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Hinge domination of cross product of special graphs

Kavitha B N*, Indrani Pramod Kelkar**
**(Department of Mathematics, Sri Venkateshwara college of Engineering, Bangalore,*
*** (Department of Mathematics,Acharya Institute of Technology, Bangalore,*

ABSTRACT
A set S of vertices in a graph G is a hinged dominating set if for every vertex u in V-S is adjacent to some vertex v in S and a vertex w in V-S such that (v,w) is not an edge in E. The hinged domination number $\gamma_h(G)$ is the minimum size of a hinged dominating set. In this paper, we find the hinge domination of cross product graphs$\mathbf{K_m \times P_n}$, $\mathbf{C_m \times P_n}$, $\mathbf{K_{m,r} \times P_n}$ $\boldsymbol{B_m}$, $\boldsymbol{B_{m,n}}$,$\boldsymbol{H_{m,n}}$and present the relation between hinge domination and domination number of these graphs.
***Keywords*–**Cross product,Domination, Hinge Domination.

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I. INTRODUCTION

The study of Cross product of graph was initiated by Imrich[9]. For structure and recognition of Cross Product of graph we refer to Imrich[10].

The study of domination in graphs has found rapid growth in the recent years. It is a highly flourishing area of research in graph theory. So far, hundreds of research articles have appeared on this topic of research in view of its growing real life IJPT| March-2016 | Vol. 8 | Issue No.1 | 3500-3510 Page 3502 applications.

All graphs considered here are finite, nontrivial, undirected with no loops, multiple edges and isolated vertices. The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it. Many variants of dominating sets are available in the existing literature. A set D of vertices in a graph G(V, E) is a dominating set of G if every vertex in V–D is adjacent to some vertex in D. In other words D is a dominating set if the closed neighborhood N[D] = V. A dominating set is called a minimal dominating set (MDS) if no proper subset D' of D is a dominating set. The domination number γ(G) of G is the minimum cardinality of a dominating set.

Hinge domination in graph was introduced by Indrani Promad Kelkar, Kavitha B N[5]. They defined the hinge dominating set as -A vertex u in V-S is said to be hinge dominated if u is adjacent to some vertex v in S and a vertex w in V-S such that (v,w) is not an edge in E. S is a hinge dominating set if every vertex in V-S is hinge dominated The hinged domination number $\gamma_h(G)$ is the minimum size of a hinged dominating set.In this article we are going to find the hinge domination number of cross product of standard graphs.

II. MAIN RESULT

Lemma 2.1: The domination number of cross product of complete graph K_m and path P_n is

$$\gamma(K_m \times P_n) = n\gamma(K_m)$$

Proof: Consider vertex $(u_i, v_r) \in V(K_m \times P_n)$. This vertex is adjacent to n-1 vertices of rth copy of K_n and two vertices $(u_i, v_{r-1}), (u_i, v_{r+1})$, hence it dominates all $(n-1)$ vertices of the rth copy of K_n for $r = 1,2,3, \ldots n$.
Consider $D = \{(u_1, v_1), (u_1, v_2), \ldots \ldots (u_1, v_n)\}$. D dominates all $m(n-1)$ vertices of $K_m \times P_n$ and is minimal as D–(u_i, v_r)cannot dominate all vertices ofr^{th} copy of K_n.
Hence domination number is$\gamma(K_m \times P_n) = n$. We know that domination number of complete graph K_m is 1, therefore we can write$\gamma(K_m \times P_n) = n\gamma(K_m)$.

Theorem.2.2: The hinge domination number of $K_m \times P_n$ is

$$\gamma_h(K_m \times P_n) = n \text{ for } n \geq 2.$$

Proof: From lemma 2.1, consider the minimal dominating set of $K_m \times P_n$ with n vertices as $D = \{(u_1, v_1), (u_1, v_2), \ldots \ldots (u_1, v_n)\}$. To check condition for hinge domination, for r=1,2,…..,n consider an arbitrary vertex (u_i, v_r) in $V - D$ where $i = 2,3 \ldots \ldots \ldots m$, Here (u_i, v_r) is adjacent to (u_1, v_1)in D and is adjacent to (u_i, v_{r-1})in V – D with (u_1, v_1) not adjacent to (u_i, v_{r-1}). Thus for all vertices in V – D, hinge domination condition is satisfied. Therefore, the minimal dominating set D is minimal hinge dominating set of $K_m \times P_n$ giving hinge domination number of $K_m \times P_n$ as

$$\gamma_h(K_m \times P_n) = n \text{ for } n \geq 2.$$

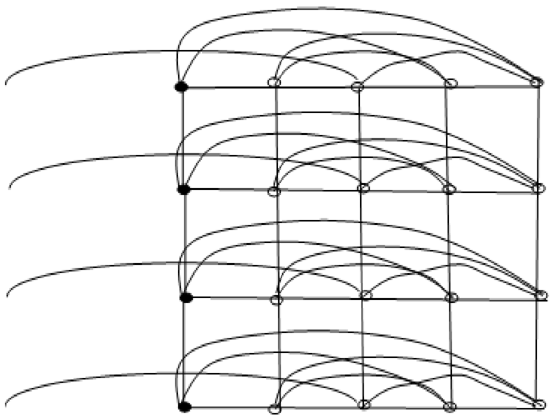

Fig 1 : Hinge dominating set of $K_5 \times P_4$

Lemma 2.3: The domination number of cross product of complete bipartite graph $K_{m,r}$ and P_n is

$$\gamma(K_{m,r} \times P_n) = n\ \gamma(K_{m,r})$$

Proof: Consider vertex set $V(K_{m,r} \times P_n)$.Based on the concept of domination in complete bipartite graph, where dominating set includes one vertex from the two parts U and V each.

Consider $D = \{(u_1, w_k), (v_1, w_k)$, for k = 1 to n}.A vertex (u_1, w_k)in D is adjacent to and hence dominates rvertices$(v_1, w_k), (v_2, w_k) \dots\dots\dots\dots (v_n, w_k)$.

Similarly, vertex (v_1, w_k) is adjacent to and hence dominates mvertices $(u_1, w_k), (u_2, w_k) \dots\dots\dots\dots (u_m, w_k)$ in the kth copy of $K_{m,r}$.There for D dominates all the vertices of $V(K_{m,r} \times P_n)$.

As no subset of D will Dominate all vertices of V, D is minimal dominating set of $K_{m,r} \times P_n$.

$\therefore$ Domination number of $K_{m,r} \times P_n$is $|D|$ giving,

$$\gamma(K_{m,r} \times P_n) = 2n = n\,\gamma(K_{m,r})$$

Theorem 2.4 : The hinge domination number of $K_{m,r} \times P_n$ is

$$\gamma_h(K_{m,r} \times P_n) = 2n \text{ for } n \geq 2.$$

Proof: From lemma 2.4.1, consider the minimal dominating set of $K_{m,r} \times P_n$ with 2n vertices as$D = \{(u_1, w_k), (v_1, w_k)$, for k = 1 to n}. To check condition for hinge domination, for i > 1, consider a vertex (u_i, w_k) in $V - D$. The vertex (u_i, w_k) is adjacent to (v_1, w_k) in D and to (u_i, w_{k+1}) in $V - D$ with (v_1, w_k) not adjacent to (u_i, w_{k+1}), when k = n replace k+1 by n -1.

Similarly, for j > 1,consider a vertex (v_j, w_k) in $V - D$. The vertex(v_j, w_k) is adjacent to (u_1, w_k) in D and to (v_j, w_{k+1}) in $V - D$ with (u_1, w_k) not adjacent to (v_j, w_{k+1}), when k = n replace k+1 by n -1.

Thus, for every vertex in V – D, hinge domination condition is satisfied, hence D is minimal hinge dominating set, giving hinge domination number as

$$\gamma_h(K_{m,r} \times P_n) = 2n \ \blacksquare$$

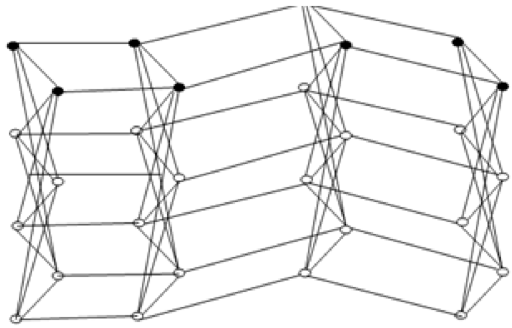

Fig 2 : Hinge dominating set of $K_{4,3} \times P_4$

Theorem 2.5 : The hinge domination number of $C_m \times P_n$ is

$$\gamma_h(C_m \times P_n) = \gamma(C_m \times P_n)$$

Proof: Consider the minimal dominating set D of $C_m \times P_n$. Suppose an arbitrary vertex (u_i, v_j) in V – D is dominated by a vertex (u_r, v_s) in D, such that

Case i) r=i-1 or i+1 and j=s, vertex from jth cycle then there exists a neighbor vertex on the ith cycle graph, (u_k, v_j) where k=i+1 or i-1 such that $(u_r, v_s)$is not adjacent to (u_k, v_j) being three consecutive vertices of jth cycle copy.

Case ii) Other choice is that i=r and s=j-1 or s=j+1 then the vertex (u_k, v_j) where k=i+1 or i-1 such that $(u_r, v_s)$is not adjacent to (u_k, v_j).

Hence from both the cases above we get that the hinge domination condition is satisfied by every vertex in V – D. Therefore, the minimal dominating set is a minimal hinge dominating set, giving hinge domination number as,

$\gamma_h(C_m \times P_n) = \gamma(C_m \times P_n)$■

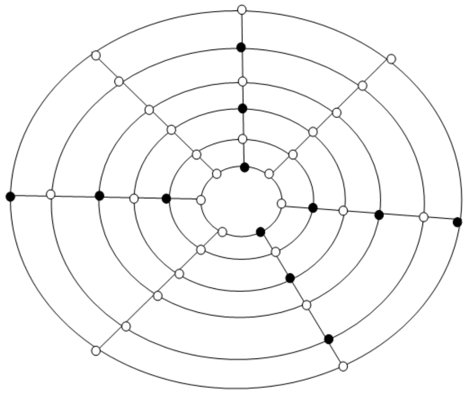

Fig 3 : Hinge dominating set of $\mathbf{C_7 \times P_6}$

Theorem 2.6: Hinge domination number of book graph is

$$\gamma_h(B_m) = 2$$

Proof: From[14] the domination number of book graph is n, $D = \{(v, w_1), (v, w_2)\}$is the minimal dominating set ofB_m. For D to behinge domination set, we need if for every vertex $u \in V - D$ is adjacent to some vertex $v\epsilon D$ and a vertex $w \in V - D$ such that (v, w) is not an edge in E. Consider$V - D = \{(u_i, w_1), (u_i, w_2), i = 1,2,3 \dots \dots \dots \dots m\}$.
$Avertex(u_i, w_1)$in $V - D$, for some i=1,2, …m is adjacent tothe vertex (v, w_1) in D and also to vertex (u_i, w_2) in $(V - D)$such that (v, w_1) is not adjacent to (u_i, w_2). Similarlyvertex(u_i, w_2) is adjacent to (v, w_2) in D and (u_i, w_1) in$V - D$ such that (v, w_2) is not adjacent to(u_i, w_2). As this is true for all values i = 1, 2, … m, it is true for all vertices in V – D. This clearly shows that D satisfies the hinge domination condition. Therefore, D is a minimal hinge dominating set of B_m giving the domination number of book graph as,$\gamma_h(B_m) = 2$■

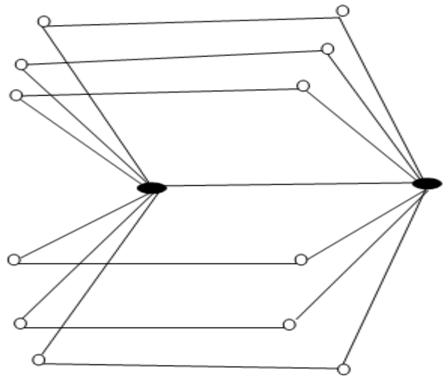

Fig 4: Hinge dominating set of$\mathbf{B_6}$

Theorem 2.7: Hinge domination number of stacked book graph is $B_{m,n}$is

$$\gamma_h(B_{m,n}) = n \text{ for } m \geq 3 \text{ and } n \geq 2$$

Proof: From[14] the domination number of stacked book is n, the minimal dominating set of stacked book graph is D=$V_C = \{(v, w_k)$ for $k = 1,2,3, \dots . n\}$. To check if D is hinge dominating set, we need to check if for every vertex $u \in V - D$ is adjacent to some vertex $v\epsilon D$ and a vertex $w \in V - D$ such that (v, w) is not an edge in $E(B_{m,n})$.
Consider
$V - D = V_O =$
$\{(u_i, w_1), (u_i, w_2) \dots \dots \dots \dots \dots \dots \dots . (u_i, w_n)\ for\ i =$ *1,2,3,…….m*
A vertex in V-D say (u_i, w_k) is adjacent to (v, w_k) in D and (u_i, w_{k+1}) in $V - D$ such that (v, w_k) is not adjacent to $(u_i, w_{k+1})$for k=1, 2, 3……..n-1.
For $k = n$, we get (u_i, w_n) is adjacent to (v, w_n) in D and (u_i, w_{n-1}) in $V - D$ such that (v, w_n) is not adjacent to (u_i, w_{n-1}).
So hinge domination condition is satisfied for all vertices in $V - D$, so D is hinge dominating set of minimum cardinality in $B_{m,n}$ giving hinge domination number as

$$\gamma_h(B_{m,n}) = n \qquad ■$$

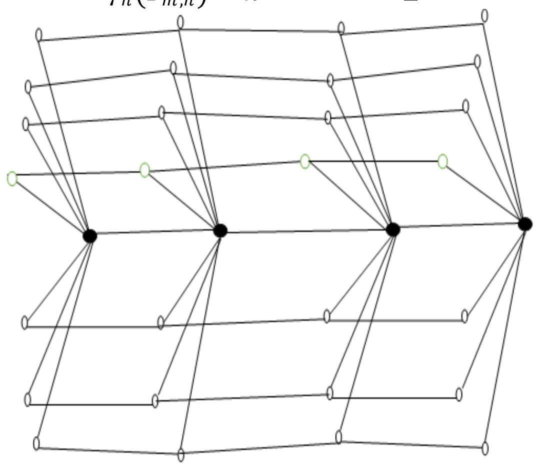

Fig 5 : Hinge dominating set of$\mathbf{B_{7,4}}$

Theorem2.8: Hinge domination number of hanging cycle graph $H_{m,n}$ is

$$\gamma_h(H_{m,n}) = n \text{ For } m \geq 3 \text{ and } n \geq 2$$

Proof: The domination number of graph $H_{m,n}$ is $\gamma(H_{m,n}) = n$ with dominating set $V_C = \{(v, w_k)\}$for $k = 1,2,3, \dots \dots \dots n$. For V_C to be a hinge dominating set, we require, if for every vertex $u \in V - D$ is adjacent to some vertex $v\epsilon D$ and a vertex $w \in V - D$ such that (v, w) is not an edge in E.
Consider

$$V - D = V_O \begin{Bmatrix} (u_i, w_1), (u_i, w_2) \dots \dots \dots (u_i, w_k) \\ for\ i = 1,2,3, \dots \dots . m \end{Bmatrix}$$

For $k = 1,2,3, \dots n - 1$, (u_i, w_k) is adjacent to (v, w_k) in D and (u_i, w_{k+1}) in $V - D$ such that (v, w_k) is not adjacent to (u_i, w_{k+1}).

For $k = n$, (u_i, w_n) is adjacent to (v, w_n) in D and (u_i, w_1) in $V - D$ such that (v, w_n) is not adjacent to (u_i, w_1).
∴ Hinge domination condition is satisfied for all vertices in $V - D$
∴ D is a hinge dominating set of minimum cardinality in $H_{m,n}$ giving hinge domination number of $H_{m,n}$ as $\gamma_h(H_{m,n}) = n$ ■

III. CONCLUSION

We defined a new domination parameter called Hinge domination number and found values of hinge domination number of standard graphs in [9]. In a graph with pendants, it is essential that all the pendants are included in the hinge dominating set.

Some important results expressingrelation between hinge domination number and domination number of product graphs are

1) $\gamma_h(\mathrm{K_m} \times \mathrm{P_n}) = \gamma(\mathrm{K_m} \times \mathrm{P_n}) = |V(\mathrm{P_n})|\gamma(\mathrm{K_m})$
2) $\gamma_h(\mathrm{K_m} \times \mathrm{P_n}) \le \gamma_h(\mathrm{K_m}) \times \gamma_h(\mathrm{P_n}) form \ge 3$
3) $\gamma_h(B_{m,n}) = \gamma_h(\mathrm{S_{m+1}} \times \mathrm{P_n}) = |V(\mathrm{P_n})|\gamma(\mathrm{S_{m+1}})$
4) $\gamma_h(\mathrm{S_{m+1}} \times \mathrm{P_n}) \le \gamma_h(\mathrm{S_{m+1}}) \times \gamma_h(\mathrm{P_n})$
5) $\gamma_\mathrm{h}(\mathrm{H_{m,n}}) = \gamma_h(\mathrm{S_{m+1}} \times \mathrm{C_n}) = |V(\mathrm{C_n})|\gamma(\mathrm{S_{m+1}})$
6) $\gamma_h(\mathrm{S_{m+1}} \times \mathrm{C_n}) \le \gamma_h(\mathrm{S_{m+1}}) \times \gamma_h(\mathrm{C_n})$
7) $\gamma_h(\mathrm{K_{m,r}} \times \mathrm{P_n}) = \gamma(\mathrm{K_{m,r}} \times \mathrm{P_n}) = |V(\mathrm{P_n})|\gamma(\mathrm{K_{m,r}})$
8) We have $\gamma_h(\mathrm{K_{m,r}}) = 2$ and $\gamma_h(\mathrm{P_n}) < n$and $\gamma_h(\mathrm{K_{m,r}} \times \mathrm{P_n}) = 2 \times n$this gives that

$$\gamma_h(\mathrm{K_{m,r}} \times \mathrm{P_n}) > \gamma_h(\mathrm{K_{m,r}}) \times \gamma_h(\mathrm{P_n})$$

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