RESEARCH ARTICLE OPEN ACCESS

# Relations among Some Polygonal Numbers and a Nasty Number of the Form Six Times a Perfect Square

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**ABSTRACT:**
In this paper, we discover some relations among polygonal numbers and a nasty number by applying the solution of the standard equation $Z^2 = 6X^2 + Y^2$
***Keywords*:** Ternary quadratic equation, integral solutions, polygonal numbers.

Date of Submission: 24-07-2017 Date of acceptance: 05-08-2017

**NOTATIONS USED:**
Polygonal number of rank $n$ with side $m$ is given by

$$T_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$$

## I. INTRODUCTION

Diophantine equations are numerously rich because of its variety. Diophantine problems have fewer equations than unknown variables and involve finding solutions in integers. There is no universal method available to know whether a Diophantine equation has a solution or finding all solutions if it exists. In this context one may refer [1-3]. In [4-12] different types of ternary quadratic equations with three unknowns are studied for its non-trivial integral solutions by applying various methods. In this paper, we present some relations among the polygonal numbers and a nasty number by employing the general solutions to the ternary quadratic equation $Z^2 = 6X^2 + Y^2$

## II. METHOD OF ANALYSIS:

Consider the Diophantine equation

$$Z^2 = 6X^2 + Y^2$$

The general solution to $Z^2 = 6X^2 + Y^2$ is given by

$$Z = 6r^2 + s^2 \quad (1)$$

$$Y = 6r^2 - s^2 \quad (2)$$

$$X = 2rs \quad (3)$$

**RELATION-I**
**Relation among pentagonal number, decagonal number and nasty number**

Let $t_{5,n}, t_{10,m}$ be the pentagonal and decagonal number of rank $n$ and $m$ respectively.
Assume that "two hundred and sixteen times pentagonal number – sixteen times decagonal number $= 6X^2$, a nasty number"

$$\Rightarrow 216t_{5,n} - 16t_{10,m} = 6X^2$$

$$\Rightarrow (18n-3)^2 - (8m-3)^2 = 6X^2$$

which implies that

$$Z^2 = 6X^2 + Y^2$$

where

$$Z = (18n-3) \quad (4)$$

$$Y = (8m-3) \quad (5)$$

From (1) and (4), we have

$$n = \frac{6r^2 + s^2 + 3}{18}$$

From (2) and (5), we get

$$m = \frac{6r^2 - s^2 + 3}{8}$$

Here, we note that $n$ and $m$ are integers when

$$r = 6k + 5,\ s = 12k + 9$$

Therefore, the values of the ranks of pentagonal and decagonal number are represented by

$$n = 20k^2 + 32K + 13$$

$$m = 9k^2 - 18k + 9$$

We observe that

$$216t_{5,n} - 16t_{10,m} = 6(144k^2 + 288k + 9)^2$$

Hence, $216t_{5,n} - 16t_{10,m}$ is a nasty number.

**RELATION-II**

**Relation among triangular number, octagonal number and nasty number**

Let $t_{3,n}, t_{8,m}$ be the triangular and octagonal number of rank $n$ and $m$ respectively.

Assume that "eight times triangular number – three times octagonal number $= 6X^2$, a nasty number"

$$\Rightarrow 8t_{3,n} - 3t_{8,m} = 6X^2$$

$$\Rightarrow (2n+1)^2 - (3m-1)^2 = 6X^2$$

which can be written as

$$Z^2 = 6X^2 + Y^2$$

where

$$Z = (2n+1) \quad (6)$$

$$Y = (3m-1) \quad (7)$$

Equating (1) and (6), we have

$$n = \frac{6r^2 + s^2 - 1}{2}$$

Comparing (2) and (7), we get

$$m = \frac{6r^2 - s^2 + 1}{3}$$

Here, we note that the values of $n$ and $m$ are integers for the following choices of $r$ and $s$

$$r = 6k,\ s = 6k + 1$$

Therefore, the values of the ranks of triangular and octagonal number are expressed by

$$n = 126k^2 + 6K$$

$$m = 60k^2 - 4k$$

We observe that

$$8t_{3,n} - 3t_{8,m} = 6(72k^2 + 12k)^2$$

Hence, $8t_{3,n} - 3t_{8,m}$ is a nasty number.

**RELATION-III**

**Relation among heptagonal number, decagonal number and nasty number**

Let $t_{7,n}, t_{10,m}$ be the heptagonal and decagonal number of rank $n$ and $m$ respectively.

Assume that "forty times heptagonal number – sixteen times decagonal number $= 6X^2$, a nasty number"

$$\Rightarrow 40t_{7,n} - 16t_{10,m} = 6X^2$$

$$\Rightarrow (10n-3)^2 - (8m-3)^2 = 6X^2$$

$$\Rightarrow Z^2 = 6X^2 + Y^2$$

where

$$Z = (10n-3) \quad (8)$$

$$Y = (8m-3) \quad (9)$$

In view of (1) and (8), we establish that

$$n = \frac{6r^2 + s^2 + 3}{10}$$

Comparing (2) and (9), we find that

$$m = \frac{6r^2 - s^2 + 3}{8}$$

Since our intension is to find the integer values for the ranks, we note that $n$ and $m$ when

$$r = 10k + 1,\ s = 20k + 1$$

Therefore, the values of the ranks of heptagonal and decagonal numbers are expressed by

$$n = 100k^2 + 16K + 1$$

$$m = 25k^2 + 10k + 1$$

We observe that

$$40t_{7,n} - 16t_{10,m} = 6(400k^2 + 60k + 2)^2$$

Hence, $40t_{7,n} - 16t_{10,m}$ is a nasty number.

**RELATION-IV**

**Relation among Heptagonal number, Hexadecagonal number and nasty number**

Let $t_{7,n}$, $t_{16,m}$ be the heptagonal and hexadecagonal number of rank $n$ and $m$ respectively.

Assume that "Forty times heptagonal number – Seven times hexadecagonal number $= 6X^2$, a nasty number"

$$\Rightarrow 40t_{7,n} - 7t_{16,m} = 6X^2$$

$$\Rightarrow (10n-3)^2 - (7m-3)^2 = 6X^2$$

which can be written as

$$Z^2 = 6X^2 + Y^2$$

where

$$Z = (10n-3) \quad (10)$$

$$Y = (7m-3) \quad (11)$$

From (1) and (10), we find the ranks of the heptagonal number by

$$n = \frac{6r^2 + s^2 + 3}{10}$$

From (2) and (11), we evaluate the ranks of the hexadecagonal number by

$$m = \frac{6r^2 - s^2 + 3}{7}$$

Here, we note that $n$ and $m$ are integers when

$$r = 35k + 4 \text{ , } s = 70k + 1$$

Therefore, the values of the ranks of heptagonal and hexadecagonal numbers are expressed by

$$n = 1225k^2 + 182K + 10$$

$$m = 350k^2 + 220k + 14$$

We observe that

$$40t_{7,n} - 7t_{16,m} = 6\left(4900k^2 + 630k + 8\right)^2$$

Hence, $40t_{7,n} - 7t_{16,m}$ is a nasty number.

**RELATION-V**

**Relation among Dodecagonal number, Icosagonal number and nasty number**

Let $t_{12,n}$ ,$t_{20,m}$ be the dodecagonal and icosagonal number of rank $n$ and $m$ respectively.

Assume that "Twenty times dodecagonal number – Nine times icosagonal number $= 6X^2$ , a nasty number"

$$\Rightarrow 20t_{12,n} - 9t_{20,m} = 6X^2$$

$$\Rightarrow (10n-4)^2 - (9m-4)^2 = 6X^2$$

which can be written as

$$Z^2 = 6X^2 + Y^2$$

where

$$Z = (10n-4) \qquad (12)$$

$$Y = (9m-4) \qquad (13)$$

From (1) and (12), we find the ranks of the dodecagonal number by

$$n = \frac{6r^2 + s^2 + 4}{10}$$

From (2) and (13), we discover the ranks of icosagonal number by

$$m = \frac{6r^2 - s^2 + 4}{9}$$

Here, we note that $n$ and $m$ are integers when

$$r = 45k + 15 \text{ , } s = 90k + 16$$

Therefore, the values of the ranks of dodecagonal and icosagonal numbers are pointed out by

$$n = 2025k^2 + 1098K + 161$$

$$m = 450k^2 + 580k + 122$$

We observe that

$$20t_{12,n} - 9t_{20,m} = 6\left(8100k^2 + 4140k + 480\right)^2$$

Hence, $20t_{12,n} - 9t_{20,m}$ is a nasty number.

**RELATION-VI**

**Relation among Tridecagonal number, Icosikaidigonal number and nasty number**

Let $t_{13,n}$, $t_{22,m}$ be the tridecagonal and icosikaidigonal number of rank $n$ and $m$ respectively.

Assume that "Eighty eight times tridecagonal number – Forty times icosikaidigonal number $= 6X^2$ , a nasty number"

$$\Rightarrow 88t_{13,n} - 40t_{22,m} = 6X^2$$

$$\Rightarrow (22n-9)^2 - (40m-9)^2 = 6X^2$$

which can be written as

$$Z^2 = 6X^2 + Y^2$$

where

$$Z = (22n-9) \qquad (14)$$

$$Y = (40m-9) \qquad (15)$$

From (1) and (14), the ranks of the tridecagonl number are given by

$$n = \frac{6r^2 + s^2 + 9}{22}$$

From (2) and (15), the ranks of the icosikaidigonal number are noted by

$$m = \frac{6r^2 - s^2 + 9}{40}$$

Since our aim is to find the integer values for the ranks, we observe that $n$ and $m$ are integers when

$$r = 440k + 4 \text{ , } s = 880k + 15$$

Thus, the values of the ranks of tridecagonal and icosikaidigonal numbers are expressed by

$$n = 88000k^2 + 2160K + 15$$

$$m = 9680k^2 - 132k - 3$$

Therefore,

$$88t_{13,n} - 40t_{22,m} = 6\left(8100k^2 + 4140k + 480\right)^2$$

Hence, $88t_{13,n} - 40t_{22,m}$ is a nasty number.

## III. CONCLUSION:

To conclude, one can investigate a variety of interesting relations between any other familiar numbers by applying the solutions of different kinds of Diophantine equations.

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International Journal of Engineering Research and Applications (IJERA) is **UGC approved** Journal with Sl. No. 4525, Journal no. 47088. Indexed in Cross Ref, Index Copernicus (ICV 80.82), NASA, Ads, Researcher Id Thomson Reuters, DOAJ.

V. Pandichelvi. "Relations among Some Polygonal Numbers and a Nasty Number of the Form Six Times a Perfect Square." International Journal of Engineering Research and Applications (IJERA) 7.8 (2017): 12-15.