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Hybrid Monte Carlo estimation of Bitcoin volatility through stochastic volatility model

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ABSTRACT
We estimate the Bitcoin volatility usingthe realized stochastic volatility model. The model parameters are determined by Bayesian inference usingthe Markov chain Monte Carlo method. We apply the hybrid Monte Carlo method for the volatility update process, which is the most time-consuming part. We investigate several integrators in the hybrid Monte Carlo method and find that the 2nd order minimum norm integrator outperforms the others. The parameterphiof the model is found to be close to one, which indicates that the Bitcoin volatility ispersistent. We test the accuracy of the estimated volatilities by using standardized returns. The distribution of the standardized returns is found to be close to the standard normal distributions, which indicates that the volatility is estimated accurately by the realized stochastic volatility model.

Keywords–Bitcoin, Hybrid Monte Carlo method, Hamiltonian Monte Carlo method, Realized volatility, Standardized return

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I. INTRODUCTION

Volatility is of great importance in empirical finance to measure and forecast risk on the financial markets. Commonly used methods to estimate volatility are model-based methods,whichuse a specific model,such as the generalized autoregressive conditional heteroskedasticity (GARCH) model [1] and the stochastic volatility (SV) model [2-4], to capture time-series properties. The original GARCH model [1]wasdesigned to capture the symmetric volatility that reacts to positive and negative returns equally. There exist various extended versions of the GARCH model that can capture the asymmetric characteristic of volatility (e.g., see[5-9]). The asymmetric volatility is especially important for equities becausethey respond asymmetrically to positive andnegative shocks, which is known as the leverage effect [10-11].

The SV model is alsocommonly usedto estimate volatility and can also capture the asymmetry of volatility. Furthermore, Ref.[12] proposedthe realized SV (RSV) model, which utilizes realized volatility (RV) data as additional information to determine the volatility. By using the additional data, the RSV model is expected to estimate more accurate volatilities.

In the model-based methods, the model's parameters are determined so that the model matches the underlying time series. For GARCH-type models, this matching process is usually performed usingthe maximum likelihood method. Bayesian inference can also be used toestimate the model'sparameters, and variousMarkov chain Monte Carlo (MCMC) methods have been examined toperform Bayesian inference GARCH-type models (e.g., see [13-21]). Becausethe likelihood function of the SV model is written in integral form and,thus,is not easily tractable in the maximum likelihood approach, Bayesian inference is often chosen for theSV model'sparameter estimations. Through several studies, anefficient MCMC scheme has been developed for the SV model [22-26].

This study focuses on the estimation ofthe volatility of Bitcoin's return time series by the RSV model.Bitcoin has attracted much interest from researchersand has been recognized as an innovative payment medium. In recent years, several studies on Bitcoin have been conducted,including on its hedging capabilities [27], bubbles [28], price clustering [29], multifractality [30], relationships with other financial assets [31], Taylor effect [32], and market efficiency[33-38]. Properties of Bitcoin volatility have also been investigated[39-44], and an interesting property called "inverted leverage effect" has beenfound in cryptocurrency markets.

To estimate Bitcoin volatility, we use the RSV model and perform Bayesian inference for this model. For the volatility update process, which is the most time-consuming part, we adapt the hybrid Monte Carlo (HMC) method [45], whichwas originally developed for lattice quantum chromodynamics (QCD) simulations and has later been utilized in various other fields. The HMC method, which is also known as the Hamiltonian Monte Carlo method,has been tested for the SV model, for which ithas been shown to samplevolatility variables more effectively than for the Metropolis method[46-47]. One of the distinct features of the HMC method is that it is easily parallelized, and its GPU computing can accelerate volatility updates in the RSV model[48-49].

The HMC method consists of a molecular dynamics (MD) simulation and aMetropolis test. In the MD simulation, Hamilton's equations of motion are solved by an appropriate integrator. In this study, wealso investigate several other integrators and compare their performance.

This paper is organized as follows. Section 2 describes the RSV model;Section 3 introduces the HMC method;Section 4 describesthe data used;Section 5 presents the results; andfinally, section 6presents the conclusions.

Realized Stochastic volatility model

The RSV modelused in this study is formulatedas follows [12].

$$R_t = \exp(h_t/2)\varepsilon_t, \quad \varepsilon_t \sim N(0,1), \ t = 1,\cdots,T \quad (1)$$

$$\ln RV_t = \xi + h_t + u_t, \ u_t \sim N(0,\sigma_u^2), \ t = 1,\cdots,T \quad (2)$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t, \eta_t \sim N(0,\sigma_\eta^2), \quad t = 1,\cdots,T-1 \quad (3)$$

$$h_1 = \mu + \eta_0, \quad \eta_0 \sim N(0, \frac{\sigma_\eta^2}{1-\phi}), \quad (4)$$

where R_t is the daily return on day t, RV_t is the daily RV with 5min sampling frequency (e.g., see [50]),and h_t is the log volatility defined by $h_t \equiv \ln(\sigma_t^2)$. The model parameters neededfor the estimationare $\mu, \phi, \sigma_\eta^2, \xi$, and σ_u^2 and we denote $\theta = (\theta_1,\cdots,\theta_5) = (\mu,\phi,\sigma_\eta^2,\xi,\sigma_u^2)$. We estimate these parameters through Bayesian inference. From the Bayes' theorem, the posterior density of h and θ is given by

$$P(\theta, h|R, RV) \sim f(R, RV|\theta, h)\pi(\theta), \quad (5)$$

where $f(R, RV|\theta, h)$ is the conditional likelihood function for the RSV model, and$\pi(\theta)$ is the prior density for θ. In this study, we use the flat prior for μ, ϕ, and ξ ,and for σ_u^2 and σ_η^2 we use $\pi(\sigma_u^2) = 1/\sigma_u^2$ and$\pi(\sigma_\eta^2) = 1/\sigma_\eta^2$, respectively.

The conditional likelihood function $f(R, RV|\theta, h)$ is expressedas

$$f(R,RV|\theta,h) = \sqrt{\frac{1-\phi^2}{2\pi\sigma_\eta^2}} \exp\left(-\frac{(h_1-\mu)^2}{2\sigma_\eta^2/(1-\phi^2)}\right) \times \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp\left(-\frac{(h_t-\mu-\phi(h_{t-1}-\mu))^2}{2\sigma_\eta^2}\right) \times \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{R_t}{2\sigma_t^2}\right) \frac{1}{\sqrt{2\pi\sigma_u^2}} \exp\left(-\frac{(\ln RV_t-\xi-h_t)^2}{2\sigma_u^2}\right). \quad (6)$$

In Bayesian inference, the parameters are obtained as expectation values through the posterior density$E[\theta_i]$:

$$E[\theta_i] = \int \theta_i P(\theta, h|R, RV) d\theta dh / Z, \quad (7)$$

where Z is the normalization constant given by $Z = \int P(\theta, h|R, RV) d\theta dh$. Because(7) is not analytically tractable, we estimate it usingthe MCMC method. For $\theta = (\mu, \phi, \sigma_\eta^2, \xi, \sigma_u^2)$, we use the standard MCMC update technique[22-24]. The most time-consuming part is the update of volatility variables h, for which we use the HMC method.

hybrid monte Carlo algorithm

We employ the HMC method to update the volatility variablesin the MCMC process. The HMC method is described as follows. First, we define the Hamiltonian H as

$$H(h,p) = \frac{1}{2}p^2 - \ln P(\theta, h|R, RV) \quad , (8)$$

where $p = (p_{1,\dots}, p_T)$ are conjugate momenta to volatility variables $h = (h_1,\cdots,h_T)$ and $p^2 \equiv \sum_{i=1}^{T} p_i^2$.Using H, (7) is rewritten as

$$E[\theta_i] = \int \theta_i \exp(-H(h,p)) d\theta dh dp / \bar{Z} \quad , (9)$$

where $\bar{Z} = \int \exp(-H(h,p)) \, d\theta dh dp$. To estimate (9)withthe MCMC method, we need to sample (h, p) with the probability density $\sim \exp(-H(h,p))$. In the HMC method, candidate variables are generated by solving Hamilton's equations of motion as

$$\frac{dh_i}{d\tau} = \frac{\partial H}{\partial p_i}, \quad (10)$$

$$\frac{dp_i}{d\tau} = -\frac{\partial H}{\partial h_i}. \quad (11)$$

To solve (10)–(11), we perform the MD simulation with an appropriate integrator. The simplest integrator for the HMC method is the 2nd order leapfrog (2LF) integrator [47]. However, the 2LF integrator is not the only choice for the HMC method, and other integrators such as high-order [51-52] and minimum-norm (MN) integrators [53] can be used. In this study, we use the 2LF, 4th order MN,and 2nd order MN (2MN) integrators and compare their performance.

In the Lie algebra formalism [54-56], Hamilton's equations of motion are given by

$$\frac{df}{d\tau} = \{f, H\}, \quad (12)$$

where f =hor p and {,} is the Poisson bracket. Defining the linear operator L(H) by

$$L(H)f = \{f, H\}, \quad (13)$$

the formal solution of (12) is given by

$$f(\tau + \Delta\tau) = exp[\Delta\tau L(H)] f(\tau). \quad (14)$$

Rewriting L(H) as

$$L(H) = L\left(\frac{1}{2}p^2\right) - L\big(lnP(\theta, h|R, RV)\big)$$

$$= T + V \quad , \quad (15)$$

the2LF integrator is given by decomposing $exp[\Delta\tau(T+V)]$ as

$$exp[\Delta\tau(T+V)] = exp\left(\frac{\Delta\tau T}{2}\right) exp(\Delta\tau V)\, exp\left(\frac{\Delta\tau T}{2}\right) + O(\Delta\tau 3). \quad (16)$$

Letting $I_2(\Delta\tau)$ denote the2LF integrator, we obtain

$$I_2(\Delta\tau) = exp\left(\frac{\Delta\tau T}{2}\right) exp(\Delta\tau V)\, exp\left(\frac{\Delta\tau T}{2}\right). \quad (17)$$

The higher-order integrators can be constructed through the 2nd order LF integrator [54,56]. The (2k+2)th order integrator $I_{2k+2}(\Delta\tau)$ is given recursively by

$$I_{2k+2}(\Delta\tau) = I_{2k}(a_1\Delta\tau)\, I_{2k}(a_2\Delta\tau)\, I_{2k}(a_1\Delta\tau) \quad , \quad (18)$$

where

$$a_1 = \frac{1}{1-2^{1/(2k+1)}}, \quad (19)$$

$$a_2 = \frac{2^{1/(2k+1)}}{1-2^{1/(2k+1)}}. \quad (20)$$

Although the higher-order integrators have higher-order error terms that inducefewererrors, the cost to implement the integratoris higherwith higher-order degrees, which makes the integrator less efficient. The cost of the n-th order integrator relative to the 2nd order LF integrator increases as$3^{\frac{n}{2}-1}$[51]. For instance, the relative cost of the 4th order integrator is $3^{\frac{n}{2}-1} = 3$ for n=4. In general, the efficiency of the higher-order integrators depends on the model used. For lattice QCD simulations, higher-order integrators are expected to be effective for models with a large system size [51].

The 2MN integratoris more attractive than other integrators becauseit is expected to havefewerintegration errors without model dependence [52,56]. The 2MN integrator $I_{2MN}(\Delta\tau)$is described as

$$I_{2MN}(\Delta\tau) = e^{\lambda\Delta\tau T} e^{\frac{\Delta\tau V}{2}} e^{(1-2\lambda)\Delta\tau T} e^{\frac{\Delta\tau V}{2}} e^{\lambda\Delta\tau T}. \quad (21)$$

The error of this integrator can be minimized at $\lambda \approx 0.193183$ [52,56]. The relative cost of the 2MN integrator is approximately2.

In the MD simulation, an appropriate integrator is repeatedly appliedk times, and h and p are integrated up to the length $l = \Delta\tau \times k$. Here, we set $l = 2$. Then, the new variables h and p are accepted with a Metropolis probability of $\sim min[1, exp(-\Delta H)]$, where $\Delta H = H(h(\tau+l, p\tau+l) - H(h\tau, p\tau)$.

II. DATA

In this study, we use Bitcoin Tick data (in dollars) traded inCoinbasefrom January 28, 2015, to January 6, 2019, downloaded from Bitcoincharts[58]. From this data set, we construct daily returns and daily realized volatilities calculated with a 5min sampling frequency. Figs.1 and 2 display time series of daily returns and daily realized volatility, respectively. Both daily returns and daily realized volatilities are used as input data to the RSV model.

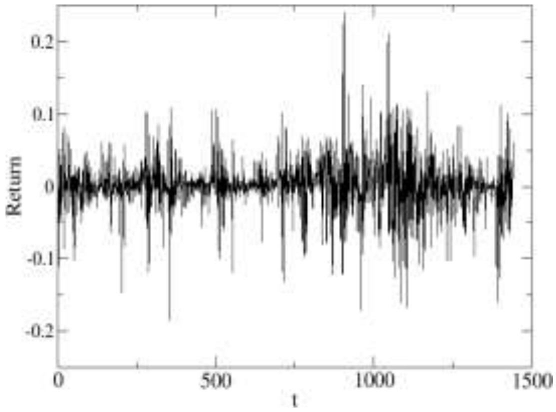

Fig.1 Daily return of Bitcoin.

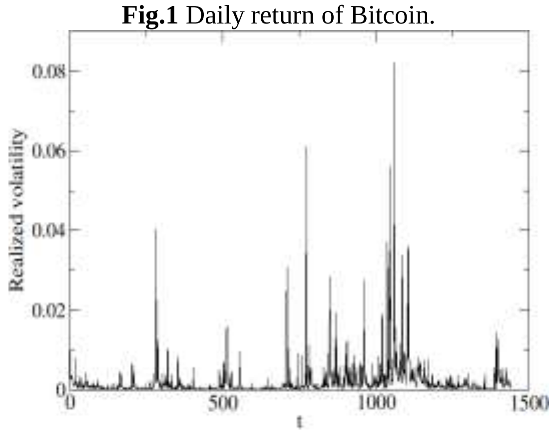

Fig.2 Daily realized volatility of Bitcoin.

III. RESULTS

First, we investigate the performance of the different integrators(2LF, 4thMN, and 2MN)used in the HMC method. More precisely, we use the integrators $I_2(\Delta\tau)$, $I_4(\Delta\tau)$, and $I_{2MN}(\Delta\tau)$ in the MD simulations. We calculate the acceptance for various step sizes$\Delta\tau$, and then define the efficiency function of the integrators [51] by

$$EFF(Acc) = \Delta\tau \times Acc(\Delta\tau), \quad (22)$$

where $Acc(\Delta\tau)$ is the acceptance at $\Delta\tau$. (22) takes a maximum value at a certain optimal acceptance. An analytical calculation [51]indicatedthat the optimal acceptance Acc_{opt} depends only on the degree of the integrator as follows:

$$Acc_{opt} = exp\left(\frac{1}{n}\right) \quad . \quad (23)$$

(23) implies that Acc_{opt} is0.61 and 0.78 for the 2nd and 4th integrators, respectively. Fig.3 shows the efficiency function $EFF(Acc)$ of the 2LF integrator as a function of step size$\Delta\tau$.It is found that $EFF(Acc)$ ismaximum around Acc=0.6–0.7, which is consistent with the result from (23). Similarly, Fig.4 shows the$EFF(Acc)$ of the 4thorder integrator,

and reveals that $EFF(Acc)$ismaximum around Acc= 0.7–0.8, which is alsoconsistent withthe result from (23).

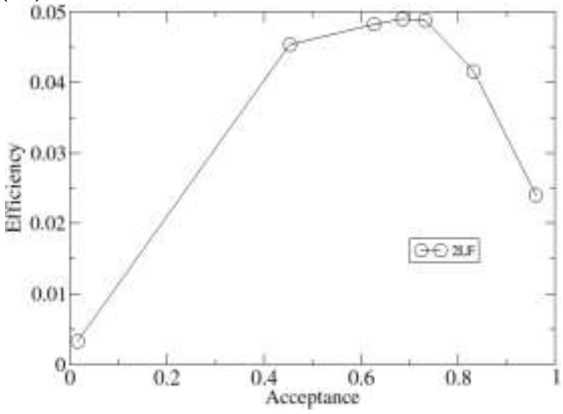

Fig.3 Efficiency ofthe 2LF integrator as a function of acceptance.

To evaluate the actual performance of the integrators, we calculate the relative efficiency ofthe 2LF integrator as $EFF_{4th}(Acc_{opt})/EFF_{2LF}(Acc_{opt})$. From Figs.3 and 4, we find $EFF_{4th}(\mathrm{Acc_{opt}}) = 0.11$ and $\mathrm{EFF_{2LF}}(\mathrm{Acc_{opt}})$=0.049. Thus, $\mathrm{EFF_{4th}}(\mathrm{Acc_{opt}})/\mathrm{EFF_{2LF}}(\mathrm{Acc_{opt}})$is approximately 2.2. This value is lowerthan that of the relative cost of the 4thorder integrator, i.e., 3, which indicates that the 4th integrator is less efficient than the 2LF integrator.

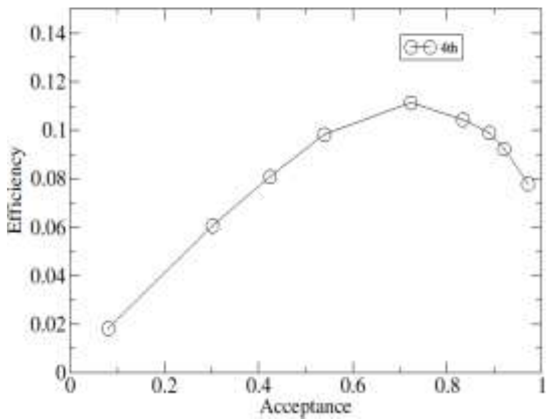

Fig.4 Efficiency ofthe 4th order integrator as a function of acceptance.

Fig.5shows EFF(Acc) of the 2MN integrator and reveals that$\mathrm{Acc_{opt}} \approx 0.7$.Because$\mathrm{EFF_{2MN}}(\mathrm{Acc_{opt}})$ is found to be approximately0.25, the relative efficiency of the 2MN integrator, i.e., $\mathrm{EFF_{2MN}}(\mathrm{Acc_{opt}})/\mathrm{EFF_{2LF}}(\mathrm{Acc_{opt}})$, is approximately 5.1.Becausethe relative cost of the 2MN integrator is 2, itsrelative efficiency is higher than the relative cost, which means that the 2MN integrator outperforms the 2LF integrator, in agreement with previous results [53].

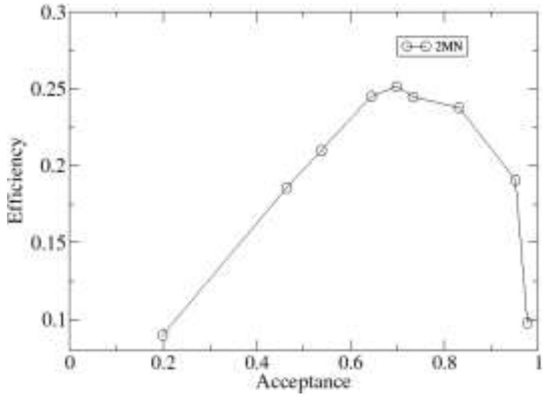

Fig.5 Efficiency ofthe 2MN integrator as a function of acceptance.

Table 1lists the results of the parameters. We use the 2MN integrator in the HMC simulation and collect 25000 samples after 5000 thermalization processes. The results in Table 1 are average values over the 25000 collected samples. The parameter ϕ is related to the persistenceof volatility; if ϕ is close to 1, the volatility time series has strong persistence. We found $\phi = 0.8255$, which is close to 1, and thus,the time series of Bitcoin volatility has the property of persistence. Fig.6 shows the Bitcoin volatility time-series estimated by the RSV model. AS shown in the figure,the Bitcoin volatility exhibits volatility clustering, i.e., the tendency of large changes in volatilities to cluster. The presence of volatility clustering is also in agreement with the volatility persistence.

Table. 1 Estimatedparameters.

μ	ϕ	σ_η^2
-7.48(2)	0.8255(6)	0.696(3)
ξ	σ_u^2	
0.206(2)	0.065(2)	

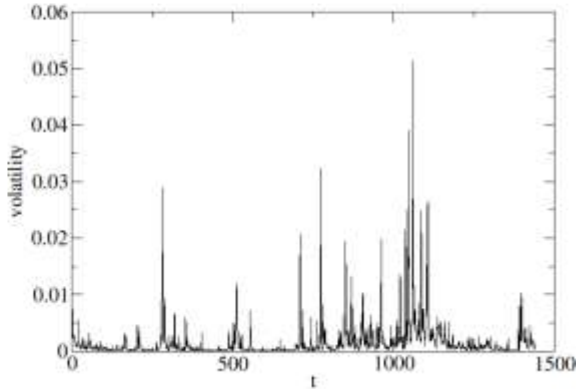

Fig.6 Volatility estimated by the RSV model. The volatility results are averages calculated from 25000 samples.

Fig.7 shows the return distribution,which is identified as leptokurtic. Table 2 lists the results of variance, kurtosis, and skewness. The kurtosis is 7.5, which exceeds the value of 3 of the normal distribution. The skewness appearsto be zero. To test the accuracy of the volatility estimated by the RSV model, we calculate the returns standardized (e.g.[59-65]) by the estimated volatilities. Let us assume that the returnR_t at day t is

$$R_t = \sigma_t \varepsilon_t, \qquad \varepsilon_t \sim N(0,1), \qquad (24)$$

and calculate the standardized return as

$$\overline{R_t} = R_t/\sigma_t. \qquad (25)$$

We use the volatility obtained by the RSV model for σ_t.If the volatility is accurately estimated, the standardized returns $\overline{R_t}$ exhibit the standard normal variables. Fig.8 shows the distribution of the standardized returns, which is foundtobeclose to the standard normal distribution.As shown in Table 2, the variance and kurtosisare found to be 0.936 and 2.63, respectively, which are consistent with those of the standard normal distribution and confirmsthat the volatility isestimated accurately by the RSV model.

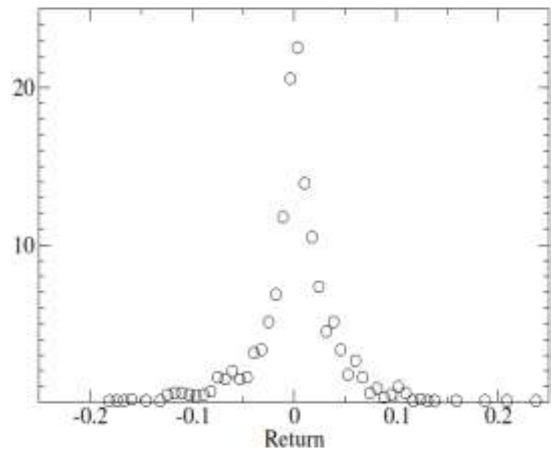

Fig.7 Original return distribution.

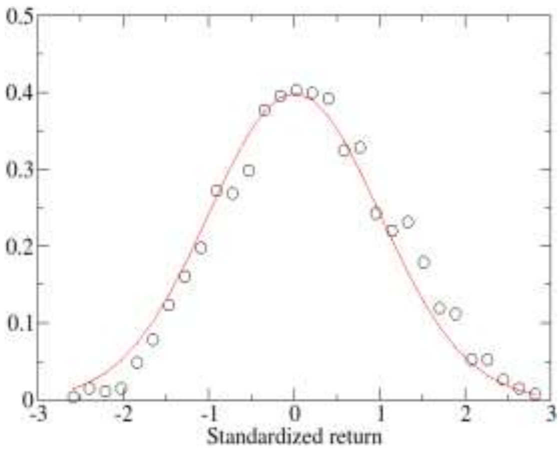

Fig.8 Standardized return distribution. Thesolid red curve is equal to$\frac{\exp\left(-\frac{r^2}{2}\right)}{\sqrt{2\pi}}$.

Table 2. Variance, kurtosis, and skewness values obtained.

	Original return	Standardized return
variance	$1.5(3) \times 10^{-3}$	0.936(3)
kurtosis	7.5(11)	2.63(5)
skewness	-0.13(30)	0.09(15)

IV. CONCLUSION

We performed Bayesian inference of the RSV model to estimate Bitcoin volatility using the HMC method for the volatility update process, which is the most time-consuming part ofthe Bayesian inference of the RSV model. We examined 2LF, 4thorder MN, and 2MN integrators and foundthat the 2MN integrator is the most efficient for the present case. The parameter ϕwasfound to be close to one, which indicates that the volatility time series waspersistent.

We testedthe accuracy of theestimated volatility by using standardized returns and found that, while the original distribution is leptokurtic, the distribution of the standardized returns is consistent with the standard normal distribution, which indicates that the RSV model estimated the Bitcoin volatilities accurately.

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