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On the validity of Kepler's laws and the enigma of dark matter

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ABSTRACT
In this study, we analyze one of the great enigmas of the Standard Model of Cosmology, which is constituted by what has been termed: Dark Matter, and which is also termed: the Missing Mass. The non-universality of the laws of traditional Celestial Mechanics that are valid in the Solar System, is established in former term. Subsequently, the appropriate mathematical expressions of the gravitational force that rules the Galactic Systems, as well as the galaxy clusters, are deduced. Finally, the true result is obtained, that it is not necessary to postulate the existence of that strange substance which is known as Dark Matter in order to justify the curve of radial galactic velocities. It is found that the experimental results, the measurements of the speeds of the stars in the galaxies, have a mathematical explanation with a different Celestial Mechanics as is established in this paper.

Keywords - Gravitation, cosmology, Kepler laws, dark matter, radial velocity, galaxy rotation.

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I. INTRODUCTION

In the year of 1687, Isaac Newton (1642-1727) published his theory on celestial mechanics that includes the law of universal gravitation [1]. In several texts where this topic is dealt with, the qualifier "universal" is found. Is it the law of gravitation, as it is commonly written, of universal application, as stated? We aim to explain that this is not the case [1]. Kepler's laws also do not seem to have that universality that could be expected in principle [2]. In this context, an interesting question turns out to be: Is the gravitational mechanics of the Solar System also valid at the level of the Galaxy or of the galaxy clusters? Here, we explain the basic differences that exist between solar systems and galactic systems, in terms of their gravitational dynamics.

In the last 85 years, in the field of cosmology, the hypothesis of the existence of an exotic and strange substance has been handled, practically in all of the structures of the universe, but particularly in the galactic gravitational systems and in the galaxy clusters [3, 4]. Within this conjecture, at least one of Kepler's laws has basically been linked to the existence of what has been called the Dark Matter [5]. This is due to the fact that in the calculations that are involved to postulate the existence of this dark matter, Kepler's laws are mainly used, with the aim of justifying the observational data of the speed of many stars in galaxies and of galaxies in clusters [2-5]. We speak of the missing mass or the mass that is not seen, a mass that would be added to the gravitational mass in practically all corners of the universe [5, 6].

Adding this mass is somewhat foreign to the traditional matter called baryonic, we try to justify the speeds that are so great that they have been measured for stars in galaxies and for galaxies in galaxy clusters. The values of the velocities that have been measured for stars in a galaxy do not, generally, agree with the predictions that result from Kepler and Newton's mechanics. The stars, according to the measurements, rotate too fast to be confined to the body of the same galaxy. In fact, galaxies should be torn apart, because the gravitational force of cohesion, which is justified by visible mass, does not seem to be sufficient. To explain this excess, in the speed with which the stars rotate and also the fact that the galaxy does not disintegrate, the existence of that strange substance that has been called Dark Matter is postulated.

The history related to these observational facts, among others, records Fritz Swicky (1898-1974) in the 1930s [3-5] and Vera Rubin (1928-2016) in the 1960s and 1970s [7-11] who, after conducting detailed studies on the subject, were forced to postulate the existence of this strange and invisible substance which they called Dark Matter. After making observations with spectroscopy, which was very advanced for their time, they tried to explain their results with the part of the celestial mechanics that they had within their reach. In particular, Vera Rubin in his work published in 2006 states that the radial velocities of stars in galaxies and galaxies in the clusters should obey a mathematical expression that can be obtained from the third law of Kepler and the Law of Universal Gravitation as postulated by Isaac Newton [1, 8-11]. Each of these two denominations (Kepler's Laws and Dark Matter) deserve attention and in detail [12, 13].

Given the lack of clear answers regarding this enigmatic part of cosmology about the missing mass, the questions that can be posed are many, and those that are addressed in this paper are: Are Kepler's Laws and the law of Newton's gravitation universally applicable? In a galaxy and in the clusters, do Kepler's laws have the same validity as in the solar system? Does Dark Matter really exist? [14-19].

With the results of this work we aim to clarify, on the one hand, that the laws of Kepler and Newton, as we know them, are not universally applicable and, on the other hand, that the mathematical expression used by Vera Rubin was not adequate to explain how the radial velocity of stars and galaxies varies [11]. Here are presented the calculations that explain the supposedly excessive speeds that have been experimentally found with stars and galaxies that are moving. It will also be possible to conclude that the postulation of the existence of the Dark Matter was a departure that was too hasty, and the existence of something inexplicable was postulated. The measured speeds have a mathematical explanation that is clearly stated in the present work.

II. THE LAWS OF KEPLER AND THE LAW OF UNIVERSAL GRAVITATION

An important part of the work of Johannes Kepler (1571-1630), in seeking to adjust platonic solids with respect to the solar system [20], resulted in the formulation of the three so-called Kepler Laws. Kepler, famous in the accounts of a part of the history of relatively modern astronomy, had vision problems and also suffered from myopia and apparently never used a telescope. In a letter that Galileo Galilei (1564-1642) sent to Kepler on some occasion, he tells him about the use of the telescope. Galileo tells him that he invited scientific figures from the Vatican to observe the heavens through his telescope. The Vatican scientists did not want to see through that diabolical tube that they claimed would only obscure the mind [20].

Kepler's most important work, however, was based on astronomical observations made by another famous astronomer Ticho Brahe (1546-1601), who had also never used a telescope. Rather, Brahe only used much more primitive observation instruments, with which he gathered a large amount of data, that was mainly about the solar system and planets that were known at the time [20].

After a great reluctance Ticho Brahe, near the time of his death, agreed to share with Kepler the treasure, consisting mainly of observational data on the solar system. The information collected by Ticho Brahe contained, in a special way, important data about the planet Mars. After Mercury and Pluto, Mars is the planet whose orbit has a relatively large eccentricity. This fact allowed Kepler to establish his first law of planetary motion: The orbits of the planets are elliptical, with the sun in one of its foci [20-22].

Kepler took approximately 5 years to develop the first laws of planetary motion. One can imagine that it presented a great opposition to the acceptance of the fact that the orbits of the planets were not circular, because before that it was affirmed by Aristotle and Nicolaus Copernicus. Aristotle considered that the heavens contained perfection [21]. The corrupt factor was based here on Earth. Therefore, the orbits of the planets should be perfect circles. Nevertheless, Kepler can be considered as the first astronomer who introduced some beauty into the mathematical description of what the heavens were, circumscribing the great amount of data of Ticho Brahe and making those data correspond with no equal harmony with the elegance of his three laws of the planetary movements. It is possible to consider Kepler's laws as phenomenological laws since they are based on observables. Later Isaac Newton deduced and analytically verified Kepler's laws, including in particular the gravitational dynamics when considering point masses [23].

On the first law of Kepler we can state the following:

Le Verrier, an astronomer of French origin, discovered in 1859 that the orbit of Mercury manifested an anomaly, since it did not go through the same point in its orbit around the sun. It should be an ellipse but its orbit does not close as would be expected. Rather, after centuries of study it has been observed that it forms a kind of rosette. Le Verrier believed that an unknown planet or the presence of an asteroid cloud could be responsible for such a precession phenomenon since Newton's theory of mechanics predicted it [24]. He was interested in calculating this deviation, which was caused by the supposed planet or an asteroid cloud, by applying Newton's dynamics, but the calculations showed an irreconcilable difference with the measurements obtained from the observations. The value that Le Verrier calculated for Mercury's shift was 38 arc seconds per century (when the value measured in the sky was 43). These results, perhaps exaggeratedly, caused great confusion in physics and astronomy at that time; it was a shadow for the mechanics of Kepler and Newton that questioned the perfection of the mechanics as a dynamic theory [24].

It is now known that an explanation of this deviation from the normal ellipse has been obtained, it is claimed, using the theory of General Relativity [23, 25]. What can be safely assured is that Mercury is too close to the sun compared to Earth. The Earth is approximately 2.5 times farther from the sun than Mercury. This means that the image that could be seen of the sun from the surface of Mercury is much greater than as seen from the earth. This situation causes both the sun and mercury to experience intense Tidal Forces. On the other hand, it is well known that the Tidal Forces also disturb the Earth-Moon interaction. Due to these forces, the earth extends the duration of the day by approximately 20 seconds every million years. More than 500 million years ago, the days here on Earth lasted approximately 21 hours. By conservation of the kinetic moment of the Earth-Moon system, the Moon moves away from the Earth a little. Therefore, the moon's orbit does not have a perfect closure either. In the case of Mercury, the Tidal Forces may well disturb its orbit and, therefore, the movement is such that the orbit does not close at the same point as Kepler's first law affirms [25, 26].

The second law of Kepler: The vector of position of any of the planets relative to the sun sweeps equal areas of its ellipse in equal times (Law of the areas).

This law is not strictly true, since in the Earth-Moon interaction the Earth reduces its kinetic moment and this reduction is reflected in the Moon increasing its kinetic moment. As a result, the moon moves away from the earth due to this increase in momentum [26]. Kepler's second law is based on the conservation of the kinetic moment. This is preserved only in the complete system of interaction. The moment of the Moon increases, but it does not remain constant.

The third law of Kepler: The square of the periods of revolution is proportional to the cube of the average distances of the planets with respect to the sun.

The enunciation of this law depends strongly on the variation of the central force that the sun exerts on the planets. The law of universal gravitation was stated as being inversely proportional to the square of the average distance of the planets from the sun. It was called universal law because it was thought of as a law with universal validity, that is, it is valid everywhere [1, 2]. This, as will be seen, is not maintained for the galactic dynamics. In a galaxy, the resulting force on a star does not always vary as does one on the distance squared. Due to the distribution of the set of stars in a galaxy, the force has another mathematical expression: it depends on the discretization of the mass and also on the shape of the density of stars. This situation will be explained in detail.

III. THE GENESIS OF THE DARK MATTER

As previously said, due to the incongruence between the data obtained in the galaxies and the calculations of Celestial Mechanics, it is postulated the existence of an exotic and strange substance that would cause the gravitational force on an important part of the stars in a galaxy was greater than that predicted by Newton and Kepler mechanics [3-5, 8-11, 27].

On the other hand (and also because it has not been possible to prove the existence of what has been called Dark Matter, despite the fact that several countries around the world have large installations with specialized detectors that seek precisely to identify the possible particles that would make up this strange substance), due to failure in detection, it is reasonable that legitimate doubts arise concerning the possible action of this strange substance on the stars in the galaxy and on the galaxies in the clusters [6, 15-17, 28] .

From all of this argumentation, what can be inferred is that the explanation of the measured (excessive) speeds could be contained within another cause. What is detected is that, in many galaxies, from a certain distance from the galactic centre, the speed of the stars practically no longer varies, thus it is no longer reduced as predicted by the Celestial Mechanics of Newton and Kepler. There is talk of a practically constant radial stellar velocity, as experimentally determined by Vera Rubin [8-11] (see Figures 1 and 2).

After a careful review, it can be stated that the way in which Vera Rubin confronted the experimental results with the theory was not adequate [11]. It is very likely that the reasoning has been more or less the following: Using Newton's law of universal gravitation we can obtain the third law of Kepler and vice versa:

Kepler's third law for any of the planets in the dynamics of the Solar System is,

$$T^2 = ka^3 \quad (1)$$

Where k is Kepler's constant. The average distance between the sun and the planet is a.

This expression can be obtained from the balance between the force of gravity exerted by the sun on the planet and the so-called centrifugal force of the circular movement. By equating the law of universal gravitation with the "centrifugal" force, one has,

$$GMm/r^2 = mv^2/r \quad (2)$$

where G is the gravitation constant, M is the mass of the central body, in this case the sun, the mass of the planet is m, and the planet is at distance r.

For a circular orbit, it is said that,

$$v = 2\pi\, r/T = 2\pi a/T \quad (3)$$

With the expression (3) substituted in (2) we have,

$T^2 = ka^3$ (4)

The third law of Kepler is reproduced with $k = 4\pi^2/GM$.

Also, we see that from expression (2) we have,

$v^2 = GM/r$ (5)

This last expression (5) that has validity in the solar system, coincides with the expression that Vera Rubin used to confront the experimental results [11]. Unfortunately, the experimental results did not coincide with the predictions of the expression (5) for many of the stars of several galaxies. Therefore, Rubin concluded that there was no agreement and she forced to accept the postulate of the existence of dark matter [11].

According to the expression (5), the speed must decrease as $1/\sqrt{r}$, as does happen for the planets of the solar system.

In many of the experimental results obtained by Vera Rubin, the speed does not decrease, in fact it remains almost constant [11]. This forced the postulation of the strange matter that would act as the missing mass that should be added to prevent the galaxy from disintegrating.

The mathematical reality is another story, as will be seen below.

IV. THE NON-UNIVERSALITY OF LAWS

Firstly, it is somewhat complicated to discuss the universality of the laws that have been established as physical laws. In fact, nature, we can affirm, has universal laws in the sense that almost everything works well in the different areas in which natural laws manifest themselves. It is to be expected that if a mathematical expression is specified for a certain law of nature, it should be clarified that the specified law is not properly the act of nature. In other words, a theory is not nature in a proper sense, but only an abstraction, and it is a representation that man makes about a natural phenomenon which, with luck, will be a good approximation. In this sense, always the representation that man makes of what he says that nature is, will have a degree of idealization. It is to draw attention that Isaac Newton has enunciated the law of gravitation in the way he did: Law of "universal" gravitation [1].

It is known that with this Newton aimed to clarify that it was a mathematical expression that could be used in the same on Earth as in the Heavens. With this Newton sought to overcome the Aristotelian precept that the earth and the heavens were governed by different laws. As aforementioned, the heavens should be perfect.

Now let us see how the force of gravity manifests itself when it comes to a galaxy, since it is a gravitational system with some differences, with respect to the Solar System, and those differences are important. There are two fundamental differences between a solar system and a galaxy.

a) The mass in a solar system is concentrated in a small part of the volume of the system, mainly in the sun. In a galaxy, the mass is distributed practically throughout the entire volume of the system.
b) The way in which the mass is distributed in the galactic volume is of great importance. That is, the shape of the mass density $\rho(r, \theta, \phi)$ is decisive, in that there will always be a distribution that must be included.

According to the experimental results of Vera Rubin, the speed of a significant number of stars in a galaxy is constant, and practically this does not vary with radial distance. Considering that they were circular orbits, taking the expression (3) for constant speed, we would have to modify Kepler's third law in the following way:

From expression (3) with $v = \text{constant}$ we have a new expression for Kepler's third law,

$T^2 = K\,a^2$ (6)

here $K = 4\pi^2/v^2$

In a gravitational system where the measured velocity is a constant, Kepler's third law would have to say that the squares of the periods of the stars would be proportional to the squares of the radial distances. It can be seen in equation (2) that Kepler's third law, as is known, can be derived from a law of gravitation that depends on $1/r^2$. Any other expression for Kepler's third law would have to be derived from a gravitational force, at least with a different radial dependence.

Next, a new expression for the law of gravitation is derived. When a massive body, such as a star, is immersed in a medium other than a vacuum, the gravitational force has another radial dependence. Let us see this, using Gauss's law for the flow of a field, in this case gravitational, for the case of a galaxy, introducing considerations of symmetry to achieve a simplification in the calculations, and derive what is obtained for the gravitational force on a significant number of stars in a galaxy with a different radial dependence. Next, we describe the details.

V. A SPHERICAL GALAXY WITH RADIAL SYMMETRY

In general, galaxies turn out to have very varied forms. The best known forms are spirals with arms, which are discs with a bulge towards the central part. In the present work, to simplify the calculations we assume a spherical galaxy with a given mass distribution. In the universe, we have a discretization of the mass, and here we will consider that the distribution is continuous and depends only on r (see Figure 1).

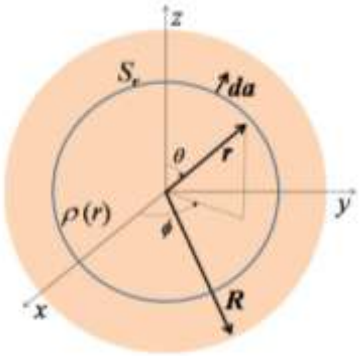

Figure 1. Representation of a spherical galaxy with a density of stars $\rho(r)$. The central part represents a Gaussian surface S_r with a normal vector $\boldsymbol{da}$

Considering spherical symmetry, to obtain the expression of the field and the gravitational force, within the mass distribution, the equation for the mass within any sphere of radius r, in spherical coordinates, is written as

$$M_r = \int_0^{\pi}\int_0^{2\pi}\int_0^{r} \rho(r)\, dv = \int_0^{r} \rho(r)\, 4\pi\, r^2\, dr \qquad (7)$$

Using Gauss's law for the flow of a vector field such as the gravitational field, the flow of the field is written as

$$\oint \boldsymbol{g_r} \cdot \boldsymbol{d\bar{a}} = 4\pi\, G \int_0^{\pi}\int_0^{2\pi}\int_0^{r} \rho(r)\, dv \qquad (8)$$

where G is the gravitational constant, in bold type $\boldsymbol{g_r}$ is the gravitational field and $\boldsymbol{d\bar{a}}$ is a vector that is perpendicular to the Gaussian surface (see Figure 1). By symmetry, in the integration the gravitational field remains constant and we get

$$g_r = G\left(\int_0^{r} \rho(r) 4\pi\, r^2\, dr\right)/r^2 = G\, M_r/r^2 \qquad (9)$$

The mass inside the Gaussian surface M_r depends on the variable r. Therefore, the gravitational field at a distance $r < R$ from the centre of the mass distribution has a strong dependence on how the density function varies $\rho(r)$, which determines M_r. Here, R would be the radius of the galaxy.

Example 1

It can be seen that with a density distribution that has the form

$$\rho(r) = \kappa/r \qquad (10)$$

where κ a constant, from equation (7) it turns out that

$$\left(\int_0^{r} \rho(r) 4\pi\, r^2\, dr\right) = M_r = 2\,\pi\,\kappa\, r^2 \qquad (11)$$

The gravitational field, equation (9) within the distribution is, using for the calculation the expression (10)

$$g_r = G\, 2\,\pi\,\kappa = cte \qquad (12)$$

With this expression for the gravitational field within the mass distribution for a galaxy, with a density varying as per the expression (10), equation (2) is written as

$$\mathrm{G M_r m}/\mathrm{r}^2 = \mathrm{G}\, 2\,\pi\,\kappa\, \mathrm{m} = \mathrm{m v}^2/\mathrm{r} \qquad (13)$$

From this, the expression for the velocity of the stars in a galaxy, which has the density that has been proposed in equation (10), turns out to be

$$\mathrm{v} = \sqrt{\mathrm{G} 2\,\pi\,\kappa\, \mathrm{r}} \qquad (14)$$

That is, the speed will be greater as we move away from the centre of the galaxy.

$$\mathrm{v} \propto \sqrt{\mathrm{r}} \qquad (15)$$

It is very likely that if one searches for this type of galaxy, one can find some galaxies whose velocity increases more or less in this way, and whose density will have approximately the variation that is proposed in equation (10).

Example 2

Among many others, an interesting case that can be treated, is one with a mass distribution that varies as

$$\rho(\mathrm{r}) = \kappa/\mathrm{r}^2 \qquad (16)$$

With this distribution of mass density in a galaxy such as Andromeda, but with more spherical symmetry, equation (7) now indicates that the mass within the radius of sphere r increases with r at the first power

$$\left(\int_0^{\mathrm{r}} \rho(\mathrm{r}) 4\pi\, \mathrm{r}^2\, \mathrm{dr}\right) = \mathrm{M_r} = 2\,\pi\,\kappa\, \mathrm{r} \qquad (17)$$

With expression (17), equation (12) is now rewritten as

$$\mathrm{g_r} = \mathrm{G}\, 2\,\pi\,\kappa\,/\mathrm{r} \qquad (18)$$

The gravitational field within the mass distribution for a galaxy, with mass density varying as the expression (16) provides the gravitational force that equates with the "centrifugal" force, equation (13) is written as

$$\mathrm{G M_r m}/\mathrm{r}^2 = \mathrm{G}\, 2\,\pi\,\kappa\, \mathrm{m}\,/\mathrm{r} = \mathrm{m v}^2/\mathrm{r} \qquad (19)$$

The radial velocity for stars inside the distribution is then left as

$$v = \sqrt{G 2\pi\kappa} = \text{constant} \qquad (20)$$

In astronomical calculations, the approximations are usually very large. Also, in the velocity measurements for the stars the radial curve obtained by Vera Rubin is not so constant. This could be a consequence of the incorporation of two things that are great approximations to reality: a) it is considered a spherical galaxy, b) the proposed density of stars within the galaxy probably does not coincide with the real density, as is proposed here with the density distribution $\rho(r)$ in equation (16).

With these calculations, and given the proposed approaches, one can provide, in large part, an account of the anomalous (constant) speeds that were obtained by Vera Rubin and other researchers.

VI. NUMERICAL CALCULATIONS

As an additional demonstration, several configurations for a galaxy were simulated numerically: from the spherical galaxy passing through ellipsoidal configurations to having a disk with a small thickness. In each case, the resultant force was calculated vectorially for several stars in radial directions. The Newtonian expression was used with only the principle of vector superposition. Once the force was calculated, it was equated with the expression for the centrifugal force and from there the velocity of the stars was calculated.

Figure 2 shows some configurations of galaxies and the corresponding star velocities that were calculated numerically. In all cases, the 79 stars (a number that is arbitrarily chosen) were distributed approximately as $\rho(r) \propto 1/r^2$.

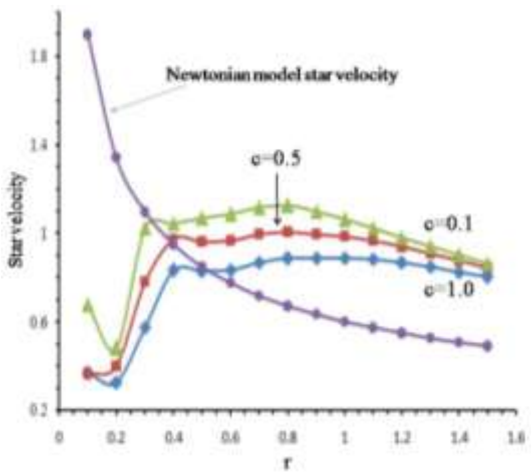

Figure 2. The velocity profiles, that are obtained numerically along a radius, for a discrete spheroidal distribution of particles, are shown. The spheroids consist of 79 particles that are located in three layers of radii 1/3, 2/3 and 1.0 on a normalised scale, and whose vertical semi-axis is c = 1.0, c = 0.5 and c = 0.1, respectively. In contrast to the velocity distribution for the Newtonian model, it is observed that the velocity profiles maintain a near-constant profile for r> 0.4, speeds that are similar to those in the work of Rubin, with radial velocities of several stars for spheroidal galactic configurations. A variation of the star density that is proportional to $1/r^2$ was assumed.

VII. COMMENTS AND CONCLUSIONS

The laws of nature, as such, seem to be applicable universally. Limitations generally appear when man performs mathematical abstractions. In this sense, there seems to be an important gap between mathematical abstraction and everyday reality. Sometimes, this leads to a postulate that mathematics is not formally a science, since it cannot always be verified experimentally. For all this, one must handle with prudence the qualifier that is relative to the universality of the laws that man has established on natural phenomena. The law of gravitation and Kepler's laws have a valid mathematical expression in the Solar System, however, a different mathematical expression is required in the Galactic System.

Unfortunately, of the mathematical expression used by Vera Rubin, expression (5) is not adequate. In fact, the force experienced by many of the stars in a galaxy has much more to do with a gravitational field given by the expression (18) instead of the expression for the traditional gravitational force of equation (2). With the expression in equation (18) the almost constant speeds determined experimentally by Vera Rubin and others can be seen.

Given this argument, the mathematical calculations and the numerical simulation, it is concluded that the postulate of the existence of that strange substance called Dark Matter is unnecessary. Using the appropriate celestial mechanics for galaxies, one can take into account, with a good approximation, the experimental results that for years remained unexplained, regarding stellar dynamics in galaxies and of galaxies in clusters.

To conclude, one can specify a maxim for the scientific method: One can use the method, but one should not introduce something incomprehensible. One should look for something, but one should not try to explain something by introducing something inexplicable. When this is done, one will have two problems: the original problem and now also the new problem. When introducing the Strange and Exotic Matter, then something inexplicable will remain in the discussion.

Here, the test has been verified in several ways: the so-called Dark Matter never existed as something cryptic.

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