

# Hinge Domination Number of a Graph

Indrani Pramod Kelkar

Professor, Department of Mathematics, Acharya Institute of Technology, Bangalore

**ABSTRACT:** A set  $S$  of vertices in a graph  $G$  is a hinged dominating set if for every vertex  $u$  in  $V-S$  is adjacent to some vertex  $v$  in  $S$  and a vertex  $w$  in  $V-S$  such that  $(v,w)$  is not an edge in  $E$ . Hence for every vertex in  $V-S$  The hinged domination number  $\gamma_h(G)$  is the minimum size of a hinged dominating set. In this paper, we obtain the exact values of  $\gamma_h(G)$  for some standard graphs and establish some general results on this parameter.

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## I. INTRODUCTION

The study of domination in graphs has found rapid growth in the recent years. It is a highly flourishing area of research in graph theory. So far, hundreds of research articles have appeared on this topic of research in view of its growing real life IJPT| March-2016 | Vol. 8 | Issue No.1 | 3500-3510 Page 3502 applications.

All graphs considered here are finite, nontrivial, undirected with no loops, multiple edges and isolated vertices. The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it. Many variants of dominating sets are available in the existing literature. A set  $D$  of vertices in a graph  $G(V, E)$  is a dominating set of  $G$  if every vertex in  $V-D$  is adjacent to some vertex in  $D$ . In other words  $D$  is a dominating set if the closed neighborhood  $N[D] = V$ . A dominating set is called a minimal dominating set (MDS) if no proper subset  $D'$  of  $D$  is a dominating set. The domination number  $\gamma(G)$  of  $G$  is the minimum cardinality of a dominating set.

A company which has different departments like manufacturing, dispatch, accounts etc has outsourced customer care. So the customer care personals need to have contact with company departments as well as customers, and depending on the requirement of customer contact respective department and complete the communication. Based on this concept of two side communication requirement, we have introduced a new domination parameter called hinge domination number. A dominating set  $D$  of a graph  $G(V, E)$  is a hinge dominating set, if for every vertex  $u \in V-S$  is adjacent to some vertex  $v \in S$  and a vertex  $w \in V-S$  such that  $(v, w)$  is not an edge in  $E$ . The hinged domination number  $\gamma_h(G)$  of  $G$  is the minimum cardinality of a hinged dominating set.

By hinge domination definition it is clear that there is a V shape edge formation between three vertices

$v, u, w$ . So if vertex  $u$  is hinge dominated it is essential that  $d(u) \geq 2$ . Hence all vertices of degree one, pendants, in a graph should be included into hinge dominating set.

## II. MAIN RESULTS

**Proposition 2.1 :** Hinge domination number of a path graph  $P_n$  is

$$\begin{aligned} \gamma_h(P_n) &= 2 && \text{if } n = 2 \\ &= k + 2 && \text{if } n = 3k \\ &= \left\lfloor \frac{n-1}{3} \right\rfloor + 1 && \text{if } n \neq 3k \end{aligned}$$

**Proof :** A path graph  $P_n$  has  $n$  vertices and  $n-1$  edges with two end vertices of degree one and all internal vertices of degree 2. For  $n=2, 3$  to satisfy the hinge domination condition we need to include all  $n$  vertices to hinge dominating set  $D$ . As we need to include both the end vertices in to  $D$ , for  $n=4$ , as shown in figure 1, hinge domination number is 2. For  $n > 4$ , for minimal hinge domination, include the middle vertex of each set of three consecutive vertices from  $v_3$  to  $v_{n-2}$ . Suppose we denote vertices of the path graph as  $V=\{v_1, u_1, w_1, v_2, u_2, w_2, v_3, u_3, w_3, \dots\}$ , then  $D=\{v_1, v_2, v_3, \dots\}$  forms a minimal hinge dominating set of  $P_n$  and so as to satisfy hinge domination condition we need to add some extra vertices to  $D$  as follows.

- i) If  $n=3k+1$  then the end vertex of path  $P_n$  need to be added to  $D$ .
- ii) If  $n=3k+2$  then last two vertices of path  $P_n$  need to be added to  $D$ .
- iii) If  $n=3k$  then last set of three vertices of  $P_n$  should be added to  $D$

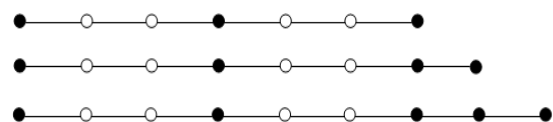


Figure 1 : Hinge dominating set of  $P_7, P_8, P_9$

Thus

$$\begin{aligned} \gamma_h(P_n) &= 2 && \text{if } n = 2 \\ &= k + 2 && \text{if } n = 3k \\ &= \left\lfloor \frac{n-1}{3} \right\rfloor + 1 && \text{if } n \neq 3k \end{aligned}$$

**Proposition 2.2** : Hinge domination number of a cycle graph  $C_n$  is

$$\begin{aligned} \gamma_h(C_n) &= k && \text{if } n = 3k \\ &= k + 1 && \text{if } n = 3k + 1 \\ &= k + 2 && \text{if } n = 3k + 2 \end{aligned}$$

**Proof** : A cycle graph  $C_n$  has  $n$  vertices and  $n$  edges with every vertex of degree 2. For  $n=3$  to satisfy the hinge domination condition we need to include any one vertex to hinge dominating set  $D$ . Suppose we denote vertices of the cycle graph as  $V=\{v_1, u_1, w_1, v_2, u_2, w_2, v_3, u_3, w_3, \dots\}$ , then  $D=\{v_1, v_2, v_3, \dots, v_k\}$  forms a minimal hinge dominating set of  $C_n$  and so as to satisfy hinge domination condition we need to add some extra vertices to  $D$  as follows.

- i) If  $n=3k+1$  then the last vertex of cycle  $C_n$  need to be added to  $D$ .
- ii) If  $n=3k+2$  then last two vertices of cycle  $C_n$  need to be added to  $D$ .

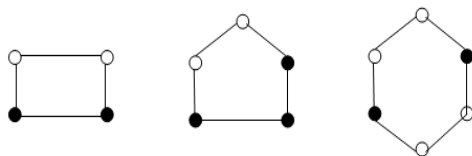


Figure 2 : Hinge dominating set of  $C_4, C_5, C_6$

**Proposition 2.3** : Hinge domination number of a star graph  $S_{n+1}$  is  $n+1$ .

**Proof** : Star graph  $S_{n+1}$  has  $n$  pendant vertices and a center vertex connected to all these pendants. As all the pendants need to be included to hinge dominating set, the center vertex being sole vertex in  $V-D$  need to be included to  $D$  as hinge domination condition will not be valid for it otherwise. Therefore  $\gamma_h(S_{n+1}) = n + 1$ .

**Proposition 2.4** : Hinge domination number of a wheel graph  $W_{n+1}$  is  $\gamma_h(C_n)$ .

**Proof** : Wheel graph  $W_{n+1}$  is a join of cycle graph  $C_n$  and a star graph  $S_{n+1}$ . It has  $n$  vertices of degree 3 and a center vertex of degree  $n$  connected to all vertices of the cycle graph. As the center vertex is connected to all the vertices of cycle graph it can not be included to hinge dominating set. Now for the remaining  $n$  vertices using proposition 2.2 it is easy to prove that  $\gamma_h(W_{n+1}) = \gamma_h(C_n)$ .

**Proposition 2.5** : Hinge domination number of a complete graph  $K_n$  is  $\gamma_h(K_n) = n$ .

**Proof** : Complete graph  $K_n$  has  $n$  vertices, where every vertex is connected to all the remaining vertices. Thus the condition for hinge domination can not be satisfied by any vertex in  $V-D$ , so all vertices should be included to  $D$ , giving  $\gamma_h(K_n) = n$ .

**Proposition 2.6** : Hinge domination number of a complete bipartite graph  $K_{m,n}$  is

$$\gamma_h(K_{m,n}) = 2$$

**Proof** : Complete bipartite graph  $K_{m,n}$  has vertex set  $V = V_1 \cup V_2$ , with  $m$  and  $n$  vertices each such that every vertex of  $V_1$  is connected to all the vertices of  $V_2$ . Thus the selection of one vertex each from  $V_1$  and  $V_2$  included in to  $D$  will satisfy the condition for hinge domination by any vertex in  $V-D$ , so  $\gamma_h(K_{m,n}) = 2$ .

**Proposition 2.7** : Hinge domination number of a tree  $T$  with  $p$  pendants  $\gamma_h(T) \geq p$

**Proof** : Let  $T$  be a tree on  $n$  vertices,  $n-1$  edges and  $p$  pendants. As every pendant vertex need to be added to hinge dominating set it is clear that  $\gamma_h(T) \geq p$ . The equality holds for a bi-star graph.

### III. CONCLUSION

Exact values of hinge domination number of standard graphs are found and lower bound for hinge domination number of tree is established in the paper.

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