RESEARCH ARTICLE OPEN ACCESS

Some allied normal spaces via gsp-open sets in topology

[1]Govindappa Navalagi and [*2]R G Charantimath
[1]Department of Mathematics KIT Tiptur 572202 ,Karnataka India
*[*2]Department of Mathematics KIT Tiptur 572202 ,Karnataka India*
Corresponding Auther: Govindappa Navalagi

ABSTRACT
Aim of this paper is to introduce and study some allied normal spaces using gsp-open sets , g^*-closed sets , gs-open sets and semipreopen sets. We, also investigate some basic properties of these allied normal spaces.

Mathematics Subject Classification (2010): 54A05,54B05,54C08 ,54D10
Keywords: semipreopen sets, gsp-closed sets, g^*-closed sets, gs-closed sets, gsp-irresoluteness, semipreirrsolute and pre-gsp-closed functions

---

Date of Submission: 20-06-2018 Date of acceptance: 09-07-2018

I. INTRODUCTION

In 1982 A S Mashhour et al[10] have defined and studied the concept of pre-open sets and pre-continuous functions in topology. In 1983 S.N.Deeb et al [7] have defined and studied the concept of pre-closed sets ,precloseropearater,p-regular spaces and pre-closed functions in topology. In 1986, D. Andrijivic [1] introduced and studied the notion of semipre open sets, semipreclosed sets ,semipreinterior operator and semipre-closed operator in topological spaces. Later many topologist have been studied these above mention sets in the literature. For the first time , N.Levine [9] has introduced the notion g-closed sets and g-open sets in topology. S P Arya et.al[2] have defined and studied the nontion of gs-closed sets and gs-open sets in 1990. In 1995 , J.Dontchev[6] has defined and studied of concept of gsp-closed sets, gsp-open sets , gsp-continuous function and gsp-irresoluteness in topology. In 2000 M.K.R.S . Veera kumar[12] has defined and studied of properties of g*-closed sets in topological spaces. In this paper , we introduce and study some allied normal spaces using gsp-open sets , g^*-closed sets , gs-open sets and semipreopen sets. We, also investigate some basic properties of these allied normal spaces.

II. PRELIMINARIES

Throughout this paper (X , τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated . If A be a subset of X, the Closure of A and Interior of A denoted by Cl(A) and Int(A)respectivly.

We give the following define are useful in the sequel :

DEFINITION 2.1 : A subset A of space X is said to be :

(i)semi-open set [8] if $A \subset Cl (Int (A))$

(ii) pre-open set [10] if $A \subset IntCl(A)$

(iii) semi-pre open set [1] if $A \subset Cl (Int (Cl(A)))$

The complement of a semiopen (resp. preopen , semipreopen) set of a space X is called semiclosed [3] (resp. preclosed [7] ,semipreclosed [1]) set in X.

The family of all semi open (resp. preopen ,semi-pre open) sets of X will be denoted by SO(X) (resp. PO(X) , SPO(X)).

Definition 2.2[4] : The intersection of all semi-closed sets of X containing subset A is called the semi-closure of A and is denoted by sCl(A).

Definition 2.3[1] : The intersection of all semipre-closed sets of X containing subset A is called the semipre-closure of A and is denoted by spCl(A).

Definition 2.4[5]: The union of all semi-open sets of X contained in A is called the semi-interior of A and is denoted by sInt (A).

Definition 2.5[1]: The union of all semipre-open sets of X contained in A is called the semipre-interior of A and is denoted by spInt(A).

Definition 2.6 : A sub set A of a space X is said to be :

(i) a generalized closed (briefly, g- closed) **[9]** set if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ)

(ii) a generalized semi-closed (briefly, gs- closed) [**2**] set if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ)

(iii) a generalized semi-preclosed (briefly, gsp-closed) [**6**] set if $spCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ)

(iv) a g^*-closed set[12] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open set in (X, τ)

Definition 2.7 : A function f:X→Y is said to be semipre-irresolute [] if $f^{-1}(U)$ is semi preopen set in X for every semipre open set U in Y

Definition 2.8[13] : A topology space X is said to be semipre-normal space if for any pair of disjoint semipre-closed sets A and B of X, there exit disjoint semipre open sets U and V such that A⊆U and B⊆V.

Definition 2.9[6]: A function f:X→Y is said to be gsp-irresolute if $f^{-1}(V)$ is gspopen in X for every gspopen set V of Y

Definition 2.10 [15] : A function f:X→Y is said to be g^*-irresolute if $f^{-1}(V)$ is a g^* -closed set of X for every g^* -closed set of Y

Definition 2.11 [14]: A function f:X→Y is said to be pre-gs closed, if for each F⊂SC(X), f(F) is gs-closed in Y

Definition 2.12 [11] : A topological space X is said to be g-normal if for every pair of disjoint g–closed sets A and B of X ,there exist disjoint open sets U and V of X such that A⊂U and B⊂V.

III. PROPERTIES OF (SP,GSP)-NORMAL SPACES

Firstly , we define and study the properties of gsp-normal spaces in the following.

Definition 3.1: A topological space X is said to be gsp-normal if for any pair of disjoint gsp-closed sets A and B, there exist disjoint open sets U and V such that A⊂U and B⊂V.

Since every g-closed set is gsp-closed set so every gsp-normal space is g-normal space.

Theorem 3.2: A topological space X is gsp-normal if and only if for any disjoint gsp-closed sets A and B of X, there exist open sets U and V of X such that A⊂U, B⊂V and Cl(U) ∩Cl(V)=∅.

Proof: Necessity: Let A and B be any disjoint gsp-closed sets of X. There exist open sets U_0 and V of X such that $A \subset U_0$, B⊂V and $U_0 \cap V = \emptyset$ hence $U_0 \cap Cl(V) = \emptyset$. Since X is gsp-normal there exist open sets G and H of X such that A⊂G, Cl(V) ⊂H and G∩H=∅, hence Cl(G)∩H=∅. Now put $U = U_0 \cap G$, then U and V are open sets of X such that A⊂U, B⊂V and Cl(U) ∩Cl(V)=∅.

Sufficiency: Obvious.

Theorem 3.3: A topological space X is said to be an gsp-normal space if and only for every closed set F and for every open set G contain F there exist gsp-open set U such that F⊂U⊂gspCl(U)⊂G.

Proof: Let F be closed set in X and G be an open set in X such that F⊂U,X−G is a closed set and (X−G)∩F=∅. Since X is gsp-normal space then there exist open sets U and V of X such that U∩V=∅, (X−G)⊂V and F⊂U U⊂(X−V).

Since every open set in gsp-open set and hence U and V are gsp-open sets of X such that gspCl(U) ⊂ gspCl(X−V)= X−V.Hence F⊂U⊂ gspCl(V) ⊂(X−V) ⊂G.

Theorem 3.4: If f:X→Y is an open gsp-irresolute bijection and X is gsp-normal, then Y is gsp-normal.

Proof: Let A and B be any disjoint gsp-closed sets of Y. Since f is gsp-irresolute,$f^{-1}(A)$ and $f^{-1}(B)$ are disjoint gsp-closed sets X. Since X is gsp-normal, then there exists disjoint open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is open and bijectivity, we obtain A⊂f(U), B⊂f(V),f(U)∩f(V)=∅ and also f(U) and f(V) are open sets of Y. This show that Y is gsp-normal.

We, define the following

Definition 3.5: A topological space X is said to be (sp,gsp)-normal if for any pair of disjoint semipreclosed sets A and B there exist disjoint gsp-open sets U and V such that A⊂U and B⊂V.

In view of definition of gsp-closed set we give the following.

Definition 3.6: A subset a of spacee (X,τ) is called gsp-open set if its complement is a gsp-closed set of (X, τ)

Lemma 3.7: A subset A of a space X is said to be gsp-open if F⊆spInt(A) Whenever F⊆A and F is closed in X.

Theorem 3.8: The following properties are equivalent for a space X.

(i) X is (sp,gsp)-normal

(ii) For any pair of disjoint semipre-closed sets A and B of X, there exist disjoint gsp-
open sets U and V such that A⊂U and B⊂V

(iii) For any semipre closed set A and any semipre-open set V containing A, there
exists gsp-open set U such that A⊂U⊂spCl(U)⊂V.

Proof: (i)⟹(ii): This proof is obvious since every semipre-open set is gsp-open set.

(ii)⟹(iii): Let A be any semipreclosed set and V an semipre-open set containing A. Since A and X−V are disjoint semipreclosed sets of X, since A and X−V are disjoint semipreclosed sets of X, then there exist gsp-open sets U, W of X such that A⊂U, X−V⊂W and U∩V=∅. By lemma 3.7.,we have X−V⊂spInt(W). Since U∩spInt(W)=∅. We have spCl(U) ∩spInt(W)=∅ and hence spCl(U)⊂X− spInt(W)⊂V.Therefore, we obtain A⊂U⊂ spCl(U)⊂V.

(ii)⟹(iii): Let A and B be any disjoint semipre-closed sets of X. Since X−B is an semipre-open set containing A, there exists a gsp-open set G, such that A⊂G⊂spCl(G)⊂X−B. By lemma 3.7.,we have A⊂spInt(G). Put U=spInt(G) and V= X−spCl(G).Then U and V are disjoint semipre-open sets and hence are disjoint gspopen sets such that A⊂U and B⊂V. Therefore , X is (sp,gsp)-normal.

We, define the following

Definition 3.9: A function f:X→Y is called pre generalized semipre-closed (brifly, pre-gsp-closed) if for each semipre-closed set F of X , f(F) is gsp-closed set in Y.

Theorem 3.10: A surjective function f:X→Y is pre-gsp-closed if and only if for each subset B of Y and semipre-open set U of X containing $f^{-1}(B)$, there exists a gsp-open set V of Y such that B⊂V and $f^{-1}(V)$ ⊂U.

Proof: Necessity: Suppose that f is pre-gsp-closed. Let B be any subset of Y and U and semipre-open set of X containing $f^{-1}(B)$. Put V=Y−f(X−U). Then V is gsp-open in Y, B⊂V and $f^{-1}(V)$ ⊂U.

Sufficiency: Let F be any semipre-closed set of X. Put B=Y−f(F), then we have $f^{-1}(B)$ ⊂(X−F) and (X−F) is semipre-open in X. There exists a gsp-open set V of Y such that B=Y−f(F)⊂V and $f^{-1}(V)$ ⊂(X−F). Therefore, we obtain f(F)=(Y−V) and hence f(F) is pre-gsp-closed in Y. This show that f is pregsp-closed.

Theorem 3.11: If f:X→Y is a semipre-irresolute pre gsp-closed surjection and X is semipre-normal. Then Y is (sp,gsp)-normal.

Proof: Let A and B be any distinct semipre-closed set of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint semipre-closed sets of X, as f is semipre- irresolute. since X is semipre-normal exist disjoint semipre open sets U and V of X such that $f^{-1}(A)$⊂U and $f^{-1}(B)$⊂V. Since f is pregsp-closed.By theorem 3.10., there exist gsp-open sets G and H. such that A⊂G, B⊂H, $f^{-1}(G)$⊂U and $f^{-1}(H)$⊂V. Since U and V are disjoint, we have G∩H=∅. This show that Y is (sp,gsp)-normal.

4.Properties of Strongly gsp - normal spaces.

Definition 4.1: A topological space X is said to be strongly gsp-normal space, if for any pair of disjoint closed sets A and B, there exists disjoint gsp-open sets U and V such that A⊂U and BA⊂V

Theorem 4.2: The following properties are equivalent for a space X.

(i) X is strongly gsp -normal space

(ii For any pair of disjoint closed sets A and B of X, there exist disjoint gsp-open sets
U and V such that A⊂U and B⊂V

(iii) For any closed set A and any open set V containing A, there exists gsp-open set
U such that A⊂U⊂spCl(U)⊂V.

Proof: (i)⟹(ii):Obvious, since every open set is gsp-open set.

(ii)⟹(iii): Let A be any closed set and V be an open set containing A, there exist gsp-open sets U, W of X such that A⊂U, X−V⊂W and U∩V=∅. By lemma 3.7.,we have X−V⊂spInt(W). Since U∩spInt(W)=∅. We have spCl(U) ∩spInt(W)=∅ have and hence spCl(U)⊂ X− spInt(W)⊂V. Therefore, we obtain A⊂U⊂ spCl(U)⊂V.

(iii)⟹(i): Let A and B be any disjoint closed sets of X. Since X−B is an open set containing A, there exists a gsp-open set G, such that A⊂G⊂spCl(G)⊂X−B. By lemma 3.7.,we have A⊂spInt(G), Put U=spInt(G) and V= X−spCl(G).Then U and V are disjoint gsp-open sets such that A⊂U and B⊂V. Therefore X is strongly gsp-normal space.

Theorem 4.3: If f:X→Y is continuous pre-gsp-closed surjection and X is strongly gsp- normal. Then Y is strongly gsp-normal.

Proof: Let A and B be any disjoint closed sets of Y. Then, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets X. As f is continuous function. Since X is strongly normal, then there exists disjoint gsp-open sets U and V of X such that $f^{-1}(A)$ ⊂U and $f^{-1}(B)$ ⊂V. Since f is pre-gsp-closed function, by theorem 6.3.4 there exists gsp-open sets G and H in Y such that A⊂G, B⊂H, $f^{-1}(G)$⊂U and $f^{-1}(H)$⊂V. Since U and V are disjoint, we have G ∩ H=∅. This show that Y is strongly gsp- normal.

Definition 4.4: A space X is said to be (g^*,gs)-normal space if for any pair of disjoint g^*-closed sets A and B of X, there exists disjoint gs-open sets U and V such that A⊂U and B⊂V.

Theorem 4.5: The following properties are equivalent for a space X

(i) X is (g^*,gs)-normal space

(ii) For any pair of disjoint g^*-closed sets A and B of X there exists disjoint gs-open
sets U and V such that A⊂U and B⊂V.

(iii) For any g^*-closed set A and any g^*-open set V containing A, there exists gs-open
set U such that A⊂U⊂sCl(U)⊂V.

Proof: (i)⟹(ii): Obvious

(ii)⟹(iii) Let A be any g^*-closed set and V be an g^*-open set containing A⊂U , X−V⊂W and U∩W=∅, by lemma 3.7, we have X−U⊂sInt(W). Since U∩ sInt(W)= ∅. We have sCl(U) ∩sInt(W) =

∅ and hence sCl(U) ⊂X- sInt(W) ⊂V. There fore we obtained A⊂U ⊂sCl(U) ⊂**V**

(iii)⟹(i): Let A and B be any disjoint g^*-closed sets of X. Since X−B is g^*-open set containing A, there exists a gs-open set G, such that A⊂G⊂sCl(G)⊂X−B. By lemma 3.7 we have A⊂sInt(G), Put U=sInt(G) and V= X−sCl(G).Then U and V are disjoint open sets such that A⊂U and B⊂V. Therefore X is (g^*,gs)-normal space.

We, define the following

Definition 4.6: A space X is said to be (g^*, s)-normal space, if for any pair of disjoint g^*-closed sets A and B of X, there exists disjoint semiopen sets U and V such that A⊂U and B⊂V.

We , recall the following.

Theorem 4.8[49]: A function f:X→Y is said to be pre-gs-closed if and only if for each subset B of Y and each U∈SO(X) containing f^{-1}(B) , there exists a gs-open set V of Y such that B⊂V and f^{-1}(V) ⊂U.

Now, we prove the following.

Theorem 4.9: If function f:X→Y is g^*-irresolute pre-gs-closed surjection and X is (g^*,s)-normal space, then Y is (g^*,s)-normal space.

Proof: Let A and B be any disjoint g^*-closed sets of Y, then f^{-1}(A) and f^{-1}(B) are disjoint g^*-closed sets of X. As f is g^*-irresolute function. Since X is (g^*,s)-normal, then there exists disjoint semiopen sets U and V such that f^{-1}(A) ⊂U and f^{-1}(B) ⊂V. Since f is pre-gs-closed function, by theorem 4.8. there exist G and H gs-open sets in Y such that A⊂U and B⊂V. f^{-1}(G) ⊂U and f^{-1}(H) ⊂V. Since U and V are disjoint , we have G∩ H=∅. This show that Y is (g^*,s)-normal space.

We, define the following

Definition 4.10: A space X is said to be (g^*,gsp)-normal space, if for any pair of disjoint g^*-closed sets A and B of X, there exists disjoint gsp-open sets U and V such that A⊂U and B⊂V.

Theorem 4.11: The following properties are equivalent for a space X

(i) X is (g^*,gsp)-normal space

(ii) For any pair of disjoint g^*-closed sets A and B of X, there exists disjoint gsp-open sets U and V such that A⊂U and B⊂V.

(iii) For any g^*-closed set A and any g^*-open set V containing A, there exists gsp-open set U such that A⊂U⊂spCl(U)⊂V.

Proof: Routine proof of the theorem is omitted.

We, define the following

Definition 4.12: A space X is said to be (g^*,sp)-normal space, if for any pair of disjoint g^*-closed sets A and B of X, there exists disjoint semipre-open sets U and V such that A⊂U and B⊂V.

Theorem 4.13: If function f:X→Y is g^*-irresolute pregsp-closed surjection and X is (g^*,sp)-normal space, then Y is (g^*,gsp)-normal space.

Proof is similar to theorem 4.3.

Routine proofs of the following theorems are omitted

Theorem 4.14: The following properties are equivalent for a space X

(i) X is (g^*,s)-normal space

(ii) For any pair of disjoint g^*-closed sets A and B of X, there exists disjoint semi-open sets U and V such that A⊂U and B⊂V.

(iii) For any g^*-closed set A and any g^*-open set V containing A, there exists semi-open set U such that A⊂U⊂sCl(U)⊂V.

Theorem 4.15: The following properties are equivalent for a space X

(i) X is (g^*,sp)-normal space

(ii) For any pair of disjoint g^*-closed sets A and B of X, there exists disjoint semipre-open sets U and V such that A⊂U and B⊂V.

(iii) For any g^*-closed set A and any g^*-open set V containing A, there exists semipre-open set U such that A⊂U⊂spCl(U)⊂V.

REFERENCES

[1]. D.Andrijevic, Semipreopen sets, Math.Vensik 38(1),(1986), 24-32.

[2]. S.P.Arya and T.Nour, Characterizations of s-normal Spaces, Indian,J.Pure Appl.Math.21(1990) 717-719.

[3]. Biswas, On characterization of semi-continuous functions, Atti.Accad.Naz.Lincei Rend.Cl.Sci.Fis.Mat.Natur 48(8)(1970),399-402

[4]. S.G.Crossely. and S.K.Hildebrand, On semi-Closure. Texas. J.Sci,22.(1971),99-112

[5]. P Das. Note on Some Application of Semi Open Sets. Progress of Math,BHU,7,(1973), 33-44

[6]. J.Dontchev,On generalizingsemi-pre open sets,Mem.Fac.Sci. Kochi.Univ.Ser.A.Math, 6(1995), 35-48.

[7]. S.N.El-Deeb, I.A. Hasanein, A.S.Mashhour and T. Noiri, On p-regular spaces,Bull Math. Soc. Sci. Math. R.S.Roumanie (N.S), 27(75), (1983), 311-315.

[8]. N.Levine, Semi-open sets and semi-continuous in Topological spaces, Amer. Math, Monthly 70(1963) , 36-41

[9]. N.Levine, Generalized closed sets in Topology,Rend.Cric. Math.Palermo,19(2)(1970), 89-96.

[10]. A.S. Mashhoour, M.E. Abd El-Monsef and S.N .El-Deeb, On Pre continuous and Weak Precontinuous Mappings, Proc. Math. Phys. Soc. Egypt, 53(1982).47-53.

[11]. B.M.Munshi.Separation axioms. Acta.Ciencia India 12 (1986). 140-144.

[12]. G. B.Navalagi, On semi-pre continuous functions and properties of generalized

semipre closed sets in topology, IJMS,29(2) (2002),85-98

[13]. Govindappa Navalagi and Mallamma Shankrikop,Semipre -regular and Semipre-normal spaces in topological spaces., The Global Journal of Applied Mathematis & Mathematical Sciences (GJ-AMMS) Vol, 2.No 1-2.(January-December 2009):pp. 27-39

[14]. T Noiri On S-Normal spaces and pre gs-closed functions. Acta Math. Hungar 80 (1-2)(1998),105-113.

[15]. M.K.R.S.Veera Kumar "Between closed sets and g-closed sets" Mem, Fac,Sci.Kochi. Uni(Math) 21(2000),1-19.

Govindappa Navalagi "Some allied normal spaces via gsp-open sets in topology "International Journal of Engineering Research and Applications (IJERA) , vol. 8, no.7, 2018, pp.21-25