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IIR System identification using Elitist Teaching-Learning-Based Optimization Algorithm

P.Eswari*, Y.Ramalakshmanna**
** (MTECH Student, Department of ECE, SRKR Engineering College, Bhimavaram, Andhra Pradesh, India.*
*** (Associate Professor, Department of ECE, SRKR Engineering College, Bhimavaram, Andhra Pradesh, India.*

ABSTRACT:
The adaptive IIR system identification has become a major area of concern in signal processing systems. The IIR filter models are nonlinear in practical, the classical methods like least mean square error and gradient based optimization techniques cannot reduce the stability problem in filter and meet the requirements of the system. The optimization algorithm decreases the stability problem. In this paper ETLBO is used for adaptive IIR system identification. The elitism concept with TLBO affects the exploration and exploitation capabilities of the algorithm. ETLBO showed better results compared to DE. ETLBO provides good convergence rate compared to DE.
Keywords: Adaptive filter, Coefficients of filter, IIR System identification, Mean square error, Optimization algorithm.

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I. INTRODUCTION

System identification is a challenging area of research due to its non-linear and recursive model structure. Digital filters play important role for signal separation and restoration in DSP applications. These filters are used in signal processing, communication, military, biomedical applications etc. The feedback of IIR filter provides infinite impulse response with finite number of coefficients. The output in FIR filter only depends on present and previous inputs, but in IIR filter the output depends not only on present input but also on previous inputs and outputs. So, IIR filter is complex in structure and requires large memory compared to FIR filter. IIR filter contains both poles and zeros. There exists stability problem if the poles move outside the unit circle during optimization process. But IIR filter can attain a particular level of performance with lesser order compared to FIR filter. IIR system identification model has non-linear error surface due to the presence of feedback polynomial. The conventional methods require continuous and differentiable cost function. They cannot work on large search space problems with ease. The conventional methods also struck in local optimum points. Taking these issues into consideration, the optimization algorithms

which are inspired from nature are used to solve these non-linear, recursive error models.

The concept of IIR system identification is to adaptively modify the coefficients of adaptive filter using optimization algorithm such that it matches with input/output configuration of unknown system. The aim of this project is IIR system identification using ETLBO algorithm. The results are compared with differential evolution (DE) algorithm. Mean square error (MSE) of output responses between the unknown system and adaptive filter is taken as fitness function. The ETLBO showed better performance compared to DE. ETLBO converges faster to best optimal solution compared to DE. It reduced the error greatly compared to DE.

II. IIR SYSTEM IDENTIFICATION

It is non-linear and recursive error model. The main task of the system identification is to make the parameters of the adaptive filter modify iteratively using optimization algorithms until it matches with the output response of unknown filter. In system identification configuration, the adaptive algorithm searches the adaptive filter coefficients such that input/output relation matches closely with the unknown filter.

The input-output relation in IIR filter is given by the difference equation:

$$y(i) + \sum_{k=1}^{n} a_k y(i-k) = \sum_{k=0}^{m} b_k x(i-k) \qquad (1)$$

where x(i) and y(i) are the input and output of the filter, the order of the filter is the greater of n or m.

The block diagram of adaptive filter for IIR system identification is shown in Fig.1.

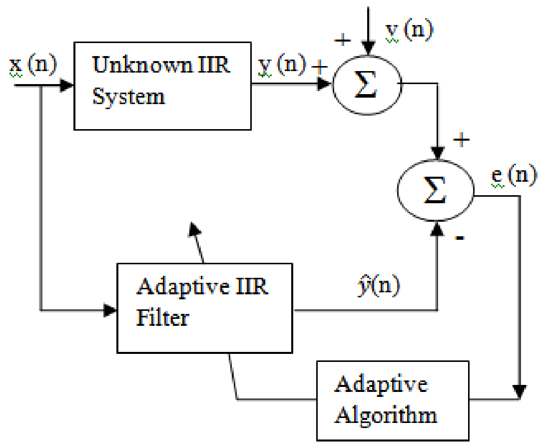

Fig.1. Block diagram of adaptive filter for IIR system identification

The transfer function in z-domain of unknown plant is given by

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1 + \sum_{k=1}^{n} a_k z^{-k}} \quad (2)$$

a_k and b_k are the coefficients of unknown filter. B(z) represents feed-forward (numerator) polynomial and A(z) represents feedback (denominator) polynomial in z-domain. The output response of unknown IIR filter is given by

$$y(n) = H(z)x(n) \quad (3)$$

The output of unknown plant in Fig.1 is given by

$$y_0(n) = y(n) + v(n) \quad (4)$$

The transfer function in z-domain of adaptive filter is given by

$$H_A(z) = \frac{\hat{B}(z)}{\hat{A}(z)} = \frac{\sum_{k=0}^{m} \widehat{b_k} z^{-k}}{1 + \sum_{k=1}^{n} \widehat{a_k} z^{-k}} \quad (5)$$

The output response of unknown IIR filter is given by

$$\hat{y}(n) = H_A(z)x(n) \quad (6)$$

e(n) represents the error function between the output response of unknown filter y(n) and adaptive filter $\hat{y}(n)$.

$$e(n) = y_0(n) - \hat{y}(n) \quad (7)$$

The mean square of the error (MSE) of error function is considered as fitness function. The MSE must be as low as possible.

$$E[e(n)] = \frac{1}{N}\sum_{p=1}^{N} e(n)^2 \quad (8)$$

N represents the total number of input samples. E(.) is the statistical expectation operator.

III. OPTIMIZATION ALGORITHM

3.1. ELITIST TEACHING-LEARNING-BASED OPTIMIZATION ALGORITHM (ETLBO)

TLBO algorithm is proposed by R.V.Rao. Unlike other population based algorithms, the performance of the TLBO depends on only common algorithm parameters like population size, length of design variables and number of iterations. The evolutionary (EA) and swarm intelligence (SI) based algorithms performance depends on both the common algorithm parameters and algorithm specific parameters. The tuning and controlling of algorithm specific parameters is complex, time consuming and affects the exploration and exploitation capabilities of the algorithm. The crossover and mutation probabilities in GA, inertial weights, acceleration rate in PSO, scaling factor in DE etc, represents the algorithm specific parameters. The performance of the algorithm depends on the proper selection of algorithm specific parameters.

The improper selection results in excessive computational strain or local optimum convergence. Compared to population based algorithm, TLBO is simple, fast, easy to implement and powerful. TLBO is algorithm specific parameter less algorithm.

TLBO is inspired by teaching-learning process in a class. It is built on the concept how the learners improve their knowledge from teacher and other learners who have better knowledge than them. The teacher is the most knowledgeable person among the learners. The total number of learners denotes the population size; the number of subjects of the learner represented the design variable ain optimization algorithm. The working process is done in two phases: 1. Teacher phase, 2. Learner phase.

Teacher phase:

In this phase the learners improve their knowledge with the help of interaction with teacher. The teacher shares his/her views in order to improve the overall success rate of the learners. The teacher tries to improve the mean of the team of learners in each subject his/her they teach. The best solution among the learner solutions is considered as teacher solution. The difference between the existing mean of the learners in each subject and teacher solution is given by,

Differencemean = *rand* * (X_{best} - T_f X_{mean}) (9)

X_{mean} represents the mean result of learners in all the subjects. T_f is the teaching factor. The value of T_f will be 1 or 2 based on rounding up criteria given.
rand is a value between [0, 1].

$$T_f = \text{round}\ [1 + (\text{rand}\ (0, 1) * (2-1))] \tag{10}$$

The existing solution is updated in teacher phase as,

$$X_{new} = X_{old} + \text{Differencemean} \tag{11}$$

The updated X_{new} is accepted only if fitness of X_{new} is better than X_{old}. The accepted solution in teacher phase becomes the input to learner phase.
Learner Phase:
The learner also improves his/her knowledge from the interaction with other learners who have more knowledge than them. Randomly select two learners X_a and X_b.
IF (fitness(X_a)> fitness(X_b))

$$X_{new} = X_{old} + r2\ (X_a - X_b) \tag{12}$$

ELSE

$$X_{new} = X_{old} + r2\ (X_b - X_a) \tag{13}$$

End
The X_{new} is accepted only if it has better fitness than X_{old}. r2 is a random number between [0, 1]. The updated X_{new} in learner phase is accepted only if fitness of X_{new} is better than X_{old}. The process continues until the termination criterion is met.
The elitism concept is included in TLBO to further improve the convergence towards optimal solution. Firstly the best solutions are stored as elite solutions. The worst solutions in the present iteration are replaced with best solutions of the previous iteration.

3.2. Differential Evolution (DE)

DE is a stochastic population based algorithm used to solve non-linear and non-differentiable problems. DE consists of three important concepts: mutation, crossover and selection. Firstly the population is initialized with N population size and with length of design variable as P. These are called target vectors (X). In mutation process, randomly three distinct vectors are selected. The donor vector is generated by adding the weighted difference of the two vectors to the third vector.
Let X_{r1}, X_{r2}, X_{r3} are three distant random vectors. 'v' represents the donor vector generated. The scaling factor F value is between 0 and 1.

$$v = X_{r1} + F*(X_{r2} - X_{r3}),\ r1 \neq r2 \neq r3 \tag{14}$$

In crossover process, the selection is done between the target vector and donor vector and trail vector is generated.

$$u_q = v_q,\ \text{if}((\text{rand} \leq CR)||(j==q)),\ q=1,2,..,P$$
$$u_q = X_q,\ \text{if}((\text{rand} > CR)||(j \neq q)) \tag{15}$$

'j' denotes random integer value between 1 and P. CR is the crossover rate and it is between 0 and 1.
In selection process, the produced trial vector is considered only if the fitness function of trial vector is greater than fitness function of target vector.
This process continues until the termination criterion is met. There are several types of DE; DE rand/1/bin scheme is used in this algorithm. The algorithm specific parameters of DE are scaling factor, crossover rate. The tuning of these parameters plays an important role in convergence of algorithm.

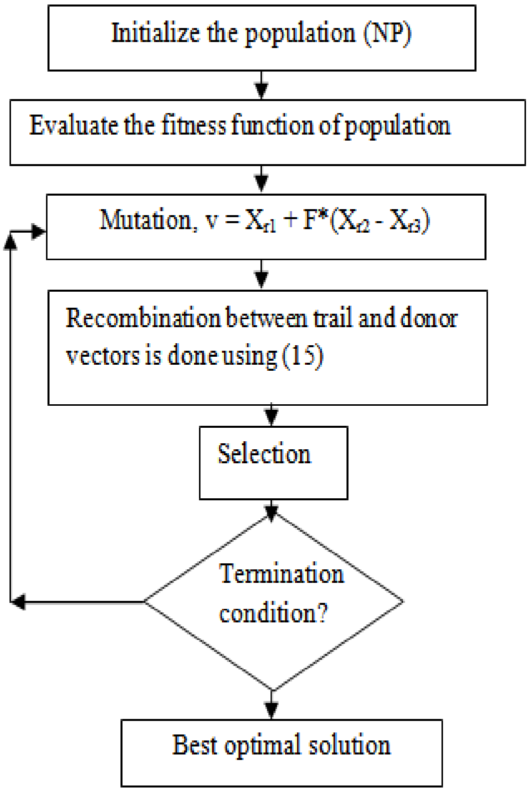

Fig2. Flowchart of DE

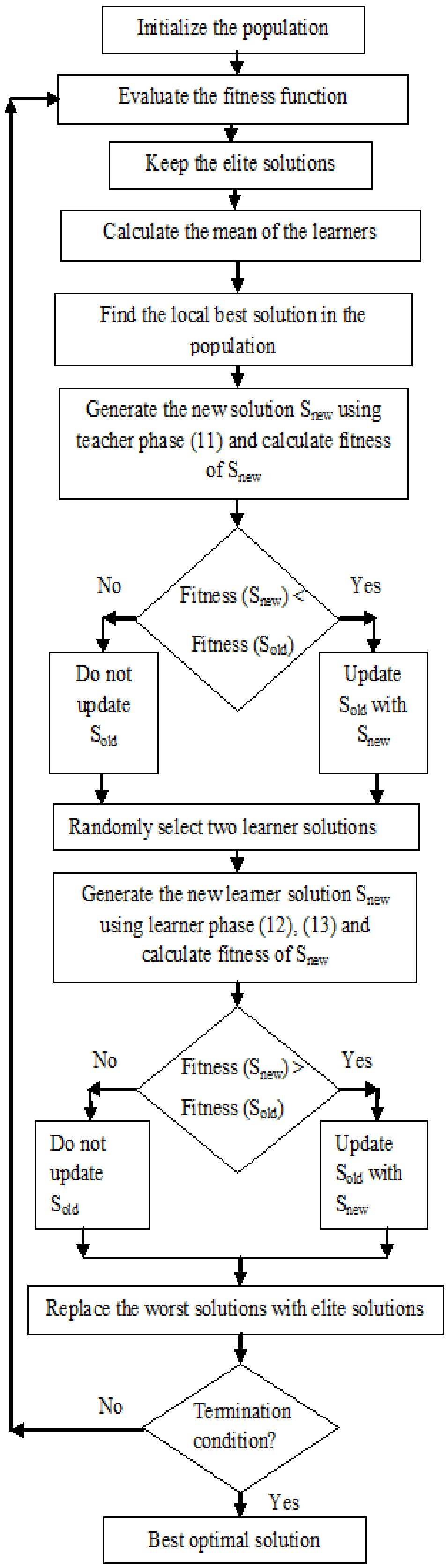

Fig3. Flowchart of ETLBO

IV. RESULTS

In this paper, IIR system identification is done using ETLBO and DE. The results are simulated using MATLAB 2018. The simulated results are used to find the performance of the algorithms in identifying the system. The random signal between [-1, 1] is given as input. The additive noise is a random noise signal.

Common algorithm parameters:
Number of iterations = 150
Population size = 25
The length of the design variables depends on order of filter and its coefficients.
ETLBO works on only common algorithm parameters. It is algorithm-specific parameter less algorithm.
Algorithm specific parameters of DE:
Scaling factor, F = 0.5
Crossover rate, CR = 0.9
The tuning of these parameters plays an important role in convergence of the algorithm.

Table-1: Optimized coefficients and MSE obtained using ETLBO and DE (2nd order IIR system)

	ETLBO					DE				
	a1	a2	b1	b2	MSE	a1	a2	b1	b2	MSE
Run1	1.2500	-0.2500	-0.3000	0.4000	3.1342e-26	1.2500	-0.2499	-0.3000	0.4000	3.4660e-09
Run2	1.2500	-0.2500	-0.3000	0.4000	9.3872e-28	1.2491	-0.2480	-0.2985	0.3983	2.8080e-06
Run3	1.2500	-0.2500	-0.3000	0.4000	5.8780e-30	1.2501	-0.2501	-0.3000	0.4001	4.0278e-09
Run4	1.2500	-0.2500	-0.3000	0.4000	6.3736e-26	1.2500	-0.2500	-0.3000	0.4000	1.1712e-13
Run5	1.2500	-0.2500	-0.3000	0.4000	2.0303e-25	1.2508	-0.2512	-0.3012	0.4018	5.1353e-06

Table-2: Optimized coefficients and MSE obtained using ETLBO and DE (3rd order IIR system)

ETLBO:	a0	a1	a2	b1	b2	b3	MSE
Run1	-0.2000	-0.4000	0.5000	-0.6000	0.2500	-0.2000	8.0917e-11
Run2	-0.2000	-0.4000	0.5000	-0.6000	0.2500	-0.2000	1.3446e-11
Run3	-0.2002	-0.4005	0.4986	-0.5976	0.2510	-0.1994	3.2494e-07
Run4	-0.2000	-0.4000	0.5000	-0.6000	0.2500	-0.2000	6.9685e-11
Run5	-0.2000	-0.4000	0.5000	-0.6000	0.2500	0.2000	2.4595e-11
DE:	**a0**	**a1**	**a2**	**b1**	**b2**	**b3**	**MSE**
Run1	-0.2029	-0.3994	0.4918	-0.5889	0.2649	-0.1964	7.3745e-05
Run2	-0.1988	-0.3972	0.5062	-0.6091	0.2460	-0.2045	6.0598e-06
Run3	-0.2002	-0.3991	0.5038	-0.6031	0.2467	-0.1999	6.6506e-06
Run4	-0.2011	-0.3999	0.4966	-0.5964	0.2518	-0.1962	7.6854e-06
Run5	-0.1994	-0.3982	0.4998	-0.5977	0.2484	-0.2007	1.6171e-05

Table-3: Optimized coefficients and MSE obtained using ETLBO and DE (4th order IIR system)

ETLBO:	a0	a1	a2	a3	b1	b2	b3	b4	MSE
Run1	0.9997	-0.8959	0.8107	-0.7259	0.0435	0.2805	-0.2076	0.1415	1.1227e-05
Run2	0.9999	-0.9000	0.8098	-0.7288	0.0401	0.2775	-0.2100	0.1401	3.8911e-08
Run3	1.0000	-0.9003	0.8099	-0.7309	0.0401	0.2774	-0.2115	0.1381	8.9245e-06
Run4	1.0000	-0.9112	0.8200	-0.7349	0.0316	0.2796	-0.2071	0.1393	7.6056e-05
Run5	0.9999	-0.8999	0.8101	-0.7289	0.0401	0.2777	-0.2099	0.1401	9.7689e-08
DE:	**a0**	**a1**	**a2**	**a3**	**b1**	**b2**	**b3**	**b4**	**MSE**
Run1	0.9991	-0.8892	0.7973	-0.7208	0.0487	0.2745	-0.2127	0.1406	1.8208e-04
Run2	0.9996	-0.9014	0.8235	-0.7289	0.0366	0.2844	-0.2033	0.1406	1.1360e-04
Run3	1.0000	-0.9000	0.8076	-0.7292	0.0406	0.2800	-0.2072	0.1428	5.0839e-05
Run4	0.9997	-0.8983	0.8264	-0.7472	0.0470	0.2962	-0.2093	0.1358	3.1541e-04
Run5	0.9885	-0.8841	0.7803	-0.7088	0.0526	0.2655	-0.2117	0.1403	7.8744e-04

4.1. Example 1:

A second order IIR system is considered from [3] with the transfer function as shown in (16)

$$H_2(z) = \frac{1.25z^{-1} - 0.25z^{-2}}{1 - 0.3z^{-1} + 0.4z^{-2}} \quad (16)$$

This second order plant is modelled using second order IIR filter $H_{a2}(z)$. The transfer function of $H_{a2}(z)$ is assumed as,

$$H_{a2}(z) = \frac{a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} \quad (17)$$

a_1, a_2, b_1 and b_2 are numerator and denominator coefficients of (17). Table-1 shows the quantitative analysis of coefficients and MSE. From the table it is evident that ETLBO reduced MSE greatly compared with DE. In [3], for 2nd order plant PSOCFIWA provided least MSE of 3.9548e-16 in 400 iterations but whereas in this paper ETLBO provided very least MSE of 5.8780e-30 for the same plant in just 150 iterations only. Considering all the outputs, it is clear that ETLBO provides very least MSE and better convergence than PSO, RGA, DE and PSOCFIWA.

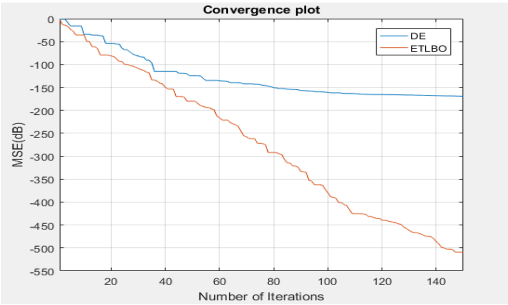

Fig.4. Convergence plot of example-1 modelled using 2nd order IIR filter

4.2. Example 2:

A transfer function of the system 3rd order system from [4] is as shown in (18)

$$H_2(z) = \frac{-0.2 - 0.4z^{-1} + 0.5z^{-2}}{1 - 0.6z^{-1} + 0.25z^{-2} - 0.2z^{-3}} \quad (18)$$

This third order plant is modelled using third order IIR filter $H_{a3}(z)$. The transfer function of $H_{a3}(z)$ is assumed as,

$$H_{a3}(z) = \frac{a_0 + a_1z^{-1} + a_2z^{-2}}{1 + b_1z^{-1} + b_2z^{-2} + b_3z^{-3}} \quad (19)$$

a_0, a_1, a_2, b_1, b_2 and b_3 are numerator and denominator coefficients of (19). Table-2 shows the quantitative analysis of coefficients and MSE for third order plant. ETLBO greatly reduced the mean square error compared with DE. In [4], CSO gives least MSE of $1.22363*10^{-5}$ in 200 iterations for 3rd plant, for the same plant ETLBO provided a MSE of $1.3446*10^{-11}$ in just 150 iterations. It is evident that ETLBO converges faster and shows better performance in identification of system when compared to CSO, PSO and GA also.

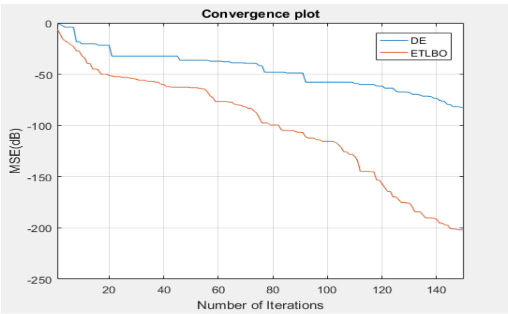

Fig.5. Convergence plot of example-2 modelled using 3rd order IIR filter

4.3. Example 3:

A 4th order system transfer function from [4] is as shown in (20)

$$H_4(z) = \frac{1 - 0.9z^{-1} + 0.81z^{-2} - 0.729z^{-3}}{1 + 0.04z^{-1} + 0.2775z^{-2} - 0.2101z^{-3} + 0.14z^{-4}} \quad (20)$$

It is modelled using fourth order IIR filter $H_{a4}(z)$. The transfer function of $H_{a4}(z)$ is assumed as,

$$H_{a4}(z) = \frac{a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3}}{1 + b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}} \quad (21)$$

a_0, a_1, a_2, a_3, b_1, b_2, b_3 and b_4 represent numerator and denominator coefficients of (21). Table-3 shows the optimized coefficients and MSE results of 4th order plant. ETLBO reduced MSE compared to DE. It almost matched all the coefficients of filter with unknown plant coefficients exactly. In [4], CSO gives least MSE of $1.39082*10^{-5}$ in 200 iterations for 4th order plant, for the same plant ETLBO provided a MSE of $3.8911*10^{-8}$ in just 150 iterations. It is obvious that ETLBO provides least MSE and converges faster than CSO, PSO and GA.

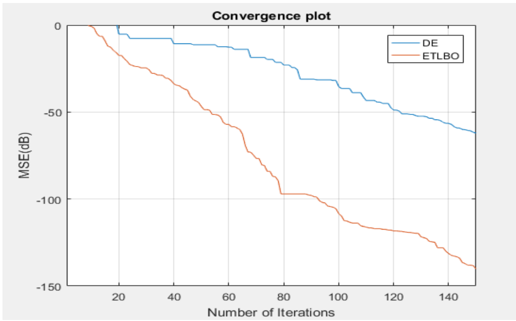

Fig.6. Convergence plot of example-3 modelled using 4th order IIR filter

V. CONCLUSION

In this paper, IIR system identification is done using ETLBO Algorithm. The results are compared with evolutionary algorithm, DE. The MATLAB is used for simulation of the results. A few benchmark transfer functions in [3], [4] are optimized using ETLBO and DE. From the results, it is clear that ETLBO provided least MSE in all second, third and fourth order systems. ETLBO algorithm showed good performance in matching the coefficients of the adaptive filter with unknown system. It performed system identification in less number of iterations when compared with CSO, PSO, RGA, PSOCFIWA and GA. ETLBO converges to the best optimal solution faster than DE. It does not require any algorithm specific parameter unlike DE.

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