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The third example of Newton's Third Law leads to inconsistent results i.e. mass becomes infinite and imaginary.

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ABSTRACT
Objective: *Newton gave three examples to explain illustrate the law after definition. The first two examples, a stone is pushed by finger or a stone is pulled by horse. Their critical study leads to inconsistent study.*
Methods/Statistical analysis: *Newton considered in two cases that forces are so applied the stones remain at rest. Thus work done is zero (W=F.0 =0). Whereas in third case Newton considered example of collisions that bodies moves after they collide. Newton further stated that change in momentum of projectile is negative of change in momentum of target.*
Findings: *Thus velocity of target can be measured from equations third examples. In many cases the velocity of the target turns out equal to c or more than c (speed of light). Thus relativistic mass becomes infinite and imaginary. This is clear inconsistency between experimental findings and theoretical deductions.*
Application/Improvements: *In physics adhoc assumptions (as in case of Michelson Morley experiments) and empirical determination of coefficient (Bethe–Weizsäcker mass formula) is taken up. In view of these if statement of Newton's is regarded as in proportionality form. Then of coefficient of proportionality is regarded as less than unity i.e. ½ (say), then consistent results are produced.*

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I. INTRODUCTION

Earlier Newton defined third law of motion in the Principia (1686) at page 20.

The original form of the Third Law of Motion as in the Principia[6] is:
To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts

Action = - Reaction (1)
or Action of first body on second = - Reaction of second body on first (1)
or Force exerted by one body = -Force exerted by other body (1)
$F_A = -F_B$

There is no other term between F_A , F_B thus forces exerted by bodies are not affected by any other factor. Thus action should always be equal to reaction. Thus action and reaction has to be always equal or force exerted by Body A on Body B must be UNIVERSALLY equal. But experimentally this deduction is not justified along with other deductions from the law. These can be easily justified.

Newton justified the law in *the Principia*. Newton further stated the third law of motion in different ways i.e. in terms of action and reaction; and force exerted by body A (F_A) on the body B (F_B) of body A .

Newton's justification of Third Law of Motion. After the statement of law Newton gave three applications of Third law of motion in qualitative way. The first two examples follow from the following statement or phrase given by Newton.
"Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone"

1.1 Newton's First example :

If you press a stone with your finger, the finger is also pressed by the stone.
Thus if a stone is pressed by finger, then finger also presses the stone. If pressing of stone by finger is action, then pressing of finger by stone is reaction.
Finger presses the stone (action) = - Stone presses the finger (reaction)
Now Newton considered the case that stone remains at rest when pressed by finger. The stone does not move at all. In this case work done is zero, as stone does not move. W =FS =0 (2)

1.2 Newton's second example:

If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone.
It is again the similar interpretation. If horse pulls the stone, then horse is also pulled backward with

same force. Thus if pulling of stone by horse is action, then pulling of horse by stone is reaction.

Pulling of stone forward by horse (action) = - backward pulling of horse (reaction) Newton considered the case when the stone remains at rest , horse is not able to set it in motion. Thus, distance travelled by stone is zero, hence work done is zero.

out of these three, two examples the body remain at rest after action and reaction. In this case work done is zero.

W =FS = F.0 =0 (2)

1.3 ewton's Third example

In third example, the bodies move after impinging or colliding. Thus work done is non zero. Thus Newton's third law of motion is applicable to stationary and moving bodies. The application of Newton's Third Law of Motion is given by the statement

"If a body impinges upon another and by its force ***change the motion of the other****, that body also (because of the quality of, the mutual pressure)* ***will undergo an equal change****, in its own motion,* ***towards the contrary part.***

The changes made by these actions are equal, not in the velocities but in the motions of bodies; that is to say, if the bodies are not hindered by any other impediments. For, because the motions are equally changed, the changes of the velocities made towards contrary parts are reciprocally proportional to the bodies. This law takes place also in attractions, as will be proved in the next scholium."

II. INTERPRETATION OF NEWTON'S THIRD APPLICATION OF THIRD LAW OF MOTION.

Till date scientists have not formulated any mathematical equation on the basis of above statement, but Newton's third law of motion is regarded as universally valid. It is being mathematically interpreted here for first time, and some limitations are self evident. Firstly the characteristics of bodies which are so significant experimentally are not taken in account. Secondly results are not consistent with Special Theory of Relativity. These are limitations of Third Law of Motion only; hence the generalization of Newton's third law of motion is necessary.

The third application of third law of motion in the Principia at page 20 implies, Change in momentum of target = - Change in momentum of projectile (2)

It is similar to original form of Newton's third law of motion,

"To every action there is equal and opposite reaction"

Reaction =- Action (1)

Sir Isaac Newton had described the application of first law of motion , in the third example of the law at page 20 of the Principia.

For target (the other body), Let body B (target) exerts force on the body B (target) of mass M which has initial velocity is $V_{initial}$. Then initial momentum of projectile is $MV_{initial}$ and forward momentum (after being hit by target) $MV_{forward}$. The change in momentum is equal to difference between final momentum and initial momentum. Change in momentum of target = $MV_{forward}$ - $MV_{initial}$ (3)

For projectile (a body), Let body A (projectile) of mass m moving with velocity $U_{forward}$ and after striking it moves with velocity $U_{backward}$. Then initial momentum of projectile is $mU_{forward}$ and final momentum $mU_{backward}$.

Change in momentum of projectile = $mU_{backward} - mU_{forward}$ (4)

According to Newton's application as in eq.(1) we get,

change in momentum of target as caused by projectile = - change in momentum of projectile which causes change in momentum of target (5)

Eq.(5) is consistent with eq.(2) which is further application of eq.(1) , change in momentum is *towards the contrary part.*

$MV_{forward} - MV_{initial} = -(mU_{backward} - mU_{forward})$ (6)

$MV_{forward} - MV_{initial} = -(U_{backward} - U_{forward})m = m\,U_{forward} - m\,U_{backward}$

$V_{forward} = (U_{forward} - U_{backward})\, m/M + V_{initial}/M$ (7)

Let target be at rest initially i.e. $V_{initial} = 0$

$V_{forward} = (U_{forward} - U_{backward})\, m/M$ (8)

Thus in eq.(8) , the velocity of target ($V_{forward}$) is reciprocal to mass of body M, in eq.(8). Thus Newton's deduction is justified.

2.1 Simple deduction from eq.(8)

In case after collision the forward velocity of target is zero i.e. does not move. Practically it may be the when smaller bodies collide with heavy body e.g. a ball collides with wall. The wall remain at rest, now eq.(8) becomes ,

$0 = (U_{forward} - U_{backward}) m/M$

or

$U_{forward} = U_{backward}$ (9)

Forward velocity of projectile (ball) = Backward velocity of projectile (ball). The velocity is vector quantity having both direction and magnitude. Thus projectile must rebound with same velocity as with it moves forward. If we regard $U_{forward}$ as action, and $U_{backward}$ as reaction, then in Reaction = Action Thus eq.(9) leads to third law of motion, the direction of action ($U_{forward}$) and reaction ($U_{backward}$) are opposite. As the velocity is vector quantity, thus to compare velocities both directions and magnitudes must be taken in account.

2.2 **Experimentally significant characteristics of bodies**

Consider three bodies of rubber , super elastic material and wool are thrown on the wall with same velocity. After striking the body of rubber will rebound with velocity v_r , body of super elastic material will rebound with velocity v_s , and velocity of cloth or wool or sponge will rebound with velocity v_w . According to eq.(9), the velocities rebound of all three substances must be equal.

Velocity of super elastic material = velocity rubber = velocity of wool In this case experimentally, the body of super elastic material will rebound with maximum velocity (v_s) . The bodies of wool, cloth or sponge will rebound with least velocity (v_w). In this case $v_s > v_r > v_w$ i.e.

velocity of super elastic material >velocity rubber>velocity of wool

Thus velocities of rebound will depend characteristics of projectiles, these are not taken in account in third law of motion. This effect can be assessed in other experiments also by choosing targets of different materials.

2.3 Interpretation in terms of velocity, momentum and kinetic energy

Further, let us consider a spring (having high spring constant) and sponge, both of mass ½ kg are thrown at same target with velocity 2m/s (7.2 km/hour). Thus both bodies will move with same velocity, momentum (1kgm/s), kinetic energy (1J).

Let both the bodies strike with wall, which remain at rest i.e. $V_{forward}$ =0. Thus equations can be interpreted on the basis of eq.(9). So spring and sponge both must rebound with same velocity or momentum, theoretically.

Thus $mU_{backward}$ (spring) = m $U_{backward}$ (sponge) (10)

As mass of the spring and sponge bodies is equal , thus

$U_{backward}$ (spring) = $U_{backward}$ (sponge) (11)

Thus theoretically according to third law of motion eq.(10) must hold good. However experimental results do not coincide with eqs. (10,11), as the sponge rebounds to small distance with infinitesimal small velocity. Whereas the spring rebounds quickly or abruptly. Thus

$U_{backward}$ (spring)> $U_{backward}$ (sponge)

The velocity is directly related with momentum and kinetic energy. Thus eq.(11) can also be interpreted as,

$P_{backward}$ (spring)> $P_{backward}$ (sponge)

$KE_{backward}$ (spring)> $KE_{backward}$ (sponge)

Newton had stated the application of third law of motion in terms of velocity and momentum, the same can be extrapolated in terms of kinetic energy.

Both theoretically and experimentally the limitations of the law become clear. For example it does not account for the significant factors e.g. inherent characteristics, nature, compositions, flexibility, rigidity, magnitude, size, elasticity, shape , distinctiveness of interacting bodies, mode of interactions, point of impact etc.

The law is universally applicable for all bodies e.g. bodies may be of steel, wood, rubber, cloth, wool, sponge, spring, typical plastic, porous material, air / fluid filled artifact, mud or kneaded flour or chewing gum specifically fabricated material etc. For all such bodies if the action is same, then the reaction must be the same. But it is not justified. Thus third law of motion has been generalized so that it may take in account the characteristics of body and other factors.

2.4 Contradiction of Special Theory of Relativity.

The third application of third law of motion as given by Newton in the Principia at page 20 i.e. "*If a body impinges upon another and by its force change the motion of the other, that body also (because of the quality of, the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part.*" leads to contradictory results. The reason is very simple. Till date the above statement is not mathematically checked specifically in all cases. Newton did not have opportunity to mathematically interpret above statement in mathematical form. It may be due to reason that in Newton's time , there was no tradition to interpret laws in terms of mathematical equations. Due to this reason Newton even did not write any equation for second law of motion and law of gravitation. The prevalent equation for second law of motion F =ma was given by Swiss Leonhard Euler in 1775, whereas equation for law of gravitation F =GmM/r2 was given lator. Newton had expressed the law of gravitation in form 12 propositions in Book III of the Principia.

The reason for above deduction is that Newton's above statement gives velocity equal to that of light i.e. $3x10^8$ m/s , which is not allowed for any body particle. In 1893 Thomson had put forth that if anybody moves with speed equal to that of light, then its mass becomes infinite. This perception is basis of Special Theory of Relativity. Even mass of numerous multiverses multiverses is finite. So nobody can move with speed more than that of light.

Further critical analysis of third application of third law of motion leads to more incorrect result that mass of body becomes imaginary which is meaningless. The reason is that Newton's third law of motion gives imaginary results. These results naturally follows from the mathematical equations based on statement of third application of third law of motion when its predicts speed of body more that speed of light i.e. speed of body>speed of light ($3x10^8$ m/s)

(a) Consider a projectile of mass 10kg, moving with velocity 30m/s (108km/hr) strikes the target of mass 10^{-6} kg, which is at rest. This is experimentally feasible case.

Thus,

m =100kg , $U_{forward}$=30m/s, $U_{backward}$ =0, M =10^{-6} kg,

When projectile of mass 10kg moving with velocity 30m/s (108km/hr) strikes with strikes a target of mass 10^{-6} kg, then it does not rebound i.e. $U_{backward}$ =0; the projectile keeps on moving with same velocity. Thus eq.(8) becomes

$V_{forward}$ = ($U_{forward}$ - $U_{backward}$) m/M (8)

$V_{forward}$ = 30x10/10^{-6} = $3x10^8$m/s =speed of light

Thus the target must move with speed of light and according to relativistic variation of mass its mass should become infinity i.e.

$$m_{rest} = \frac{m_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_{rest}}{\sqrt{1-\frac{c^2}{c^2}}} = \infty \quad (12)$$

It is physically inconsistent result even mass of multiverse is finite, nobody can move with speed of light. This situation can be practically observed in many cases, but mass never become infinity. This limitation of Newton's deduction is being interpreted for first time quantitatively.

(b) Consider a projectile of mass 10kg, moving with velocity 30.0001m/s (108km/hr) strikes the target of mass 10^{-6} kg. This set of observations can be experimentally checked, these conditions are feasible.

Thus,

m =100kg , $U_{forward}$=30m/s, M =10^{-6} kg,

$V_{forward}$ = ($U_{forward}$ - $U_{backward}$) m/M

When projectile of mass 10kg moving with velocity 30.0001m/s strikes with strikes a target of mass 10^{-6} kg, then it does not rebound i.e. $U_{backward}$ =0

$V_{forward}$ = 30.0001x10/10^{-6} = 3.0001x10^8m/s

If all parameters remain same but u_i becomes 30.0001 m/s then

$$m_{rest} = \frac{m_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_{rest}}{\sqrt{1-1.0000066}} = \frac{m_{rest}}{\sqrt{-0.0000066}} = \text{Imaginary} \quad (13)$$

(c) Similar results contradicting theory of relativity and experiments, can be obtained in many other experimentally feasible cases. For this consider a projectile of mass 100kg moves with velocity 30m/s strikes with body of mass 0.1mgm (10^{-5} kg), then final velocity of body or target is given by eq.(8)

$V_{forward}$ = ($U_{forward}$ - $U_{backward}$) m/M = 30 x 100/10^{-5} kg m/s = $3x10^8$ m/s

Thus third application of Newton's third law of motion as given at page 20 of the Principia leads to inconsistent results. This aspect is not discussed in existing physics.

2.5 Alternate way of writing above equations.

We have eq.(6) as

$MV_{forward}$ -$MV_{initial}$ = - ($mU_{backward}$ –$mU_{forward}$) (6)

When a projectile of mass 10kg or 100kg or more moving with speed 30m/s (108km/hr) strikes with target of very -2 small mass then projectile keeps on moving in the forward direction with same velocity. However in third application of Newton's third law of motion, the velocity in backward direction is required. Thus we can write $U_{backward} = -U_{forward}$ (14)

With help of eq.(14) , eq.(6) becomes, $MV_{forward} - MV_{initial} = -(-mU_{forward} - mU_{forward})$ (6)
Let initially target is at rest i.e. $V_{initial} = 0$

$MV_{forward} = (mU_{forward} + mU_{forward})$

$V_{forward} = (U_{forward} + U_{forward})m/M$

(a) Consider a projectile of mass 5 kg, moving with velocity 15m/s (54km/hr) strikes the target of mass 10^{-6} kg, which is at rest. This is experimentally feasible case. Thus,

$V_{forward} = (U_{forward} + U_{forward})m/M$ = $(15+15)x10/10^{-6} = 30x10x10^{6} = 3x10^{8}$ m/s

Thus under this condition also body moves with speed of light. Now according to relativistic variation of mass ,

$$m_{motion} = \frac{m_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_{rest}}{\sqrt{1-\frac{c^2}{c^2}}} = \infty \quad (15)$$

It is physically inconsistent result, nobody can move with speed of light. This situation can be practically observed in many cases, but mass never become infinity.

(b) Similarly if velocity of projectile is slightly more than speed of light, then mass becomes imaginary. Thus these are inconsistent predictions from Newton's deduction.

2.4 Tradition of introduction of adhoc hypotheses and empirical evaluation

When inconsistent results are obtained then one of way to explain them is adhoc hypothesis and empirical evaluation.

A step to explaining the Michelson – Morley experiment's null result was found in the FitzGerald –Lorentz contraction hypotheses or adhoc assumption now simply called length contraction or Lorentz contraction first proposed by George FitzGerald (1889) and Hendrik Lorentz (1892). According to it law all objects physically contract by L $\sqrt{1-\frac{v^2}{c^2}}$.

Likewise the semi-empirical mass formula states that the binding energy will take the following form

$$E_B = a_v \; {}_{-}a_5 \; A^{2/3} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z))$$

Each of the terms in this formula has a theoretical basis, as will be explained below. The coefficients a_v a_δ ,a_c , A_a a and a coefficient appears in the formula for $\delta(A,Z))$ are determined empirically.

We should try to get consistent results from the statement. Only method is to change its definition. Now the definition should be generalized. The statement must be stated in terms of proportionality form.

"*If a body impinges upon another and by its force change the motion of the other, that body also (because of the quality of, the mutual pressure) will undergo an proportional change , in its own motion, towards the contrary part.*"

It is third application of Newton's Third Law of Motion. The statement must be stated in terms of proportionality form.

$MV_{forward} - MV_{initial} = -f(mU_{backward} - mU_{forward})$ 16)

Now equation equivalent to eq.(8) can be written as

$V_{forward} = f(U_{forward} - U_{backward})\, m/M$ (17)

or $V_{forward} = f\, U_{forward}\, m/M$ (18)

Now substituting various values

$V_{forward} = f\, 3x10^{8}$

Now empirically the value of f is always less than one such that $V_{forward}$ is always less than speed of light. So consistent results are obtained. Let value of f is ½ , then

$V_{forward} = v = ½$

$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-\frac{1}{4}} = 0.866$$

$$m_{motion} = \frac{m_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} = 1.154 m_{rest}$$

Thus empirical determination of value of f leads to consistent results. However third application of Newton's third law of motion lead to inconsistent results.

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