RESEARCH ARTICLE OPEN ACCESS

Sonar Target Characterization and Echo Separation Using the Fractional Fourier Transform and its Relation to the Wigner Distribution

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ABSTRACT
In this paper, we apply the relationship between the Fractional Fourier Transform (FrFT) and the Wigner Distribution (WD) to identify and separate echoes from an active sonar based upon the characteristics of different targets. Plotting the received echo using the WD, which is a time-frequency plane representation, allows us to identify the characteristics of the environment, since targets will have different time and frequency domain responses to the chirp or tone pulses that will manifest in the WD. A chirp (or tone) can also be easily separated from the cluttered background, hence enabling extraction of targets, such as submarines, from the clutter. The received signal is rotated to a new time axis 't_a', with rotational parameter 'a', such that the chirp becomes a tone and can be extracted or notched easily, without requiring a filter. Echoes with just slight differences in Doppler can be easily separated. Furthermore, echoes from strong targets, such as submarines, can be easily notched to extract the weaker targets, such as schools of fish. We demonstrate the algorithm by simulation, using several target types and chirp or tone transmit pulses. We show that a weak signal up to 20 dB below a strong one can be easily extracted using the FrFT by notching the strong target first.

Keywords: Fractional Fourier Transform, Sonar, Target Extraction, Wigner Distribution

I. INTRODUCTION

The Fractional Fourier Transform (FrFT) is a versatile tool that has been applied to problems in numerous fields, including quantum mechanics, optics [1], image processing [2], signal processing for communications ([2] and [3]), and radar [4]. The FrFT is a very useful tool for separating the echo from a target-of-interest (TOI) from interference in a non-stationary environment [2]. The problem of separating multiple moving radar targets of differing power levels received by a monostatic radar system in clutter lends itself nicely to implementation by the FrFT because moving target echoes are chirp signals, which become tones in the proper FrFT domains, and hence can be easily separated ([4] and [5]). Previous works applying the FrFT to radar include extracting a single target in clutter using time delay correlation methods [6] and using clutter map cancellation [7]. For sonar, matched filtering is the conventional method for detecting a sonar echo and is described in many works, e.g. [8].

In this paper, we seek improved methods for active sonar echo estimation and target type identification using the FrFT. We propose to use the FrFT to separate the TOI, which is a continuous wave (CW) tone or chirp pulse echo, from the clutter by rotating to the proper axis 't_a' using rotational parameter 'a', in which the TOI becomes a tone. By simply searching for the maximum peak over all values of 'a', we easily find the correct axis, utilizing the technique described in [4]. The contribution of this paper is to apply the radar target separation method in [4] to isolate and help identify sonar targets; additionally, we use signal strength and Doppler variations in the reflected signals as an indicator of target type. CW tones and chirps project as single tones so all but one value will be notched to extract the tone and suppress the clutter; alternatively, we can notch the tone to enhance and detect weaker signals. However, because there could be leakage of the SOI energy in adjacent FrFT domains, i.e. $a \pm \Delta a$, where Δa is the chosen search step size, the strongest signal will not be perfectly notched. This limits our ability to detect very weak signals more than 20 dB below a strong echo, such as small gas bubbles which may be 40 dB or more below a strong echo, and will be discussed.

The paper outline is as follows: Section II briefly reviews the FrFT and its relation to the Wigner Distribution (WD), which is a useful tool for visualizing the FrFT and specifically for characterizing sonar target echoes. Section III discusses the properties of sonar targets, such as fish and submarines as well as the difficulties in distinguishing them in the frequency domain only. Section IV describes the sonar signal model, which is a tone or chirp. Section V discusses the method of using the FrFT and its relation to the WD to separate and characterize reflections from the different targets. Section VI presents example simulation results for both sets of signals and the multiple target types. Conclusions and remarks on future work are given in Section VII.

II. BACKGROUND: FRACTIONAL FOURIER TRANSFORM (FRFT)

In discrete time, we write the N × 1 FrFT of an N × 1 vector **x** as

$$\mathbf{X}_a = \mathbf{F}^a \mathbf{x}, \tag{1}$$

where $\mathbf{F}^a$ is an N × N matrix whose elements are ([2] and [9])

$$\mathbf{F}^a[m,n] = \sum_{k=0, k\neq(N-1+(N)_2)}^{N} u_k[m] e^{-j\frac{\pi}{2}ka} u_k[n], \tag{2}$$

and where $u_k[m]$ and $u_k[n]$ are the eigenvectors of the matrix S defined by [9]

$$\mathbf{S} = \begin{bmatrix} C_0 & 1 & 0 & \dots & 1 \\ 1 & C_1 & 1 & \dots & 0 \\ 0 & 1 & C_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & C_{N-1} \end{bmatrix}, \tag{3}$$

and

$$C_n = 2cos(\frac{2\pi}{N}n) - 4. \tag{4}$$

The Wigner Distribution (WD) is a time-frequency representation of a signal - a generalization of the Fourier Transform. The WD of a signal *x*(t) is defined as

$$W_x(t,f) = \int_{-\infty}^{\infty} x(t+\tau/2)x^*(t-\tau/2)e^{-2\pi j\tau f}d\tau. \tag{5}$$

An important relationship is that the projection of the WD of a signal *x*(t) onto an axis t_a gives the energy of the signal in the FrFT domain ‘a’, $|\mathbf{X}_a(t)|^2$ (see e.g. [3] or [10]). Letting α = aπ/2, we write this as

$$|\mathbf{X}_\alpha(t)|^2 = \int_{-\infty}^{\infty} W_x(tcos(\alpha)-fsin(\alpha), tsin(\alpha)+fcos(\alpha))df. \tag{6}$$

In discrete time, the WD of a signal x[n] is written as [11]

$$W_x[\frac{n}{2f_s}, \frac{kf_s}{2N}] = e^{j\frac{\pi}{N}kn} \sum_{m=l_1}^{l_2} x[m]x^*[n-m]e^{j\frac{2\pi}{N}km}, \tag{7}$$

where $l_1 = max(0,n - (N - 1))$ and $l_2 = min(n,N - 1)$.

III. SONAR TARGET PROPERTIES AND CHARACTERIZATION LIMITATIONS

Underwater targets possess unique characteristics that determine their target strength (TS) as a function of frequency and time. Existing techniques, however, are limited in their ability to separate targets based on frequency domain characteristics alone, because target echoes overlap in frequency. We consider several examples of targets in this section and discuss their unique backscattering properties which make them more easily separable using the WD. Targets such as fish and submarines are usually moving, so the Doppler properties vs. time can be exploited; air/gas bubbles and seafloor sediment is not moving. Specifically, fish will usually move slower than submarines, resulting in a smaller frequency variation. This translates to a different FrFT rotational parameter ‘a’ for each one to be notched or isolated. We can further combine this with the unique scattering properties of each target. Even though both submarine and fish usually have a TS that is flat with frequency, the TS of submarines will be much stronger (15 to 25 dB[1]) [8] than fish (−30 to −45 dB). Using the standard definition, we can write TS as [12]

$$TS = 10 \cdot log_{10}\frac{I_r}{I}, \tag{8}$$

where I_r is the intensity of the return echo from the target, typically taken at a normalized distance of 1 meter (m) away, and I is the intensity of the transmitted signal as it hits the target. Since this is measured at a distance of 1 m, note that TS can be positive.

The TS of small air bubbles increases with frequency but is weak (e.g. −80 to −100 dB); the TS of large air bubbles is flat versus frequency but is stronger (e.g. −40 to −60 dB) [8]. The TS of sediment is strong (TS 5 to 25 dB) and depends on frequency, the roughness of the particles, and the angle of incidence.

Because of the small scale variations, we can model sediment as a Gaussian pulse with random amplitude and phase. Smaller particles will produce larger backscatter (larger TS). Plotting the reflected signal in the WD plane allows these properties to be seen more clearly. If the target is too weak, a strong target cannot be notched enough to pull out the weak ones. However, a collection of weak targets can be identified with knowledge of these properties. If it is desired to study multiple strong targets more closely, then the WD can provide a clear picture of their distribution with time, by enabling nulling of the strongest targets, such as submarines to pull out weaker ones, like fish. Finally, reverberation echoes, which are undesired echoes from reflections of the target echo as it travels through the propagation medium, will also have a weaker strength and lower Doppler than the potential target, e.g. a submarine. It is difficult to separate reverberation echoes from the target using frequency domain techniques alone [8], but we will demonstrate how it can be done easily using the FrFT.

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[1] Conventional notation is dB but this is really dB/m^2

IV. SONAR SIGNAL MODEL

For a CW (continuous wave, i.e. tone), or a chirp, we assume the transmitted signal takes the form

$$x_1(t) = cos(2\pi[f_0(t-t_r)+\frac{1}{2}K_1(t-t_r)^2])rect(\frac{t-t_r}{T}), \quad (9)$$

where f_0 = 10 kHz, t_r = 0.6T is the chirp delay necessary to account for the rectangular pulse delay, T = 0.006 seconds is the length of the CW or chirp pulse; for a tone, we set $K_1 = 0$, and for a chirp we set $K_1 = B/T$, where B = 4,000 Hz is the chirp bandwidth. Rect(·) is a rectangular pulse [13]. The received signal takes the form

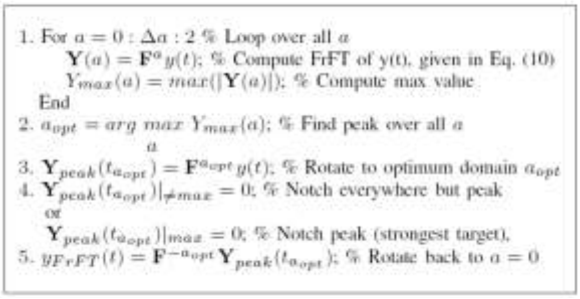
$$y(t) = A_1 x_1(t - t_{d_1}) + \sum_{i=2}^{K} A_i x_i(t) + n(t), \quad (10)$$

where $t_{d1} = 2R_1/c$ is the time delay between the transmit and receive pulse due to the target range, is the received amplitude, and c = 1,484 is the speed of sound in water. The terms $x_i(t)$, i = 2,3,...,K, are interference terms which could be reverberation echoes, or fish backscatter, modeled similarly to Eq. (9), but with unknown and different amplitudes A_i and chirp frequencies K_i. The different chirp rates result from different Doppler for reverberation echoes or speeds, e.g. from a moving target such as fish. The amplitudes are chosen based on the target strength discussed in the previous sections. We set each value of A_i based on the relative strength of target i to the desired target i = 1, using a carrier-to-interference ratio (CIR_i) value. For example, if target i = 1 has $TS_1 = -10$ dB and target i = 2 is fish with TS = −25 dB, we set CIR_2 = 15 dB, so that target 2 is 15 dB weaker than target 1, and hence we compute, in general, $A_i = \sqrt{1/(2 \cdot 10^{CIR_i/10})}$. We could also adjust the CIR to model size difference between the two targets and different distances from the receiving sonar, which would all affect the CIR in practice. The noise is modeled as AWGN or filtered AWGN, whose amplitude is set to give a desired signal-to-noise ratio (SNR). We assume K = 2 or 3 for our simulations, but in practice K could be larger.

V. PROPOSED CHARACTERIZATION OF TARGETS USING THE FRFT AND WD

We first assume a highly reflective object such as a submarine is the desired target. Then the received signal will consist of a chirp component, with unknown Doppler shift that we can detect with the FrFT. We search over ‘a’ and choose the axis ‘t_a’ that gives a maximum value of energy, where the chirp becomes a tone. By rotating to that axis we can easily notch the other target echoes (e.g. fish, sediment). Alternatively, we can notch the strong submarine to examine the remaining patterns to identify the nature of the environment using the characteristics of targets as described above. The algorithm is similar to that in [4] for separating radar echoes from multiple targets, but it either extracts a single strong target echo or notches (zeroes) it to study the rest of the environment. The algorithm is shown in Table I, and we demonstrate its use with some examples in the next section.

TABLE I
Proposed Strong Sonar Target Extraction (or Suppression) Algorithm



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1. For $a = 0 : \Delta a : 2$ % Loop over all $a$
       $\mathbf{Y}(a) = \mathbf{F}^a y(t)$; % Compute FrFT of y(t), given in Eq. (10)
       $Y_{max}(a) = max(|\mathbf{Y}(a)|)$; % Compute max value
   End
2. $a_{opt} = \underset{a}{arg\ max}\ Y_{max}(a)$; % Find peak over all $a$
3. $\mathbf{Y}_{peak}(t_{a_{opt}}) = \mathbf{F}^{a_{opt}} y(t)$; % Rotate to optimum domain $a_{opt}$
4. $\mathbf{Y}_{peak}(t_{a_{opt}})|_{\neq max} = 0$; % Notch everywhere but peak
   or
   $\mathbf{Y}_{peak}(t_{a_{opt}})|_{max} = 0$; % Notch peak (strongest target).
5. $y_{FrFT}(t) = \mathbf{F}^{-a_{opt}} \mathbf{Y}_{peak}(t_{a_{opt}})$; % Rotate back to $a = 0$
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VI. SIMULATIONS

We plot examples of the Wigner Distribution before and after applying the FrFT, i.e. on y(t) and $y_{FrFT}(t)$ from Table I, to notch interference. First, we assume the strongest signal, e.g. from a submarine, is the desired target and we use the FrFT to notch out the rest of the environment, plotting the WD before and after to demonstrate its performance to suppress interference and clutter by 20 dB or more (see Figs. 1 - 3).

In the last example, shown in Fig. 4, we notch the submarine echo to pull out echoes from what are modeled as fish, identified as such due to the slower, time varying Doppler and smaller target strength, as discussed previously. Here, the CIR between the fish, which is now the desired target i = 1, and the interfering submarine i = 2, is $CIR_2 = -20$ dB. Recall that the TS difference between these two targets could be about −60 dB, so we have assumed that it is a large school of fish that is much closer to the transmitting sonar device than the submarine. From the second plot, we see that there is another signal, e.g. from another group of fish, has been enhanced 20 dB or more because we were able to apply our algorithm to notch the submarine echo; this target echo is not visible in the first plot due to the strong submarine echo. Note from plotting the submarine echo before and after notching, we see that due to leakage, the strongest signal is nulled about 20 dB, hence this limits the amount that weaker signals can be amplified. This example shows our ability to use the FrFT to enhance weak targets, but also demonstrates the limitations in pulling out very weak signals, such as gas bubbles, which can be up to 100 dB lower than the strong signals. If a strong signal is not present, however, then very weak gas bubbles could be enhanced for analysis.

VII. CONCLUSION

This paper shows how the Fractional Fourier Transform and its relation to the Wigner Distribution can be used to characterize an underwater environment from a received sonar signal and potentially isolate the various targets of interest. We can easily notch chirp signals by rotating to the axis where those chirps become tones. We can further determine the nature of the targets by the target strength (TS) as a function of frequency and time. This is a powerful tool for studying and characterizing sonar environments as well as enhancing received echoes from targets of interest. Further research includes determining ways of pulling out signals that are very weak, i.e. more than 20 dB below a strong interferer and applying the method to collected data to test its use in identifying targets and characterizing a real underwater environment.

ACKNOWLEDGMENTS

The author thanks The Aerospace Corporation for funding this work and Alan Foonberg for reviewing the paper.

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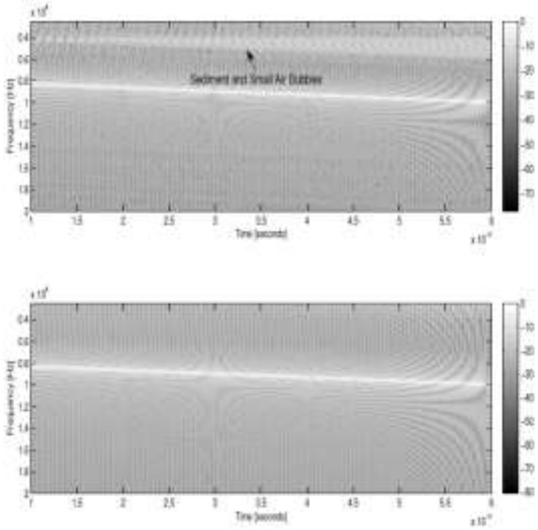

Fig. 1. Chirp (desired), filtered noise and Gaussian pulse interferer; SNR=5 dB; CIR_2=0 dB

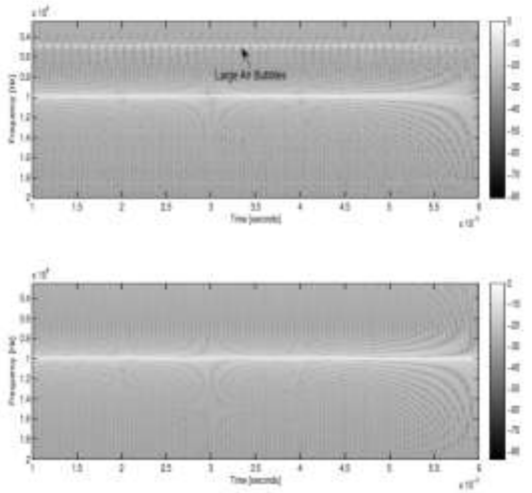

Fig. 2. Tone (desired), filtered noise and Gaussian pulse interferer; SNR = 5 dB; CIR_2 = 0 dB

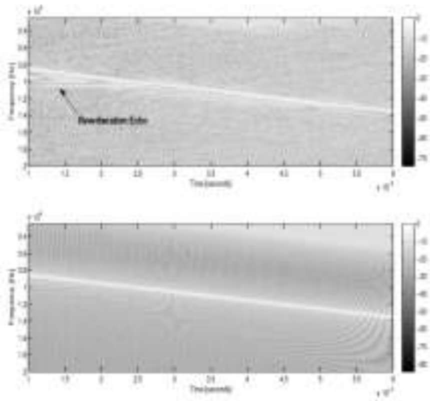

Fig. 3. Chirp (desired), noise and reverberation echo interferer; SNR = 8 dB; CIR_2 = 4 dB

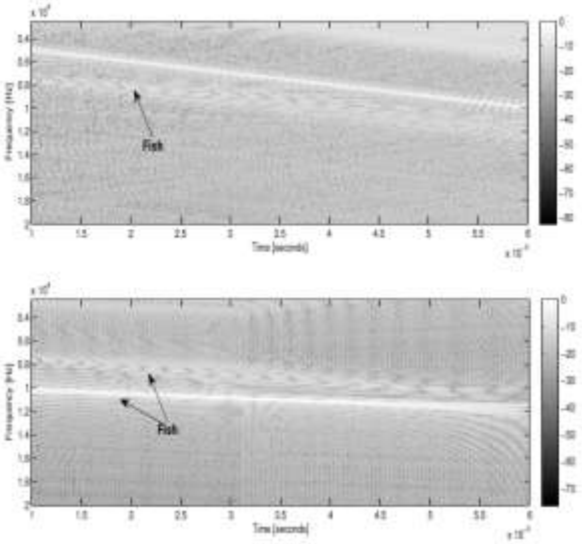

Fig. 4. Weak chirp from fish (desired), noise and strong chirp interferer; SNR=15 dB; CIR_2=−20 dB

Seema Sud " Sonar Target Characterization and Echo Separation Using the Fractional Fourier Transform and its Relation to the Wigner Distribution" International Journal of Engineering Research and Applications (IJERA) 7.8 (2017): 86-90