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Modified one-Parameter Lindley Distribution and Its Applications

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ABSTRACT
In this paper a new one-parameter lifetime distribution named “Modified One-Parameter Lindley Distribution” which is a modification of Lindley Distribution, with an increasing hazard rate for modeling lifetime data has been suggested. Its first four moments about origin and mean have been deduced and expressions for mean, variance, coefficient of variation, skewness, kurtosis and index of dispersion have been obtained. Various mathematical and statistical properties of the proposed distribution including its survival function, hazard rate function, mean deviations, and Bonferroni and Lorenz curves have been discussed. Estimation of its parameter has been obtained using the method of maximum likelihood and the method of moments. The applications and goodness of fit of the distribution have been discussed with two real lifetime data sets and the fit has been compared with other one-parameter lifetime distributions including Akash, Lindley and Exponential distributions.

Key words: Lifetime distributions, Akash distribution, Lindley distribution, mathematical and statistical properties, estimation of parameter, goodness of fit.

I. INTRODUCTION

The analysis and modeling of lifetime data play a crucial role in all branches of applied sciences including engineering, medicine, economics and insurance. There are a number of continuous distributions for modeling lifetime data such as exponential, Lindley, gamma, log-normal and Weibull. Of these, exponential, Lindley and Weibull distributions gained popularity in modeling lifetime data as compared to gamma and log-normal distributions since their survival functions do not require numerical integration. Besides, in the recent past a number of new class of one parameter lifetime distributions have evolved in statistical literature, which are in general extensions or modifications or generalizations of Lindley distribution that was used in the context of fiducial and Bayesian statistics (Lindley 1958). The range of such distributions include Akash distribution (Shankar R, 2015a), Shankar distribution (Shankar R,2015b), Sujatha distribution (Shankar, 2015c), Amarendra distribution (Shankar R, 2016), Akshaya distribution (Shankar R, 2017) and Improved Second-Degree Lindley distribution (Karuppusamy S et al, 2017) and each of these distributions has its own advantages and disadvantages in modeling lifetime data.

As a continuation of these models, in this paper, as an extension of Lindley distribution, we have proposed a new one parameter continuous distribution namely Modified One-Parameter Lindley Distribution (MOPLD) and it has been shown that it is better than exponential, Lindley, and Akash distributions for modeling life time data. We have also discussed various statistical properties including its shape, moment generating function, moments, skewness and kurtosis, hazard rate function, mean and variance, mean deviations, Bonferroni and Lorenzcurves, of this new distribution. Finally, the method of maximum likelihood and method of moments are discussed for estimating its parameter. We have also included the goodness of fit of the proposed distribution for two data sets using maximum likelihood estimation and the fit is compared with the ones that are obtained by other distributions.

II. MODIFIED ONE-PARAMETERLINDLEY DISTRIBUTION

The probability density function (p.d.f) and the cumulative density function (c.d.f) of Lindley distribution (1958) are given by

$$f_1(x;\lambda) = \frac{\lambda^2}{\lambda+1}(1+x)e^{-\lambda x};\ x>0, \lambda>0 \quad (2.1)$$

$$F_1(x;\lambda) = 1-\left[1+\frac{\lambda x}{\lambda+1}\right]e^{-\lambda x};\ x>0, \lambda>0 \quad (2.2)$$

A detailed discussion of Lindley distribution, its mathematical properties, estimation of parameter and application showing advantages of Lindley distribution over exponential distribution can be found in Ghitnay et al (2008).

A modified version of Lindley distribution by the name Akash distribution was given by Shankar, R

(2015) with probability distribution function and cumulative density function as

$$f_2(x;\lambda)=\frac{\lambda^2}{\lambda^2+1}(1+x^2)e^{-\lambda x};x>0,\lambda>0 \quad (2.3)$$

and

$$F_2(x;\lambda)=1-\left[1+\frac{\lambda x(\lambda x+2)}{\lambda^2+2}\right]e^{-\lambda x};x>0,\lambda>0 \quad (2.4)$$

A detailed discussion of this distribution and important mathematical properties shall be found in Shankar. R (2015a).Yet another one parameter modification of Lindley distribution by the name Shankar distribution was introduced by Shankar (2015b) with probability density function and cumulative density function as

$$f_3(x;\lambda)=\frac{\lambda^2}{\lambda^2+1}(\lambda+x)e^{-\lambda x};x>0,\lambda>0 \quad (2.5)$$

and

$$F_3(x,\lambda)=1-\frac{(\lambda^2+1)+\lambda x}{\lambda^2+1}e^{-\lambda x};x>0,\lambda>0 \quad (2.6)$$

and a detailed discussion on this distribution shall be found in Shankar (2015b).

As a continuation of these distributions,Karuppusamy S et al (2017) have come up with another distribution by the name "Improved Second-Degree Lindley Distribution" (ISLD) with probability density function and cumulative density function as

$$f(x;\lambda)=\frac{\lambda^3}{\lambda^2+2\lambda+2}(1+x)^2e^{-\lambda x};x>0,\lambda>0 \quad (2.7)$$

$$F(x)=1-\left[1+\frac{\lambda^2x^2+2(\lambda^2+\lambda)x}{\lambda^2+2\lambda+2}\right]e^{-\lambda x};x>0,\lambda>0 \quad (2.8)$$

and a detailed discussion on this distribution including its mathematical properties shall be found in Karuppusamy.S et al (2017) .

In this current paper as an extension of Akash Distribution and Second Degree Lindley distribution we have introduced a new distribution by the name "Modified One Parameter Lindley Distribution" (MOPLD) with probability density function (p.d.f) and cumulative density function (c.d.f) as,

$$f(x;\lambda)=\frac{\lambda^4}{\lambda^3+6}(1+x^3)e^{-\lambda x};x>0,\lambda>0 \quad (2.9)$$

and

$$F(x;\lambda)=1-\left[1+\frac{\lambda^3x^3+3\lambda^2x^2+6\lambda x}{\lambda^3+6}\right]e^{-\lambda x};\ x>0,\ \lambda>0 \quad (2.10)$$

Clearly, the Modified One Parameter Lindley Distribution is a mixture of Exponential (λ) and Gama distribution (4, λ) with mixing proportions as,

$$\frac{\lambda^3}{\lambda^3+6} \text{ and } \frac{6}{\lambda^3+6}.$$

In terms of Exponential distribution (λ) and Gama distribution (4, λ) the MOPLD shall also be written as,

$$f(x;\lambda)=p\,f_1(x)+(1-p)\,f_2(x)$$

Where $p=\frac{\lambda^3}{\lambda^3+6}$, $f_1(x)=\lambda e^{-\lambda x}$,

$1-p=\frac{6}{\lambda^3+6}$ and $f_2(x)=\frac{\lambda^4x^3e^{-\lambda x}}{\Gamma(4)}$

The graphs of the p.d.f and c.d.f of MOPLD for different values of λ are as follows.

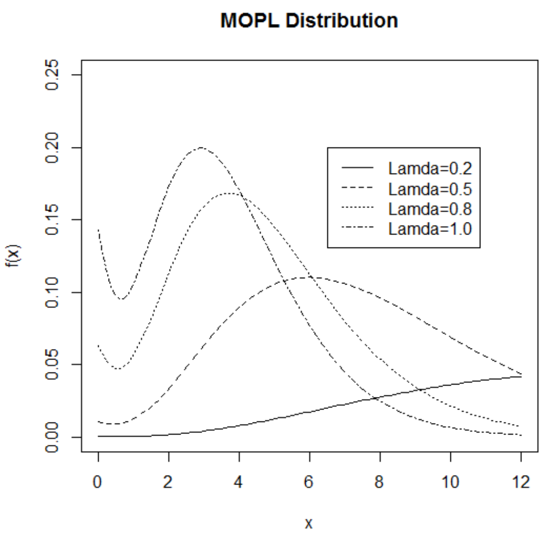

Figure 1

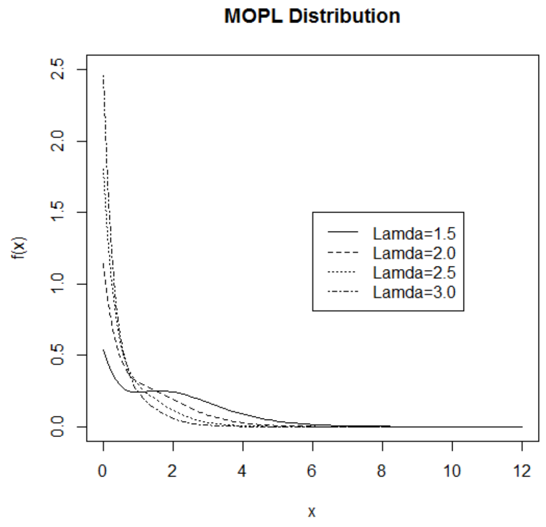

Figure2

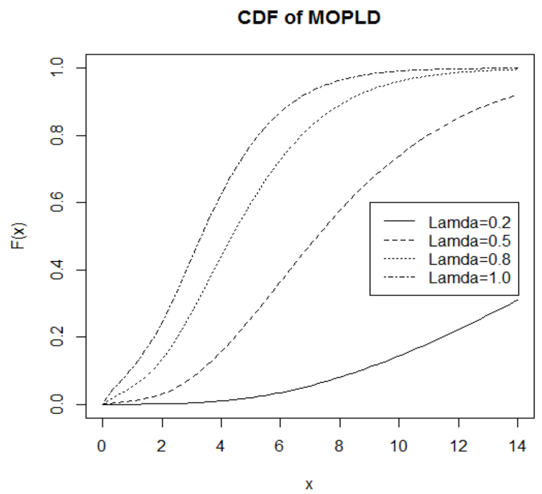

Figure 3

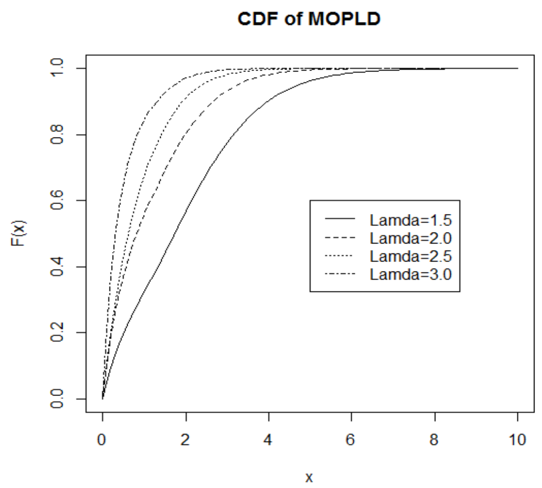

Figure 4

III. MOMENTS AND RELATED MEASURES

The r^{th} moment about origin for MOPLD shall be obtained as,

$$\mu_r' = \frac{r!\left[\lambda^3 + (r+1) + (r+2)(r+3)\right]}{\lambda^r(\lambda^3+6)} \qquad r = 1,2,3,4,\ldots$$

The first four moments about origin of MOPLD are thus deduced as,

$$\mu_1' = \frac{\lambda^3+24}{\lambda(\lambda^3+6)} \quad ,$$

$$\mu_2' = \frac{2(\lambda^3+60)}{\lambda^2(\lambda^3+6)}$$

$$\mu_3' = \frac{6(\lambda^3+120)}{\lambda^3(\lambda^3+6)}$$

$$, \ \mu_4' = \frac{24(\lambda^3+210)}{\lambda^4(\lambda^3+6)}$$

Using the first four moments about origin, the moments about mean of MOPLD shall be obtained as,

$$\mu_1 = \frac{\lambda^3+24}{\lambda(\lambda^3+6)} \tag{3.1}$$

$$\mu_2 = \frac{\lambda^6+84\lambda^3+144}{\lambda^2(\lambda^3+6)^2} \tag{3.2}$$

$$\mu_3 = \frac{2(\lambda^9+198\lambda^6+324\lambda^3+864)}{\lambda^3(\lambda^3+6)^3} \tag{3.3}$$

$$\mu_4 = \frac{9(\lambda^{12}+312\lambda^9+2304\lambda^6+10368\lambda^3+10368)}{\lambda^4(\lambda^4+6)^4} \tag{3.4}$$

Again, using moments about mean of MOPLD the coefficient of variation (C.V), coefficient of skewness($\sqrt{\beta_1}$), coefficient of kurtosis (β_2), and index of dispersion (γ) of MOPLD shall be deduced as,

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\lambda^6+84\lambda^3+144}}{\lambda^3+24} \tag{3.5}$$

.

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2(\lambda^9+198\lambda^6+324\lambda^3+864)}{(\lambda^6+84\lambda^3+144)^{3/2}} \tag{3.6}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{9(\lambda^{12}+312\lambda^9+2304\lambda^6+10368\lambda^3+10368)}{(\lambda^6+84\lambda^3+144)^2} \tag{3.7}$$

$$\gamma=\frac{\sigma^2}{\mu_1'}=\frac{\lambda^6+84\lambda^3+144}{\lambda(\lambda^3+6)(\lambda^3+24)} \quad (3.8)$$

It can be shown that MOPLD is over-dispersed ($\mu<\sigma^2$), equi-dispersed ($\mu=\sigma^2$), and under-dispersed ($\mu>\sigma^2$) for λ=1.95 while Akash distribution, Lindley distribution and Exponential distribution are over-dispersed, equi-dispersed and over-dispersed for their parametric values 1.52, 1.17, and 1.00 respectively.

IV. HAZARD RATE FUNCTION AND MEAN RESIDUAL LIFE FUNCTION

The hazard rate function and mean residual function of MOPLD are obtained as,

$$h(x)=\frac{f(x)}{1-F(x)}=\frac{\lambda^4(1+x^3)}{\lambda^3x^3+3\lambda^2x^2+6\lambda x+(\lambda^3+6)} \quad (4.1)$$

$$m(x)=\frac{1}{1-F(x)}\int_x^\infty[1-F(t)]dt$$
$$=\frac{\lambda^3x^3+6\lambda^2x^2+18\lambda x+(\lambda^3+24)}{\lambda[\lambda^3x^3+3\lambda^2x^2+6\lambda x+(\lambda^3+6)]} \quad (4.2)$$

From (4.1) and (4.2) it can be easily verified that,

$$h(0)=\frac{\lambda^4}{\lambda^3+6}=f(0)\ and\ m(0)=\frac{\lambda^3+24}{\lambda(\lambda^3+6)}$$

The graphs of hazard rate function h(x) for different values of λ are as follows:

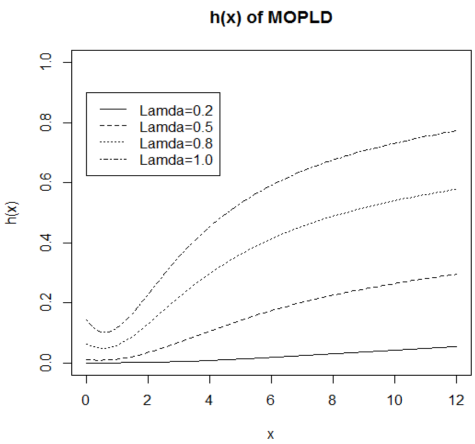

Figure-5

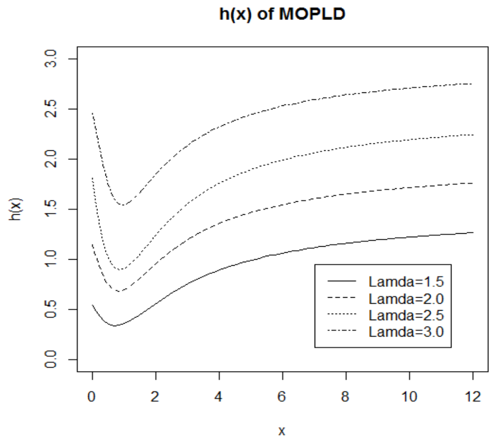

Figure-6

V. STOCHASTIC ORDERING

Stochastic ordering of positive continuous random variables is an important tool to judge the behaviour of a distribution. A random variable X is said to be smaller than a random variable Y in the

i. Stochastic order $X \underset{st}{\le} Y$ if $F_X(x)\ge F_Y(x)$ for all x

ii. Hazard rate order $X \underset{hr}{\le} Y$ if $h_X(x)\ge h_Y(x)$ for all x

iii. Mean residual life order $X \underset{mrh}{\le} Y$ if $m_X(x)\le m_Y(x)$ for all x

iv. Likelihood ratio order $X \underset{lr}{\le} Y$ if $\frac{f_X(x)}{f_y(x)}$ decreases in x

The relationship of the above said properties of a distribution is as follows:

$(a)\quad X \underset{lr}{\le} Y \Rightarrow X \underset{hr}{\le} Y \Rightarrow X \underset{mre}{\le} Y$

And $(b)\quad X \underset{lr}{\le} Y \Rightarrow X \underset{st}{\le} Y$

The Modified Lindley Distribution is ordered with respect to the strongest likelihood ratio ordering by which the other ordering of distribution follows:

Theorem: Let $X \sim MLD(\lambda_1)$ and $Y \sim MLD(\lambda_2)$. If for $\lambda_1 \ge \lambda_2$ we have $X \underset{lr}{\le} Y$ then $X \underset{hr}{\le} Y$, $X \underset{mre}{\le} Y$ and $X \underset{st}{\le} Y$

Proof: we have

$$\frac{f_X(x)}{f_y(x)}=\frac{\lambda_1^4}{\lambda_1^4(\lambda_1^3+6)}(\lambda_2^3+6)\ e^{-(\lambda_1-\lambda_2)x} \quad x>0$$

$$\Rightarrow \log_e\left[\frac{f_X(x)}{f_y(y)}\right] = \log_e\left[\frac{\lambda_1^4(\lambda_2^3+6)}{\lambda_2^4(\lambda_1^3+6)}\right] - (\lambda_1 - \lambda_2)\,x$$

$$\Rightarrow \frac{d}{dx}\log_e\left[\frac{f_X(x)}{f_y(y)}\right] = -(\lambda_1 - \lambda_2) < 0$$

Indicatingthat $\frac{f_X(x)}{f_y(x)}$ is a decreasing function for $\theta_1 \geq \theta_2$. This means that $X \underset{lr}{\leq} Y$ and consequently $X \underset{hr}{\leq} Y$, $X \underset{mrl}{\leq} Y$ and $X \underset{st}{\leq} Y$ follows.

VI. MEAN DEVIATIONS

The mean deviation about the mean and the mean deviation about the median measuresthe amount of scatter in a population. The mean deviation about the mean $[\delta_\mu(X)]$and mean deviation about the median $[\delta_M(X)]$for a distribution with p.d.f are defined respectively as

$$\delta_\mu(X) = \int_0^\infty |x-\mu|\, f(x)\,dx$$

and
$$\delta_M(X) = \int_0^\infty |x-M|\, f(x)\,dx$$

where μ=E(X) and M = Median (X)

The measures $\delta_\mu(X)$ and $\delta_M(X)$ are calculated using the relationships

$$\delta_\mu(X) = 2\mu F(\mu) - 2\int_0^\mu x\, f(x)\,dx \qquad (6.1)$$

and
$$\delta_M(X) = \mu - 2\int_0^\mu x\, f(x)\,dx \qquad (6.2)$$

Now for the MLD we have,

$$\int_0^\mu x\; f(x)dx = \mu - \frac{\left[\lambda^4(\mu+\mu^4)+\lambda^3(1+4\mu^3)+12\lambda^2\mu^2+24\lambda\mu+24\right]e^{-\lambda\mu}}{\lambda(\lambda^3+6)} \qquad (6.3)$$

Using (1) and (3) with F(μ)

$$\delta_\mu(X) = \frac{2[\lambda^3\mu^3+6\lambda^2\mu^2+18\lambda\mu+(\lambda^3+24)]\,e^{-\lambda\mu}}{\lambda(\lambda^3+6)} \qquad (6.4)$$

Similarly,

$$\int_0^M x\; f(x)dx = \mu - \frac{\left[\lambda^4(M+M^4)+\lambda^3(1+4M^3)+12\lambda^2M^2+24\lambda M+24\right]e^{-\lambda M}}{\lambda(\lambda^3+6)} \qquad (6.5)$$

Using (2),

$$\delta_M(X) = \frac{2[\lambda^4(M+M^4)+\lambda^3(1+4M^3)+12\lambda^2M^2+24\lambda M+24)]\,e^{-\lambda M}}{\lambda(\lambda^3+6)} - \mu \qquad (6.6)$$

VII. ORDER STATISTICS

Let $X_1, X_2, \ldots, X_n$ be a random of sample of size n from MLD ( ). Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ denote the corresponding order statistics. Then the pdf and cdf of the kth- order statistics, say Y = $X_{(k)}$ are given by

$$f_Y(y) = \frac{n!}{(k-1)!(n-k)!}\,F^{k-1}(y)\,\{1-F(y)\}^{n-k}\; f(y)$$

$$= \frac{n!}{(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\,F^{k+l-1}(y)\; f(y) \qquad (7.1)$$

and
$$F_Y(y) = \sum_{j=k}^{n}\binom{n}{j}F^j(y)[1-F(y)]^{n-j}$$

$$= \sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l\,F^{j+l}(y) \qquad (7.2)$$

Respectively for k = 1, 2, 3, . . . , n.

Using (7.1) and (7.2) the p.d.f and c.d.f of kth order statistics are given by,

$$f_Y(y) = \frac{n!}{(k-1)!(n-k)!}\cdot\frac{\lambda^4}{(\lambda^3+6)}(1+x^3)\,e^{-\lambda x}\cdot\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\left[1-\frac{(\lambda^3x^3+3\lambda^2x^2+6\lambda x+\lambda^3+6)\,e^{-\lambda x}}{\lambda^3+6}\right]^{k+l-1}$$

and

$$F_Y(y) = \sum_{j=k}^{n}\sum_{l=0}^{n-k}\binom{n}{j}\binom{n-j}{l}(-1)^\lambda\left[1-\frac{(\lambda^3x^3+3\lambda^2x^2+6\lambda x+\lambda^3+6)\,e^{-\lambda x}}{\lambda^3+6}\right]^{j+l}$$

VIII. BONFERRONI AND LORENZ CURVES

Bonferroni and Lorenz curves (1930) and Bonferroni Gini indices have applications in the fields like economics, demography, insurance and medicine. The Bonferroni and Lorenz curves are defined as,

$$B(p)=\frac{1}{p\mu}\int_0^q x\,f(x)\,dx=\frac{1}{p\mu}\left[\mu-\int_q^\infty x\,f(x)\,dx\right] \quad (8.1)$$

and

$$L(p)=\frac{1}{\mu}\int_0^q x\,f(x)\,dx=\frac{1}{\mu}\left[\mu-\int_q^\infty x\,f(x)\,dx\right] \quad (8.2)$$

respectively, where, μ = E(X) and q = F^{-1}(p) using B(p) and L(p) the Bonferroni and Gini indices are defined as,

$$B=1-\int_0^1 B(p)\,dp \qquad (8.3)$$

and $$G=1-2\int_0^1 L(p)\,dp \qquad (8.4)$$

using pdf (2.9), we get

$$\int_q^\infty x\,f(x)\,dx=\frac{[\lambda^4(q+q^4)+\lambda^3(1+4q^3)+12\lambda^2q^2+24\lambda q+24)]\,e^{-\lambda q}}{\lambda(\lambda^3+6)} \qquad (8.5)$$

Using (8.5) in (8.1) and (8.2) we get

$$B(p)=\frac{1}{p}\left\{1-\frac{[\lambda^4(q+q^4)+\lambda^3(1+4q^3)+12\lambda^2q^2+24\lambda q+24)]\,e^{-\lambda q}}{\lambda^3+24}\right\} \qquad (8.6)$$

and $$L(p)=1-\frac{[\lambda^4(q+q^4)+\lambda^3(1+4q^3)+12\lambda^2q^2+24\lambda q+24)]\,e^{-\lambda q}}{\lambda^3+24} \qquad (8.7)$$

Now using equations (8.6) and (8.7) in (8.3) and (8.4) we get

$$B=1-\frac{[\lambda^4(q+q^4)+\lambda^3(1+4q^3)+12\lambda^2q^2+24\lambda q+24)]\,e^{-\lambda q}}{\lambda^3+24}$$

and

$$G=-1+\frac{2[\lambda^4(q+q^4)+\lambda^3(1+4q^3)+12\lambda^2q^2+24\lambda q+24)]\,e^{-\lambda q}}{\lambda^3+24}$$

IX. ESTIMATION OF PARAMETERMAXIMUM LIKELIHOOD ESTIMATION

Let {X_1, X_2, . . . ,X_n} be an iid random sample from MOPLD. Then, the likelihood function L of MOPLD is given by,

$$L=\left(\frac{\lambda^4}{\lambda^3+6}\right)^n\prod_{i=1}^{n}(1+x_i^{\,3})\,.\,e^{-\lambda\sum_{i=1}^{n}x_i}$$

and the log-likelihood function is obtained as,

$$\log L=4n\log\lambda-n\log(\lambda^3+6)+\sum_{i=1}^{n}\log(1+x_i^{\,3})-\lambda\sum_{i=1}^{n}x_i$$

and $$\frac{d}{d\lambda}(\log L)=\frac{4n}{\lambda}-\frac{3\lambda^2 n}{\lambda^3+6}-n\,\bar{x}$$

Where $\bar{x}$ is the sample mean.

The maximum likelihood estimate of λ, $\hat{\lambda}$ is the solution to the equation $\frac{d}{d\lambda}(\log L)$ and it can be obtained by solving the polynomial equation,

$$\bar{X}\lambda^4-\lambda^3+6\bar{X}\lambda+24=0 \qquad (9.1)$$

X. METHOD OF MOMENT (MOM) ESTIMATION

Equating the population mean of MLD to the corresponding sample mean, the method of moment (MOM) estimate $\tilde{\lambda}$ of λ is the same as given by the equation (9.1).

XI. APPLICATION OF MODIFIED ONE-PARAMETER LINDLEY DISTRIBUTION

The MLD has been fitted to two real data sets and compared its goodness of fit with the one parameter exponential, Lindley and Aakash distributions.

Data set 1: The first data set is the data on strength of glass of aircraft window reported by Fuller et al., (1994).
18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.50, 25.52, 25.80, 26.69, 26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381

Data set 2: The second data set is the data on endurance of deep grove ball bearing measured in millions of revolutions before failure from lawless (1982, p-228).
17.88 28.92 33.00 41.52 42.12 45.60 48.80 51.84 51.9654.12 55.56 67.8068.44 68.64 68.88 84.12 93.12 98.64 105.12 105.84 127.92128.04 173.40

Table – 1: MLE's, -2lnL, AIC, AICC, BIC and K-S statistics of the fitted distributions for data sets 1 and 2.

	Model	Parameter estimate	-2 lnL	AIC	AICC	BIC	K-S Statistic
Data 1	MOPLD	0.1298	232.8	234.8	234.9	236.2	0.2213
	Aakash	0.0971	240.1	242.7	242.8	244.1	0.2664
	Lindley	0.0630	254.0	256.0	256.1	257.4	0.3332
	Exponential	0.0325	274.5	276.7	276.7	277.9	0.1264
Data 2	MOPLD	0.0554	226.1	228.1	228.3	229.2	0.1230
	Aakash	0.0415	227.1	229.1	229.3	230.2	0.1044
	Lindley	0.0273	231.5	233.5	233.7	234.6	0.1500
	Exponential	0.0138	242.9	244.9	245.1	246.0	0.2600

The results from the table clearly indicates that the fitting of MOPLD provides much closer fit to the data sets 1 and 2 as compared to Akash, Lindley, and Exponential distributions. The results also suggests that MOPLD is preferable as compared to Akash, Lindley, and Exponential distributions in modeling life-time data from engineering and bio-medical fields.

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