with its momentum per unit of volume given by ρV. In first instance, due to its importance, interaction by internal friction is considered. Fractional gradient is expressed by $\nabla_M^\beta \rho V$, where ρ is the mass density, V the velocity, β the spatial occupation index, and M the measure of mixing of the different spatial directions. Momentum diffusivity is the α - kinematic viscosity ν_α , so Darcy's momentum flow is defined by equation (1):

$$q_D = -\nu_\alpha \nabla_M^\beta \rho V \quad (1)$$

Then the rate of momentum change per unit of time is the negative divergence, or convergence, of Darcy's flow:

$$\frac{d}{dt}\rho V = -\nabla \cdot (-\nu_\alpha \nabla_M^\beta \rho V) \quad (2)$$

In second instance, pressure variations contribution to fluid momentum change through the force that causes the pressure gradient is taken into consideration, in such a way that the composition of the viscous friction stress and the hydrostatic pressure forms the tensor $\rho T = \nu_\alpha \nabla_M^\beta \rho V - pI$ and gives rise to the deformation law. Thus the divergence of the stress $\rho T$is the change of momentum $\frac{d}{dt}\rho V$ per unit of volume $\nabla \cdot \rho T = \frac{d}{dt}\rho V$.

Next, an external potential force, per unit volume, of the type $-\nabla \rho \phi$ is incorporated. Next, incompressibility is taken into account. Material derivative that constitute the local variation with the advective is doing explicit. But the objectivity requirement needs the invariance under coordinate changes, so advective contribution must be modified and the vorticity term arises. Finally, if contribution to the inertial force of the term that contains vorticity is written in the right side of the equation as $V \times rotV$, this can be imagine as originated in an external force that dynamizes velocity fields evolution through its vorticity, entering into contradiction with the viscous force, while the other term is interpreted as a restriction that, along current lines, contains the Bernoulli equation:

$$\frac{\partial}{\partial t}V = \nu_\alpha \nabla_M^\alpha V + V \times rotV - \nabla\left(\frac{1}{2}(V \cdot V) + \frac{p}{\rho} + \phi\right) \quad (3)$$

Authors also call the coefficient ν_α as the fractional viscosity, due to its units, and it can be compared with the turbulent viscosity of Boussinesq or the eddy-viscosity, and in this research it will be establish a proportionality between alpha-viscosity and eddy-viscosity. Those units are, $[\nu_\alpha] \sim \frac{cm^\alpha}{s}$, so

$\left[\nu_\alpha \nabla_M^\beta \rho V\right] \sim \frac{cm}{s^2}$. Therefore, indexed Reynolds number emerges $R_\beta = \frac{u l^\beta}{\nu_\alpha}$, being β the spatial occupation index [1], but the traditional Reynolds number is recovered through the relationship: $R_\beta = \frac{ul}{\nu_\alpha l^{1-\beta}} = \frac{ul}{\nu_2}$, that is, defining the relationship: $\nu_\alpha = \frac{\nu_2}{l^{1-\beta}}$, where the parameter l is understood as the average size of the vortices or the average distances between singularities.

In the approaches field, equations that describe the boundary layer are obtained from the Navier-Stokes fractional equations by simplifications that are induced from the premise of a relatively thin thickness.

Two-dimensional boundary layer equation is considered in its stationary or permanent version, together with mass conservation in its null divergence form:

$$\mathrm{u}\,\partial_{\mathrm{x}}\mathrm{u} + \mathrm{v}\,\partial_{\mathrm{y}}\mathrm{u} = \nu_\alpha\,\partial_{\mathrm{y}}^{\alpha}\mathrm{u}$$

$$\partial_{\mathrm{x}}\mathrm{u} + \partial_{\mathrm{y}}\mathrm{v} = 0 \quad (4)$$

Potential form for the main velocity is given, downstream, representing it through the velocity potential as $\Psi(u,v)$ as $u = \partial_y \Psi$, $v = -\partial_x \Psi$; and then the sub potential function $g(\xi)$ emerges as a solution of the fractional Blasius differential equation [2]. Stress is calculated by $\tau_{xy} = \mu_\alpha \partial_y^\beta \partial_y \Psi(u,v)$, being $\mu_\alpha = \rho \nu_\alpha$ and $B_\beta = \left(\frac{1+\beta}{\beta}\right) l^{\beta-1} \partial_y^{\beta-1}\left(g''(\xi)\right)/_{\xi=0}$ and it turns out to be as shown below:

$$\tau_{xy} = \left(\frac{\beta}{1+\beta}\right) B_\beta \left(\frac{1}{R_{l\beta}}\right)^{\frac{\beta}{1+\beta}} (\rho l U^2) x^{-\frac{1}{1+\beta}} \quad (5)$$

Friction force per unit of binormal or transversal-horizontal length is calculated as $F_f = 2\int_0^l \tau_{xy}\, dx$. With the fractional Blasius coefficient and the indexed Reynolds number it can be written as:

$$F_f \approx 2B_\beta \left(\frac{1}{R_{l\beta}}\right)^\theta \rho l U^2,\ C_{f\beta} \approx 2B_\beta \left(\frac{1}{R_{l\beta}}\right)^\theta,\ f_\beta \approx 8B_\beta \left(\frac{1}{R_{l\beta}}\right)^\theta,\ R_{l\beta} = \frac{U l^\beta}{\nu_\alpha} \quad (6)$$

'Hadamard linear forms' are defined by expressions such as $H_b(x) = \frac{x^{b-1}}{\Gamma(b)}$, $b > 0$ and $\Gamma(b)$; Euler gamma function, can act on locally integrable functions and have the property of semigroup, which allows them to keep the form under the convolution operation in exchange for adding their orders.

If the stress expression τ_{xy} (5) is reintroduced on friction force (6), it can be described by Hadamard's linear forms, which in turn allows its representation as a fractional derivative. Defining $\gamma = \frac{\beta}{1+\beta}$ and $C(\beta) = \gamma B_\beta \Gamma(\gamma)$, and denoting by $*_l$ the product of convolution with $l$as integration limit; friction force, per unit of binormal width, can be described as a fractional derivative of the type:

$$F_f = 2(\rho l U^2) C(\beta) \frac{x^{\gamma-1}}{\Gamma(\gamma)} *_l \left(\frac{1}{R_{x\beta}}\right)^\sigma = 2(\rho l U^2) C(\beta) D_x^\gamma \left(\frac{1}{R_{x\beta}}\right)^\sigma \quad (7)$$

Where, after derivation, it should be evaluate in $x = l$. Then, σ it is such that $\theta = \beta\left(\sigma + \frac{1}{1+\beta}\right)$, being θ the Blasius exponent and part of the coefficient has been denoted as $C(\beta)$; or well: $\theta = \beta\left(\frac{3}{2} + \frac{1}{1+\beta}\right)$, with the following ends: the viscous, $\theta \to 1$, $\beta \to 1$, the turbulent $\theta \to 0$, $\beta \to 0$. Thus Blasius exponent is a manifestation of the spatial occupation index. Therefore, drag coefficient is described by $C_f = 2C(\beta) D_x^\gamma \left(\frac{1}{R_{x\beta}}\right)^\sigma$.

An important approach in hydraulics, particularly in the problem of gauging, is the dynamic Saint-Venant equation, where the deformation tensor described in equation (8):

$$\rho \boldsymbol{T} = \left(\frac{\nu_2}{l^{1-\beta}}\right) \nabla^{1+\beta} \rho \boldsymbol{v} - p\boldsymbol{I} \quad (8)$$

suffers the hydrostatic approach under the hypothesis that gradients are small and/or the length is too large, and is also known as '*gradually varied flow'*. But in addition, a second approach is that of the diffusive wave in Saint-Venant equation, in which it is reduced to the equality of the negative gradient of the water depth with the fractional hydraulic slope. And, in this context, four models for friction slope linked to Authors names are worth mentioning: Chézy, Weisbach-Darcy, Manning and Hagen-Poiseuille [7].

It is an experimental result that the ratio between frictional shear stress (τ_f) and eddy-viscosity (ν_ε) is proportional to the ratio between shear velocity (U_*) and the distance of the boundary ($l_\perp = y$), in the turbulent motion regime, $\frac{\tau_f}{\nu_\varepsilon} \propto \frac{U_*}{l_\perp} = \frac{\sqrt{\frac{\tau_0}{\rho}}}{y}$, being $k = 2.5$ the proportionality constant, ρ the fluid density, τ_0 the same frictional stress at the boundary [8].

We rely on an argument of scales variation and we contract the densities scale until locating us in a fluid of *'ideal gas type'* where viscosity is due to the product of an average velocity by a separation average length between molecules or average free path [8]. If then, in an inverse process, we expand densities scale until we locate ourselves again in a liquid, eddy-viscosity can be understand as the product of the mean velocity $(rms, v_{rms}^{'})$ by the length of vortices average size *(l)*. If we now propose a proportionality between the eddy-viscosity and the alpha-viscosity $\frac{\nu_\alpha}{l^\beta} l = \frac{\nu_\alpha}{l^{\beta-1}}$, we arrive at a formulation of the fractional character of frictional stress.

Therefore, the ratio of frictional stress on density has the dimensions of squared velocity, and by doing it correspond with the quadratic velocity: $\left(\frac{\nu_\alpha}{l^\beta}\right)^2 = \left(\frac{\nu_2}{l}\right)^2$; then, the fractional quality of frictional stress mentioned is highlighted: $\sqrt{\frac{\tau_0}{\rho}} = \frac{\nu_\alpha}{l^\beta} = \frac{\nu_2}{l}$, or: $\tau_0 = \rho\left(\frac{\nu_\alpha}{l^\beta}\right)^2$.

While fractional character of experimental result is reflected in, $\frac{\tau_{f\beta}}{\nu_\varepsilon} = k\frac{\nu_\alpha/l^\beta}{l_\perp}$, that in the turbulent end produces $\beta \to 0$, $\frac{\tau_{f0}}{\nu_{\varepsilon 0}} = k\frac{\nu_1}{l_\perp} \approx k\frac{U_*}{l_\perp}$, where experimental result, previously mentioned, is recovered. At the other end: $\beta \to 1$, $\frac{\tau_{f1}}{\nu_{\varepsilon 1}} = k\frac{\nu_2/l}{l_\perp} \approx k\frac{U_*}{l_\perp}$, $\tau_{f\beta} = k(v_{rms}^{'})\frac{\nu_2}{l_\perp}$; and with von Kármán constant $\kappa = 0.4$, fractional frictional effort remains:

$$\tau_{f\beta} = k\nu_\varepsilon\frac{\nu_\alpha/l^\beta}{l_\perp} = \frac{1}{\kappa}\frac{\left(\nu_\alpha/l^\beta\right)^2}{l_\perp} \quad (9)$$

In [9], we review the experimental result of Levi concerning a vortex generation by means of an oblique jet in water at rest in a container. Vortex size was obtained as a proportional average between $h\frac{d}{D}$ and $H\frac{D}{D}$ where (h, d, H, D) are the height and the diameter of the jet, the water height at rest and the size of the container, and we also carry out a scale process up to the Kolmogorov length.

In the transfer of energy between vortices, at the end of the limit$\beta \to 0$, we have that the velocity $v_{rms}^{'}$ is too large with respect to the velocity at the other end $\beta = 1$ where viscous dissipation starts; so $v_{rms}^{'} \gg U_*$.

On the contrary, at the end: $\beta = 1$, we have that $U_* = \frac{\nu_2}{l}, l = l_c$, where $l_c$is the characteristic length of the viscous sublayer.

If it is here $v_{rms}^{'} \approx \frac{\nu_2}{l}$and the vortices size is small enough that it can be compared with the Kolmogorov length, it turns out that$v_{rms}^{'} \approx \frac{\nu_2}{\left(\nu_2^{3/4}\right)/\varepsilon_K^{1/4}} = (\nu_2\varepsilon_K)^{1/4}$. Reciprocally, if $v_{rms}^{'} \approx (\nu_2\varepsilon_K)^{1/4}$and this is comparable with the shear velocity $U_* = \frac{\nu_2}{l_c}$, then the characteristic length results$l_c = \frac{\nu_2}{(\nu_2\varepsilon_K)^{1/4}} = \frac{\nu_2^{3/4}}{\varepsilon_K^{1/4}} = \eta$, which is the length of Kolmogorov.

Within the context of gauging and with the fractional Weisbach-Darcy model, friction factor can be considered as an operator that alters the power of the hydraulic slope, which produces the result [2], $J_\beta = \frac{1}{8gR_h}f_\beta V^2$, then, $J_\beta = B_\beta\left(\frac{\nu_\alpha}{l^\beta}\right)^\theta \frac{1}{gR_h}V^{2-\theta}$, it is dimensionless with the velocity $\sqrt{gR_h}$ and results:

$$J_\beta = B_\beta\left(\frac{\nu_\alpha/l^\beta}{\sqrt{gR_h}}\right)^\theta\left(\frac{V}{\sqrt{gR_h}}\right)^{2-\theta} \quad (11)$$

which can also be represented by $J_\beta = B_\beta\left(\frac{\nu_2/l}{\sqrt{gR_h}}\right)^\theta\left(\frac{V}{\sqrt{gR_h}}\right)^{2-\theta}$; then when evaluating in the characteristic length it is obtained:

$$J_\beta = B_\beta\left(\frac{U_*}{\sqrt{gR_h}}\right)^\theta\left(\frac{V}{\sqrt{gR_h}}\right)^{2-\theta} \quad (12)$$

It is inferred that hydraulic slope exponent reflects the regime of the fluid movement and effects of friction and turbulence are synthesized in it. Particularly in the Chézy model, the slope exponent is $2 - \theta \to 2$. If the Chézy flow resistance formula is applied and its constant is expressed through the Manning roughness coefficient, n, then, discharge is determined by equation (13) where slope exponent is equal to $1/2$, [10]:

$$Q_{(0)} = \frac{AR_h^{\frac{2}{3}}}{n}J_{(0)}^{1/2} \quad (13)$$

where: $A$is the hydraulic area in m^2; R_h the hydraulic radius in m; J is the geometric slope of the river section, n the Manning roughness coefficient in $m^{1/6}$ and Q the discharge in m^3/s.

By means of Q_m we denotes the discharge in established regime, measured in m^3/s, and with Q_r the real discharge in a non-established regime and also in m^3/s, we want to estimate the discharge ratio Q_r/Q_m, [5].

If the general formula of flow resistance is applied, discharge ratio by slopes ratio is determined, with J_r that of the water surface for Q_r, and with J_m for Q_m; but with exponent given by $e(\theta) = \frac{1}{2-\theta}$ and $0 < \theta < 2$, instead of $1/2$, and is expressed in equation (14):

$$\frac{Q_r}{Q_m} = \left(\frac{J_r}{J_m}\right)^{\frac{1}{2-\theta}} \quad (14)$$

However, when an avenue occurs, the water surface slope J_r is determined as expressed in equation (15), [5]:

$$J_r = J_m - \frac{1}{U}\frac{dh}{dt} \quad (15)$$

where: U is the wave velocity (m/s), J_r the hydraulic slope of the wave, J_m the channel slope for uniform flow, and $\frac{dh}{dt}$ the variation of the water depth with respect to time.

U wave celerity is obtained by means of equation (16), [5],

$$U = V_1 + \sqrt{g\frac{A_2h_2 - A_1h_1}{A_1\left(1-\frac{A_1}{A_2}\right)}} \quad (16)$$

being: V_1 the average velocity in section 1; h_1 and h_2 water depths at gravity center of sections 1 and 2; A_1 and A_2 areas of sections 1 and 2; and g the acceleration of gravity, in m/s^2.

With the substitution of equation (15) in equation (14), and with necessary simplifications, equation (17) is obtained

$$\frac{Q_r}{Q_m} = \left(1 - \frac{1}{UJ_m}\frac{dh}{dt}\right)^{e(\theta)} \quad (17)$$

In equation (17), J_m is calculated by means of the equation (13) with some support stations and the estimation of the Manning roughness coefficient n; the measured discharge Q_m can be obtained from the $h - Qcurves$ for each recorded water depth. In addition, dh/dt is the slope of water depths against time curve for the considered instant.

It is customary to use the change in water depth that occurs during the hour preceding the moment for which it is necessary to value the discharge. Therefore, in equation (17) we only need to know the celerity U. Although in the traditional model the exponent is taken equal to $1/2$[5], in the present proposed model is $e(\theta) = \frac{1}{2-\theta}$.

III. RESULTS AND DISCUSION

As already mentioned, celerity is determined by means of equation (16) and then replaced in (17). Equation (18) has been obtained from equation (17), and shows in its exponent the consequences of the fractional aspect of Navier-Stokes equations that allows us to adjust the discharge according to fluid movement state, once changes in the water depths - discharges curve have been determined:

$$\frac{Q_r}{Q_m} = \left(1 - \frac{1}{UJ_m}\frac{dh}{dt}\right)^{\frac{1}{2-\theta}} \quad (18)$$

Based on results of previous development, an application in the area of surface hydrology is studied by means of the following example.

3.1 Case study

The stated methodology was applied in the estimation of the hydrograph for the Carrizal River, Tabasco, Mexico, at the Angostura gauge station, Chiapas. In Table 1, the record of hourly water depths is shown in column 1; readings are doing in 1-hour intervals. In column 8, and according to equation (15), the wave celerity calculation is shown.

Because the Angostura gauge section is too wide and has an almost rectangular shape, the water depth to the center of gravity is considered equal to half the water depth. Column 4 is similar to 6 and column 2 to 5, but displaced in the time interval of one hour, for the present case.

Table 1. Wave velocity calculation.

Height (m)	A1 (m^2)	Water depth (m)	y1 (m)	A2 (m^2)	y2 (m)	v1 (ms^{-1})	U
420.246	103.56	1.40	0.700	150.920	1.015	1.912	6.847
420.821	150.92	2.03	1.015	170.160	1.140	2.306	7.149
421.096	170.16	2.28	1.140	260.200	1.710	2.621	9.087
422.246	260.20	3.42	1.710	268.000	1.770	3.843	10.016
422.346	268.00	3.54	1.770	196.280	1.315	3.940	8.595
421.446	196.28	2.63	1.315	162.760	1.090	3.042	7.467
420.996	162.76	2.18	1.090	139.120	0.935	2.519	6.616
420.696	139.12	1.87	0.935				

On the other hand, in Table 2, the real discharge calculation Q_r is shown, after doing the correction to the discharge obtained from the water $h - Qcurve$ in the established regime Q_m, (column 6), with the slope J_m equal to $0.00203 m/m$.

In the range of the exponent $0 < \theta < 2$, negative of the pressure gradient is proportional to a power of the velocity, power that is expressed by $2-\theta$; the value found experimentally by Blasius was of $1+3/4$, of what results $\theta = 0.25$[8]. So the exponent in equation (17) results$\frac{1}{2-\theta} = 0.5714$, and with this value in Table 2, discharge ratio Q_r/Q_m is determined; but if Q_m is already known, the discharge Q_r is obtained.

In Table 2, calculation of the real discharge after the correction doing to the discharge Q_m corresponding to the established regime is shown.

Table 2. Real discharge calculation Q_r, after knowing the established regime Q_m.

Height (m)	h (m)	(Δh/Δt) (m/s)	(1/UJm)*(Δh/Δt)	Qr/Qm	Qm (m^3/s)	Qr (m^3/s)
420.246	0.575	1.597	-0.115	1.067	198	211
420.821	0.275	0.764	-0.053	1.031	348	358
421.096	1.150	3.194	-0.173	1.101	446	489
422.246	0.100	0.278	-0.014	1.008	1000	1,008
422.346	-0.900	-2.500	0.143	0.911	1056	967
421.446	-0.450	-1.250	0.082	0.950	597	568
420.996	-0.300	-0.833	0.062	0.962	410	395

In Figure 1, discharges determined by means of the $h-Q\ curve$ for the established regime are shown, as well as discharges calculated through the model Q_r that results from the application of the Navier-Stokes fractional equations.

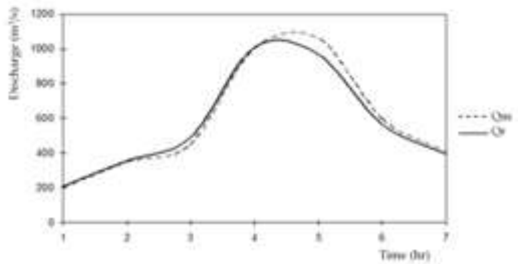

Figure 1. Hydrographs for Carrizal River. Discharges (m^3/s) as a function of time (hr).

IV. CONCLUSIONS

Viscous friction force cannot be properly described by a local operator as it is a derivative of integer order; but, on the contrary, it must be described by a non-local operator as is the derivative of a fractional order. So viscous force gives a fractional character to the Navier-Stokes equation.

Hydraulic slope synthesizes both the turbulence model as the friction model, and therefore, the fractional character resurfaces in it.

Turbulence and friction effects remains implicit in the Blasius exponent θ, therefore changes induced by a wave are manifested in variations of this exponent, which approaches zero for fully developed turbulence, and approaches 1 for viscous sublayer of laminar movement.

Alpha-viscosity and eddy-viscosity are proportional and depend on the motion state and vortices sizes or distances between singularities.

The proposed model in its original formulation, and in spite of so many years gone by, it's possible to update it with new mathematical developments, to represent natural phenomena with physical sense, such as the case of regime change variation.

Water depth-discharge curve $(h-Qcurve)$ is determined for permanent and uniform regime, which indicates that any value outside the curve is attributed to the regime change, obtaining values with changes of $\pm 10\%$. Based on results obtained, it is recommended that hydrometric stations be operated and managed with more modern technologies.

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