

Validation of Simulated Hydraulic Jump Models

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ABSTRACT

This research work is aimed at validating simulated hydraulic jump models with specific objectives which includes simulating a model that can efficiently analyze and predicts the characteristics of hydraulic jump, compare the result with computations from known models. The method of comparison used is Pearson's product moment of correlation. The results obtained from the developed model showed that critical depth(y_c), Froude number (F_r), Subsequent depth y_2 , height of jump (H_j), Length of jump (L_j), Strength of jump (S_j), velocity after the jump, specific energy (E), specific energy after the jump (E_2), energy loss (H_l), efficiency of the jump (E_{fj}), Power dissipated (P_d), contraction width (b_c) are 0.86m, 2.97,1.55m, 1.14m, 6.81m, 3.72m, 1.61m, 2.25m, 1.68m, 0.57m, 0.75, 50,329.9, 1.04m respectively. The program output gave same results as that of the empirical computation with Pearson's coefficient of correlation of $r = 0.99$. Hence, the developed model is much better than existing hydraulic model because it can be operated easily, fastly, accurately, effectively and efficiently. Federal government should implement policies for the adoption of the developed optimization models to optimally allocate resources in multi-purpose and multi-objective river basin projects as per the requirement of ISO 9001:2015. and ISO 14001:2015 requirements.

KEYWORD: Simulation, Validation, Hydraulic Jump, Models. Correlation, Critical Velocity, Supercritical Flow, Froude Number, Multipurpose Project

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I. INTRODUCTION

Hydraulic engineering problem cannot be over emphasized. This research seeks to solve identified hydraulic engineering problems using hydraulic jump model. Circles of constraints exist in waterways which includes inadequate availability of hydraulic characteristics data for easy navigation on the water ways, fishing, recreational activities, hydroelectric power generation, erosion at the banks of rivers, jetty's, hydraulic structures and spill ways, increased casualties; destruction of goods and properties during navigation on water ways, insufficient head of water for navigation, high cost of mixing chemicals and aerate water in water and waste water treatment plant and cumbersomeness in manual computation of hydraulic engineering problems which involves so much iteration that results to waste of time and cost (Akpan and Ledogo, 2018).

Hydraulic jump has certain characteristics which include length of jump, height of jump, strength of jump, energy loss (Youngkyu, Gyewoon,

Hyoseon and Seongjoon ,2015), power dissipated (Mahmoud, 2016), critical velocity and depth of flow (Veysel, Oguz , Nazire and Mevlut, 2016),. initial depth and subsequent depth of flow (Ming-Jyh and Sam,2016), initial and final velocity of flow, critical, subcritical, super critical flow (Mohit and Lodhi, 2015), Froude number (Hang and Hubert , 2015), and contraction width (Yadav , Ahmad and Asawa ,2012).

Scholars have worked on several hydraulic jump scenarios. Peter, (2015) describes a new model of internal hydraulic jumps in two-layer systems that place no restrictions (such as the Boussinesq approximation) on the densities of the two fluids. Steinrück , Schneider and Grillhofer, (2013) did an extensive research on multiple scales analysis of the undular hydraulic jump in turbulent open channel flow. Lopardo, (deweiver osla (2013 eerf woleb snoitautculf yticolev emertxe eht .spmuj ciluardyh The study of Karbasi and Azamathulla,(2016) presented new application of Gene Expression Programming (GEP) to predict

characteristics of a hydraulic jump over a rough bed. No scholar except the work of Akpan and Ledogo (2015) has simulated hydraulic jump models in visual basic programming language. Hence this research seeks to validate simulated hydraulic jump models developed by Akpan and Ledogo(2015) .

II. MATERIALS AND METHODS

The materials and method used to executing this research are based on the work of Akpan and Ledogo (2015).

The case study is a rectangular channel of about 3-6m wide, having a depth of 0.4167m and

discharging 9.0 m³/s of water with a velocity 6.m/s as shown in figure 2.1 below. Using the case study, the results of the manual analysis was carefully calculated and recorded. Simulation on the model's program output which is the program results was equally recorded and comparison of the manual analysis and developed model compared. 100% accuracy is expected. In this research determination of the properties of hydraulic jump models, simulation of the hydraulic jump model, and validating the models developed using Pearson product moment correlation was done.

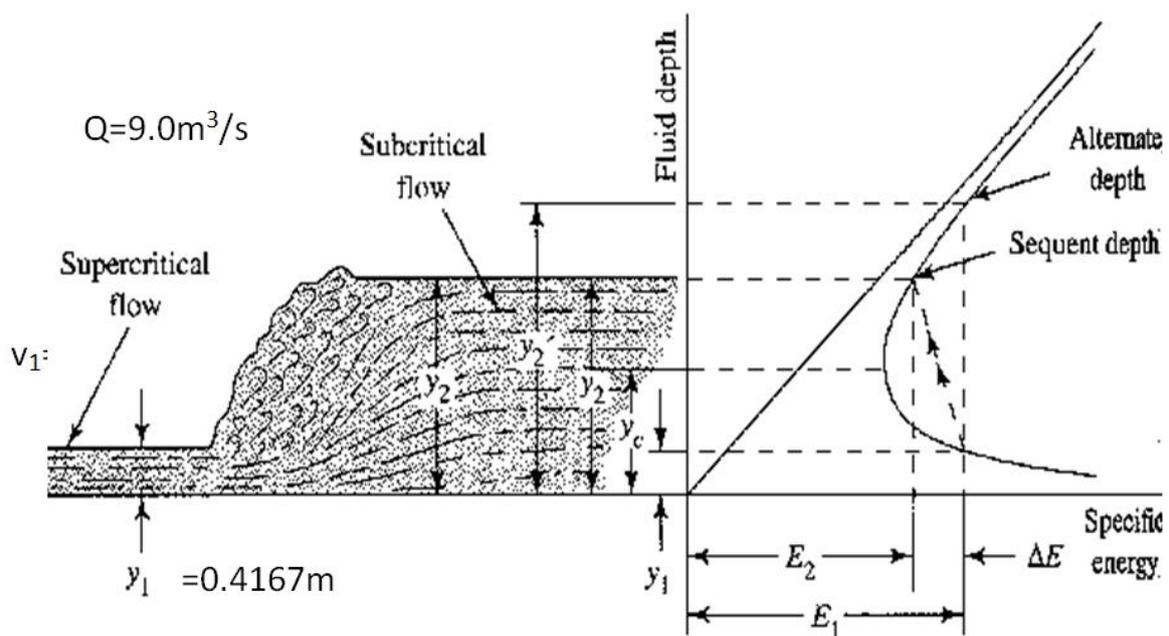


Figure 2.1: Case study for manual computation and program output

Having developed the model in view of the thoughts of Akpan and Ledogo, (2015) and applying the case study in figure 2.1 above, a rectangular channel of about 3-6m wide discharging 9.0 m³/s of water with a velocity 6.m/s all the characteristics of the jump was analyzed manually by calculating and compare the result with the modeled program. The

input parameter used in the case study includes the discharge, initial dept of flow, velocity, and width of the channel are 9.0m³/s, 0.4167m, 6.0m/s and 3.6 respectively. The method of running the developed model is presented in the work of Akpan and Ledogo, (2015).

III. ANALYSIS OF HYDRAULIC JUMP BASED ON THE CASE STUDY ABOVE IN FIGURE 2.1

1 Depth of water before jump, $y = \frac{Q}{b \times v_1} = \frac{9.0}{3.6 \times 6} = 0.4167.m(Ans)$

Discharge per unit width = $\frac{Q}{b} = \frac{9.0}{3.6} = 2.5 m^3 s \text{ per } m(Ans)$

2 Critical depth, $y_c = \left[\frac{q^2}{g} \right]^{1/3} = \left[\frac{2.5^2}{9.81} \right]^{1/3} = 0.86m(Ans)$ Since , $y_1 < y_c$, a jump would occur.

3 Froude number ahead of jump.

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{6}{\sqrt{9.81 \times 0.4167}} = 2.967(\text{Ans}) \text{ Since, } Fr_1 > 1, \text{ the flow is a shooting flow}$$

4 Subsequent depth $y_2 = \frac{v_1}{2} \left[\sqrt{1 + 8(fr)^2} \right]$

$$\text{Depth of water downstream the jump } y_2 = \frac{0.4167}{2} \left[\sqrt{1 + 8 \times 2.967^2} - 1 \right] = 1.5525m(\text{Ans})$$

5 Height of jump $H_j = y_2 - y_1 = 1.5525 - 0.4167 = 1.1358m(\text{Ans.})$

6 Length of jump, $L_j = 6(y_2 - y_1) = 6 \times 1.1358 = 6.8148m(\text{Ans.})$

7 Strength of jump $S_j = \frac{y_2}{y_1} = \frac{1.5525}{0.4167} = 3.726m(\text{Ans.})$

Velocity before jump, $v_1 = 6m/s$

8 Velocity after jump, $v_2 = \frac{q}{y_2} = \frac{2-5}{1.5525} = 1.61m(\text{Ans})$

9 Specific Energy before the jump $E_1 = y_1 + \frac{v_1^2}{2g} = 0.4167 + \frac{6^2}{2 \times 9.81} = 2.25m(\text{Ans})$

10 Specific Energy after the jump E_2

$$E_2 = y_2 + \frac{v_2^2}{2g} = 1.5525 + \frac{1.61^2}{2 \times 9.81} = 1.68m(\text{Ans})$$

11 Energy loss, Lose of energy in the jump, $E_1 = E_2 = 2.25 - 1.68 = 0.57m(\text{Ans.})$

$$\left[\text{Altrnatively, } E_l = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(1.5525 - 0.4167)^3}{4 \times 1.5525 \times 0.4167} = 0.57m \right]$$

12 Efficiency of the jump $= \frac{(8fr^2+1)^{3/2} - [4fr^2+1]}{8fr^2(2+fr^2)} \times 100$

$$= \frac{(8 \times 2.967^2 + 1)^{3/2} - (4 \times 2.967^2 + 1)}{8 \times 2.967^2(2 + 2.967^2)} \times 100$$

$$= \frac{(8 \times 8.8031 + 1)^{3/2} - (4 \times 8.8031 + 1)}{8 \times 8.8031(2 + 8.8031)} \times 100$$

$$= \frac{603.63 - 36.21}{760.81} \times 100 = 75\%$$

or

$$\frac{E_2}{E_1} \times 100 = \frac{1.68 \times}{2.25} \times 100 = 75\%$$

13 Power dissipated, $p = wQEL, P = 9810 \times [(b \times y)V_1] \times E_L,$
 $P = 9810 \times [(3.6 \times 0.4167)6] \times 0.57 = 50,329.24w(\text{Ans})$

14 Width of contraction $bc = \frac{Q}{\sqrt{\frac{2 \times g \times E^3}{3}}} = \frac{9.0}{\sqrt{\frac{2 \times 9.81 \times 2.25^3}{3}}} bc = \frac{9.0}{\sqrt{\frac{224.08}{3}}} \frac{9.0}{8.64} bc = 1.04m$

3.1 VALIDATION OF THE SIMULATED MODEL

The Pearson product moment of correlation was used to validate the simulated model. The data consist of result obtained from observed and predicted model.

The observed data were obtained from empirical calculation using known models while the predicted data were also derived from the simulated model presented in the work of Akpan and Ledogo (2015). The table below is the data presented to test the simulated data.

Table 3.1. Table of the Observed and Predicted Data

S/NO	DESCRIPTION	OBSERVED DATA(X)	PREDICTED DATA(Y)
1	Dept of water Y_1 (m)	0.4167	0.4167
2	Discharge per unit width (Q/d) (m)	3.6000	3.6000
3	Critical depth d_c (m)	0.8605	0.8605
4	Critical velocity V_c (m/s ⁻¹)	2.9054	2.9054
5	Froude number Fr_1	2.9675	2.9675
6	Subsequent depth Y_2 (m)	1.5525	1.5525
7	Height of the jump H_j (m)	1.1358	1.1361
8	Length of the Jump L_j (m)	6.8140	6.8140
9	Strength of the Jump S_j (m)	3.7260	3.7264
10	Velocity before the Jump V_1 (m/s ⁻¹)	6.0000	6.0000
11	Velocity after the Jump V_2 (m/s ⁻¹)	1.6100	1.6090
12	Specific Energy before the Jump E_1 (m)	2.2500	2.2510
13	Specific Energy after the Jump E_2 (m)	1.6800	1.6840
14	Energy Loss E_L (m)	0.5700	0.5660
15	Efficiency of the Jump Q_j (%)	75	75
16	Power dissipated P_d (kw)	50.329	50.125
17	Width of contraction (b_c)	1.0400	1.0400

Calculating the Pearson coefficient of correlation r for the above

$$x = X - \bar{x} \quad \text{and} \quad y = Y - \bar{Y}$$

where x and y represent the deviation from the mean of X and Y

Also, Calculating for the mean of the X correspondence $\bar{x} = \frac{\sum X}{n}$, $\bar{x} = \frac{162.25}{17}$, $\bar{x} = 9.5563$

Calculating for the mean of the Y correspondence, $\bar{y} = \frac{\sum Y}{n}$, $\bar{y} = \frac{162.25}{17}$, $\bar{y} = 9.5444$

Using the relation below to calculate the Pearson's product moment coefficient. of correlation

$$r = \frac{\sum xy}{\sqrt{(\sum x^2)(\sum y^2)}} = \frac{6758.74}{\sqrt{(6751.88)(6834.45)}} = \frac{6758.74}{\sqrt{46145386}} = \frac{6758.74}{6793} = 0.99$$

See table3.2 below for details

Table 3.2. Correlation Table of the Observed and Predicted Data

X	Y	$X - \bar{x}$	$(Y - \bar{Y})$	xy	x^2	y^2
0.42	0.42	-9.58	-11.58	111.01	91.84	134.17
3.60	3.60	-6.40	-8.40	53.76	40.96	70.56
0.86	0.86	-9.14	-11.14	101.81	83.53	124.09
2.91	2.91	-7.09	-9.09	64.52	50.33	82.71
2.97	2.97	-7.03	-9.03	63.52	49.46	81.59
1.55	1.55	-8.45	-10.45	88.26	71.36	109.15
1.14	1.14	-8.86	-10.86	96.30	78.57	118.02
6.81	6.81	-3.19	-5.19	16.52	10.15	26.89

3.73	3.73	-6.27	-8.27	51.91	39.36	68.45
6.00	6.00	-4.00	-6.00	24.00	16.00	36.00
1.61	1.61	-8.39	-10.39	87.18	70.39	107.97
2.25	2.25	-7.75	-9.75	75.55	60.06	95.04
1.68	1.68	-8.32	-10.32	85.83	69.22	106.42
0.57	0.57	-9.43	-11.43	107.82	88.92	130.74
75.00	75.00	65.00	63.00	4095.00	4225.00	3969.00
50.33	50.13	40.33	38.13	1537.54	1626.43	1453.52
1.04	1.04	-8.96	-10.96	98.20	80.28	120.12
162.46	162.25	-7.54	-41.75	6758.74	6751.88	6834.45

Base on the computed result from the observed and predicted data above, the Pearson's coefficient of correlation $r = 0.99$. This means that data obtained from the hydraulic models and the simulated model are similar.

IV. RESULTS AND DISCUSSIONS

The results are presented and discussed below .

4.0 Result of the Simulation Using Case Study in Figure 3.1 Above

The program output is the result of the case study. The same parameter used for the manual computation are also used as input data for the simulated model and the results are presented in figure 4.1 below

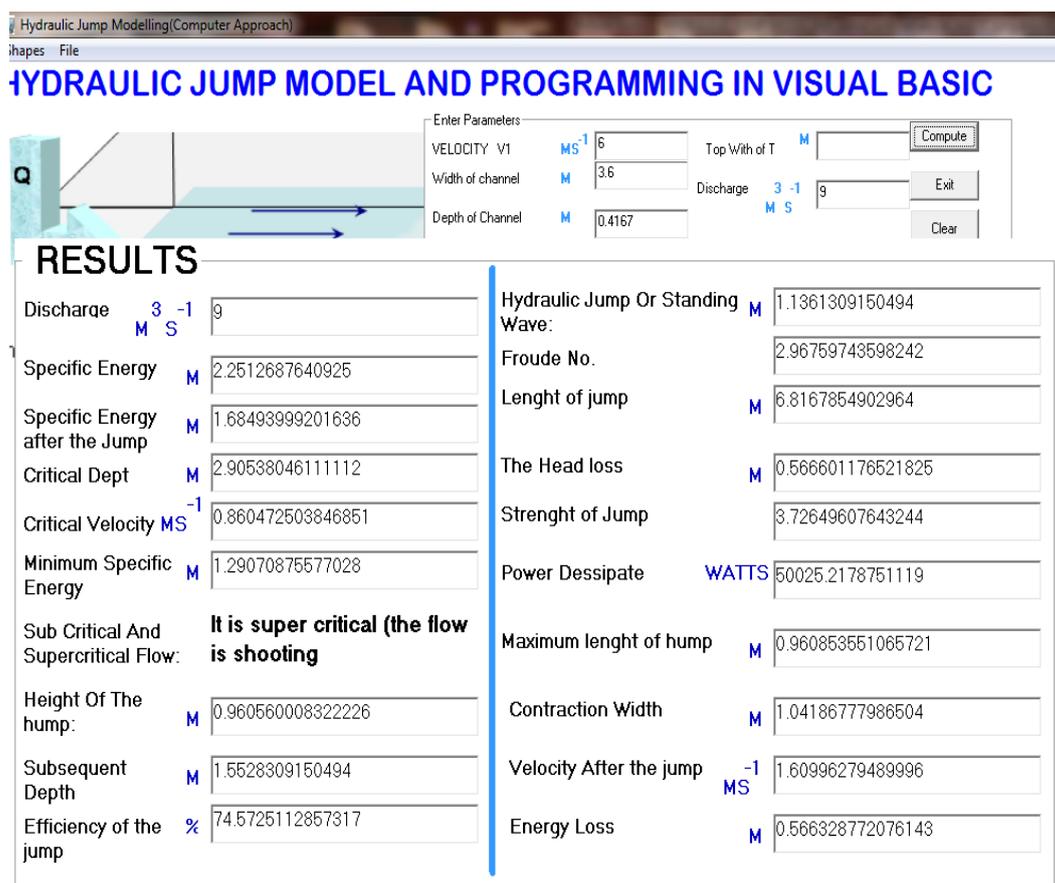


Figure 4.1. Results of Hydraulic Jump Characteristics Using Case Study in Figure 2.1(Akpan and Ledogo, 2015)

4.1 Discussion of results in figure 4.1 above

- i. The program output gave same results as that of the empirical computation.
- ii. The results obtained from the new model's computation of the characteristics of hydraulic jump for critical depth(y_c), Froude number (F_r), Subsequent depth y_2 , height of jump (H_j), Length of jump (L_j), Strength of jump (S_j), velocity after the jump, specific energy (E), specific energy after the jump (E_2), energy loss (H_l), efficiency of the jump (E_f), Power dissipated (P_d), contraction width (b_c) are 0.86m, 2.97,1.55m, 1.14m, 6.81m, 3.72m, 1.61m, 2.25m, 1.68m, 0.57m, 0.75, 50,329.9, 1.04m respectively.

- iii. The program is written and compile with visual basic 6.0. Also it is written to a storage device (CD –ROM). The program can be access directly from the CD-ROM drive and can run on any machine install with Microsoft windows (Operating System), e.g., windows XP, Window 7, Window Vista etc. Also the program is user friendly (users can make use of standard keyboard and mouse).

4.2 Results of Correlation Coefficient of the Observed and Predicted Data

Base on the result computed in table 4.1 below the Pearson's coefficient of correlation $r = 0.99$. This means that data obtained from the hydraulic models and the simulated model are similar.

Table 4.1. Correlation table for observed and predicted data

X	Y	$\bar{X} - x$	$(Y - \bar{Y})$	xy	x^2	y^2
0.42	0.42	-9.58	-11.58	111.01	91.84	134.17
3.60	3.60	-6.40	-8.40	53.76	40.96	70.56
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1.14	1.14	-8.86	-10.86	96.30	78.57	118.02
6.81	6.81	-3.19	-5.19	16.52	10.15	26.89
3.73	3.73	-6.27	-8.27	51.91	39.36	68.45
6.00	6.00	-4.00	-6.00	24.00	16.00	36.00
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1.04	1.04	-8.96	-10.96	98.20	80.28	120.12
162.46	162.25	-7.54	-41.75	6758.74	6751.88	6834.45

4.3 Discussion of results in table 4.1 above

- i. The Pearson product moment of correlation was used to test the results of hydraulic jump models and simulated model.
- ii. The observed and predicted data were computed from known hydraulic jump models and simulated hydraulic jump model respectively.
- iii. Two methods were used to compute the Pearson's value

$$r = \frac{n \sum XY - \sum X - \sum Y}{\sqrt{[n \sum X^2 - (\sum X)^2][n \sum Y^2 - (\sum Y)^2]}} \quad \text{and} \quad r = \frac{\sum xy}{\sqrt{(\sum x^2)(\sum y^2)}}$$

iv. Two Pearson product moment coefficient of correlation models used produced same result as $r = 0.99$

V. CONCLUSION

Conclusively, results showed that both models gave same values of the characteristics of hydraulic jump for critical depth (y_c), Froude number (F_r), Subsequent depth y_2 , height of jump (H_j), Length of jump (L_j), Strength of jump (S_j), velocity after the jump, specific energy (E), specific energy after the jump (E_2), energy loss (H_l), efficiency of the jump (Eff), Power dissipated (P_d), contraction width (b_c) as 0.86m, 2.97, 1.55m, 1.14m, 6.81m, 3.72m, 1.61m, 2.25m, 1.68m, 0.57m, 0.75, 50, 329.9, 1.04m respectively.

Finally, the program output gave same results as that of the empirical computation with Pearson's coefficient of correlation of $r = 0.99$. This implies that the simulated hydraulic jump model is validated and can be used at ease to execute work efficiently.

VI. RECOMMENDATIONS

The following are recommended.

Ministry of marine transport should implement policies for the use of this simulated hydraulic jump model as it will analyze automatically and produce the desired Froude number that can enable hydraulic engineers determine the nature of the flow.

Nigeria Institute of Maritime and safety agency (NIMASA) should implement the use of the model to forecast the velocity of flow required for safe navigation.

Ministries of River Basin Authorities should adopt the model in determination of the head of flow required to produce the desired jump for Hydroelectric power, Irrigation, Navigation, Fishing, Water supply, and Recreation projects.

Waste management society of Nigeria should drive the implementation of the model since it can be used to determine the jump that can mix, aerate and purify water and waste water for effective treatment. Ministry of works should implement the use of the simulated model in analysis, design, construction and maintenance of Jetties in riverine areas of the Niger delta because power dissipated from supercritical flows can be easily derived from the model to prevent erosion, uplift of foundation and scour velocity.

Rivers state river basin authorities should implement the new model since it can be used to determine the characteristics of hydraulic jump which include critical depth (y_c), Froude number (F_r), Subsequent

depth y_2 , height of jump (H_j), Length of jump (L_j), Strength of jump (S_j), velocity after the jump, specific energy (E), specific energy after the jump (E_2), energy loss (H_l), efficiency of the jump (Eff), Power dissipated (P_d)

VII CONTRIBUTION TO KNOWLEDGE

Research like this will ginger engineering graduates to explore simulations of models using programming language since it will help in executing work speedily with accuracy and at ease. It will help engineers to effectively minimize time and maximize profit in executing projects.

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