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Mass Transfer Effects on Stokes Problem for an Infinite Vertical Plate in a Rotating Fluid.

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ABSTRACT
An exact solution to the unsteady free convection flow of viscous incompressible fluid, in the presence of foreign mass, past an impulsively started infinite vertical isothermal plate in a rotating fluid, has been derived by Laplace-Transform technique. Axial and transverse velocity profiles are shown on graphs and numerical values of skin friction are listed in a table. It is observed that the non dimensional rotational parameter Rc increases there is fall in axial velocity profiles for all prandtl numbers because the coriolis forces oppose the fluid flow , hence the motion gets slow down . As Rc < 10^{-3} the flow field becomes unstable and flow is converted to the turbulent flow for all Prandtl numbers (i.e. Pr = .71 for air when Ma<<1 and Pr = 7 for water). The flow of water may become unstable at large values of time t. Increase in Schmidt number leads to decrease in axial velocity for air and water. The diffusion parameter ,N, increases leads to rise in axial velocity because the buoyancy flow forces assist the flow and the transverse skin friction increases for both air and water, the axial skin friction decreases for air and increases for water.
Keywords: Mass transfer, incompressible fluid, Coriolis forces, free convection.

I. INTRODUCTION

Stokes [12] first presented an exact solution of Navier-Stokes equation which was concerned for the flow of viscous incompressible fluid past an infinite horizontal impulsively started plate, in a stationary mass of fluid. Natural convection flow of viscous incompressible fluid past an infinite vertical plate was studied by many authors and many papers were published and these are discussed in many books on heat transfer e.g. Gebhart [5], Bejan [1, 3]. If the impulsive motion is given to an isothermal vertical plate surrounded by stationary mass of fluid, how the motion takes place? This was first studied by the Laplace transform technique by Soundelgeker [11] who studied the effect of free convection current near the plate. The assumption of the vertical plate being surrounded by a stationary mass of fluid is rather restricted one. In many geophysical applications the fluid is found to be always rotating due to rotation of the earth, such a situation almost occurs everywhere. If plate is given motion in rotating fluid how the flow takes place? This has been discussed by Batchelor [2]. Lezius & Johnston [8] experimentally examine the flow instability caused by coriolis forces. The effect of rotation and free convection currents on the motion of the fluid near an impulsively started infinite vertical plate has been studied by R.M.Lahurikar [6]. In all these papers the fluid is assumed to be pure air or water, but in nature or in chemical industries multi component mixture produces mass transfer process and heat and mass transfer occur simultaneously. Convection mass transfer alone constitutes the backbone of many operations in the chemical industries. In an industrial area atmospheric circulation acts as carrier for the many exhaust streams put out by factories in to the atmosphere. Mass transfer processes abound in the world around. Park and Lau [9]) and Park et.al. [10] Study the effect of rotational Coriolis force experimentally on mass transfer distribution. Recently mass transfer effect on transient flow past and infinite vertical plate in a rotating fluid studied is by Bhalerao and Lahurikar [4].

Our aim is to study mass transfer effects on a rotatary fluid past an infinite vertical plate, which has not been studied in the literature so far, hence the motivation to understand this physical situation .As the problem is governed by couple of linear partial differential equation, and we solved it by Laplace transform technique. In section 2, mathematical analysis is presented and in section 3, conclusions are set out.

II. MATHEMATICAL ANALYSIS:

Consider an infinite vertical plate surrounded by an infinite mass of stationary viscous incompressible fluid situated at z’ = 0 .Suppose the x’ axis is taken along the plate in a vertical upward direction and let the y’ axis be in horizontal direction assumed to right angle to x’ axis, then the z’ axis is taken normal to the x’-y’ plan. (Ref. Fig.1).

Initially, plate and fluid are assumed to be stationary and are at the same temperature T∞'. Then at time t'>0, the plate is given an impulsive motion in vertically upward direction with a velocity U_0 ,the fluid starts rotating about the z' axis very slowly with a uniform angular speed Ω' , simultaneously the plate temperature is raised to T'_w , which is then maintained constant ,causing free convection current ,the concentration level raised to C'_w. At the plate is infinite extent all the physical variables are functions of z' and t'. The concentration level of foreign mass is assumed to be slow and hence Soret and Dufour effects are negligible. When the fluid moves due to rotation of the earth steadily with the bulk of fluid , the coriolis forces (linear in velocity) oppose the displacement of fluid element and right angle to both the axis of rotation & the local velocity vector and the centrifugal forces must acts on the fluid in the Eulerian specification of the flow field. If the system rotating very slowly the square and higher order terms in the centrifugal forces may be neglected.

Under usual Boussinesq's approximations the unsteady free convection flow can be shown to be governed by a system of coupled partial differential equations,

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial z'^2} \quad (2.1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \nu\frac{\partial^2 v'}{\partial z'^2} \quad (2.2)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K\frac{\partial^2 T'}{\partial z'^2} \quad (2.3)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial z'^2} \quad (2.4)$$

$t \le 0$: u' = 0, v' = 0, T' = T'∞ , C' = C_∞ , for all z'
t' > 0 : u' = U_0 , v' = 0, T' = T'_w , C' = C'_w at z' = 0 (2.5)
: u' = 0, v' = 0, T' = T'∞ , C' = C_∞ , as z'→ ∞

Here (u', v') are the velocity components along x' and y' axis respectively ,g the acceleration due to gravity , β the coefficient of volume expansion , β^* the coefficient of volume expansion with concentration ,T' the fluid temperature near the plate , C_∞ the mass concentration in the fluid far away from the plate, ν the kinematic viscosity , C_p the specific heat at constant pressure , K the thermal conductivity , D the mass diffusivity , U_0 is the impulsive velocity of the plate, ρ the density.

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{U_0} \;;\; v = \frac{v'}{U_0} \;;\; t = \frac{t' \; G \; U_0^2}{\nu} \;;\; z = \frac{z' \; U_0 \; \sqrt{G}}{\nu} \;;\; Pr = \frac{\mu \; Cp}{K} \;; \quad (2.6)$$

$$G = \frac{\nu \; \beta \; g \; (T'_w - T'_\infty)}{U_0^3} \;;\; \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)};\quad Rc = \frac{\Omega' \nu}{G\, U_0^2} = \frac{1}{G\, R_O \; R_e} \;;$$

$$Sc = \frac{\nu}{D} \;;\; C = \frac{(C' - C'_\infty)}{(C'_w - C_\infty)} \;;\; N = \frac{\beta^* \; (C'_w - C_\infty)}{\beta \; (T'_w - T'_\infty)}.$$

Where R_C is the Rotating parameter due to coriolis forces , R_o is the Rossby number and R_e is the Reynolds number ,θ is the non dimensional temperature, Sc is the Schmidt number , C is the non dimensional mass concentration , G is the Grashof number , N is known as the ratio of buoyancy forces .

Equations (2.1) to (2.5) reduce to the following non dimensional form :

$$\frac{\partial u}{\partial t} - 2 R_c v = \frac{\partial^2 u}{\partial z^2} + \theta + N C \quad (2.7)$$

$$\frac{\partial v}{\partial t} + 2 R_c u = \frac{\partial^2 v}{\partial z^2} \quad (2.8)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} \quad (2.9)$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial z^2} \quad (2.10)$$

The initial and boundary conditions are
u= 0, v= 0, $\theta = 0$, C=0 for all z, $t \le 0$
u= 1, v= 0 , $\theta = 1$, C=1 at z= 0 , t >0 (2.11)
u= 0, v= 0, $\theta = 0$, C=0 as $z \to \infty$,t >0

To facilitate the solution we can combine equations (2.7) and (2.8) by introducing q = u + i v, which then reduced to

$\frac{\partial q}{\partial t} + 2i\, R_c\, q = \frac{\partial^2 q}{\partial z^2} + \theta + N\, C$ (2.12)

The solutions of equations (2.9) to (2.12) satisfying initial and boundary conditions are derived by the usual Laplace- transform technique:

$\theta = \text{erfc}\,(\eta\sqrt{Pr})$

$C = \text{erfc}\,(\eta\sqrt{Sc})$

$q = \frac{1}{2}\{e^{-2\eta\sqrt{t\,b_1}}\ \text{erfc}(\eta - \sqrt{t\,b_1}) + e^{2\eta\sqrt{t\,b_1}}\ \text{erfc}(\eta + \sqrt{t\,b_1})\}$
$- \frac{1}{2b_1}\{e^{-2\eta\sqrt{tb_1}}\ \text{erfc}(\eta - \sqrt{t\,b_1}) + e^{2\eta\sqrt{t\,b_1}}\ \text{erfc}(\eta + \sqrt{t\,b_1})\}$
$+ \frac{1}{2b_1}e^{ta}\ \{e^{-2\eta\sqrt{t(a+b_1)}}\ \text{erfc}(\eta - \sqrt{t\,(a+b_1)}) + e^{2\eta\sqrt{t(a+b_1)}}\ \text{erfc}(\eta + \sqrt{t(a+b_1)})\} + \frac{1}{b_1}\text{erfc}(\eta\sqrt{Pr}) - \frac{1}{2b_1}e^{ta}\ \{e^{-2\eta\sqrt{t\,aPr}}\ \text{erfc}(\eta\sqrt{Pr} - \sqrt{t\,a}) + e^{2\eta\sqrt{Pr\,t\,a}}\ \text{erfc}(\eta\sqrt{Pr} + \sqrt{a\,t})\ - \frac{N}{2b_1}\{e^{-2\eta\sqrt{t\,b_1}}\ \text{erfc}(\eta - \sqrt{t\,b_1}) + e^{2\eta\sqrt{t\,b_1}}\ \text{erfc}(\eta + \sqrt{t\,b_1})\} + \frac{N}{2b_1}e^{ct}\ \{e^{-2\eta\sqrt{t(c+b_1)}}\ \text{erfc}(\eta - \sqrt{t\,(c+b_1)}) + e^{2\eta\sqrt{t(c+b_1)}}\ \text{erfc}(\eta + \sqrt{t(c+b_1)})\} + \frac{N}{b_1}\text{erfc}(\eta\sqrt{Sc}) - \frac{N}{2b_1}e^{tc}\ \{e^{-2\eta\sqrt{t\,cSc}}\ \text{erfc}(\eta\sqrt{Sc} - \sqrt{tc}) + e^{2\eta\sqrt{Sc\,t\,c}}\ \text{erfc}(\eta\sqrt{Sc} + \sqrt{c\,t})\}$

where

$\eta = \frac{z}{2\sqrt{t}}$; $b_1 = 2\,i\,R_c$; $a = \frac{b_1}{Pr-1}$; $c = \frac{b_1}{Sc-1}$

In order to gain physical insight, we have separated q into real and imaginary parts with the help of formulae given by Lahurikar [7] and computed numerical values of u and v. We assumed the fluid is to be viscous incompressible i.e. air (Pr =0 .71, when Match number Ma << 1) and water (Pr = 7.). The values of Sc, Schmidt number are given by Bejan [1] as follows

Pr	Species at low concentration 1atm,app 25^0C	Sc
0.71	Hydrgen,H_2 Water vapour,H_2O Ammonia,NH_3 Carbon dioxide,CO_2 Methanol,CH_3OH	0.22 0.60 0.78 0.94 0.99
7.0	Hydrogen,H_2 Oxygen,O_2 Carbon dioxide,CO_2 Chlorine , Cl Calcium chloride , $CaCl_2$	152 356 453 617 750

The velocity profiles are shown on Figs. 2-8. On Fig.2 these are shown for different values of Schmidt number, with other parameters held constant. It is observed that an increase in Sc leads to a decrease in the axial velocity of both air and water (Fig.5). An increase in Sc shows instability in the transverse velocity profiles of air and water. (Fig. 6). We observe from Fig.3 an increase in N, the ratio of chemical species diffusion to thermal diffusion, leads to an increase in the axial velocity because for N>0, the buoyancy forces assist the flow of both air and water. As N increases the magnitude of the transverse velocity increases of air but the transverse velocity of water is unstable. It is observed that the rotating parameter Rc, increases the axial velocity decreases of both air and water .Physically it is true that the coriolis forces opposes to the motion of the fluid. It is also observed that when Rc $<10^{-3}$ the flow due to the axial velocity is unstable because there is a point of inflection on the axial velocity profiles.(Figs.3,7,9) . Hence the flow turns to be turbulent. With same conclusion the flow due to the transverse velocity is unstable when Rc $<10^{-2}$.When $R_c < 1$, coriolis forces are dominated by inertia forces, hence the product of the non dimensional Robssby number and the Reynolds number is large. As time t increases the axial velocity and the magnitude of the transverse velocity increases of air but the axial velocity of water shows a point of inflection so that the flow of water may become unstable. It is obvious that as the Prandtl number Pr increases the flow gets slow down. From the velocity field, the skin friction is obtained in non- dimensional form as:

$2\sqrt{t}\ \tau = -\frac{dq}{d\eta}|\eta = 0$

$= -(\tau_x + \tau_y)$

Where $\tau = \frac{\tau'\sqrt{G}}{\rho\, U_0}$

$$- \frac{dq}{d\eta|_{\eta=0}} = \left\{ \frac{2}{\sqrt{\pi}} e^{-tb_1} + 2\sqrt{t\,b_1}\ \mathrm{erf}(\sqrt{t\,b_1}) \right\} \left(\frac{1}{b_1} - 1 \right) - \frac{2}{b_1\sqrt{\pi}} - e^{ta}\frac{1}{b_1} \left\{ \frac{2}{\sqrt{\pi}} e^{-t(a+b_1)} + 2\sqrt{t\,(b_1+a)}\ \mathrm{erf}(\sqrt{t(a+b_1)}) \right\} + e^{ta}\frac{1}{b_1} \left\{ \frac{2}{\sqrt{\pi}} e^{-ta} + 2\sqrt{tPra}\ \mathrm{erf}(\sqrt{at}) \right\} + \frac{N}{b_1} \left\{ \frac{2}{\sqrt{\pi}} e^{-tb_1} + 2\sqrt{t\,b_1}\ \mathrm{erf}(\sqrt{t\,b_1}) \right\} - \frac{N}{b_1}\frac{2}{\sqrt{\pi}} - e^{tc}\frac{N}{b_1} \left\{ \frac{2}{\sqrt{\pi}} e^{-t(c+b_1)} + 2\sqrt{t\,(b_1+c)}\ \mathrm{erf}(\sqrt{t(c+b_1)}) \right\} + e^{tc}\frac{N}{b_1} \left\{ \frac{2}{\sqrt{\pi}} e^{-tc} + 2\sqrt{t\,Sc\,c}\ \mathrm{erf}(\sqrt{t\,c}) \right\}$$

Where $b_1 = 2\,i\,Rc$; $a = \frac{b_1}{Pr-1}$; $c = \frac{b_1}{Sc-1}$

We have separated τ into real and imaginary parts as axial skin friction, τ_x and the transverse skin friction, τ_y using formulae given by Lahurikar [7] . We have computed values of τ_x and τ_y and these are listed in table 2.1. We observe from this tables an increase in Sc leads to increase in the axial skin friction for of water and air except for species CO_2 and methanol , also the transverse skin friction decreases of water and air except the species methanol, Ammonia . As buoyancy force parameter, N increases the transverse skin friction decreases of both air and water but the axial skin friction increases of air and water. This happens for positive or negative N. As coriolis force parameter Rc , increases the axial and the transverse skin friction increases of both water and air. When Rc $\rightarrow 0$ there is irregular change in the values of the skin friction which is due to the turbulent flow. Increase in time t, leads to decrease in the axial skin friction of both air and water but the transverse skin friction increases of air and water. Increase in Pr leads to increase in the axial skin friction.

III. CONCLUSIONS:

Rotating parameter Rc, increases the axial velocity decreases of both air and water. As R c < 10^{-3} the flow field becomes unstable and flow is converted to the turbulent flow for all Prandtl numbers (i.e. Pr = .71 for air when Ma<<1 and Pr = 7 for water). The flow of water may become unstable at large values of time t. Increase in Schmidt number leads to decrease in axial velocity for air and water. As coriolis force parameter Rc , increases the axial and the transverse skin friction increases of both water and air.

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Table 2.1 Values of τ

t	Pr	Rc	N	Sc	τ_x	τ_y
0.2	0.71	0.3	0.2	0.22	0.8427932	0.160239
				0.60	0.8525669	0.160239
				0.78	0.8545648	0.1602303
				0.94	0.8492011	0.1595165
				0.99	0.8177584	0.1606362
		0.5		0.60	0.8608529	0.2464438
		0.7			0.8704922	0.3312964
		0.3	0.4	0.60	0.8033836	0.1660973
			-0.2		0.9509334	0.1484161
			-0.4		1.000117	0.1425224
0.4	0.71	0.3	0.2	0.60	0.5739768	0.3310597
0.2	0.71	10^2	0.2	0.60	8.138048	8.15717
		10^0			0.8872596	0.4562419
		10^{-2}			0.8433148	3.24058E-02
		10^{-4}			0.8432706	2.75878E-02
		10^{-6}			0.8471296	-0.015625
0.2	0.1	0.3	0.2	0.60	0.7624003	0.1608727
0.2	7.0	0.3	0.2	152	1.014259	0.130624
				356	1.01641	0.1305345
				453	1.016885	0.1305152
				617	1.017423	0.1304939
				750	1.017725	0.1304818
		0.5	0.2	356	1.026855	0.2132367
		0.7			1.039155	0.2947768
		0.3	0.4	356	1.012027	0.1307041
			-0.2		1.025175	0.1301953
			-0.4		1.029558	0.1300257
0.4	7	0.3	0.2	356	0.9085204	0.2613055
0.2	7	10^2	0.2	356	8.144774	8.145532
		10^0			1.060703	0.414719
		10^{-2}			1.004907	8.787155E-03
		10^{-4}			1.004441	4.02832E-03
		10^{-6}			1.005107	-0.015625
0.2	100	0.3	0.2	356	1.096472	0.126489

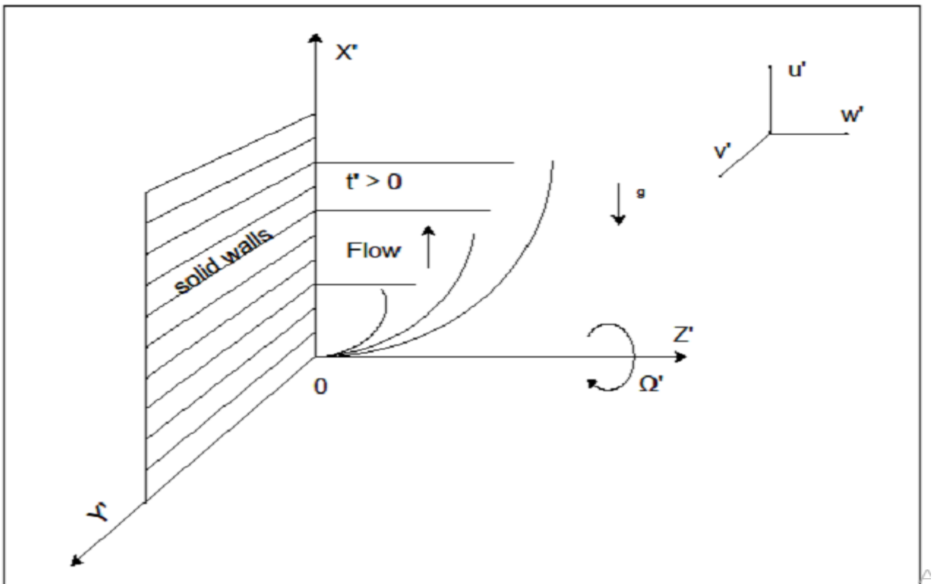


Fig.1 Geometry of the problem

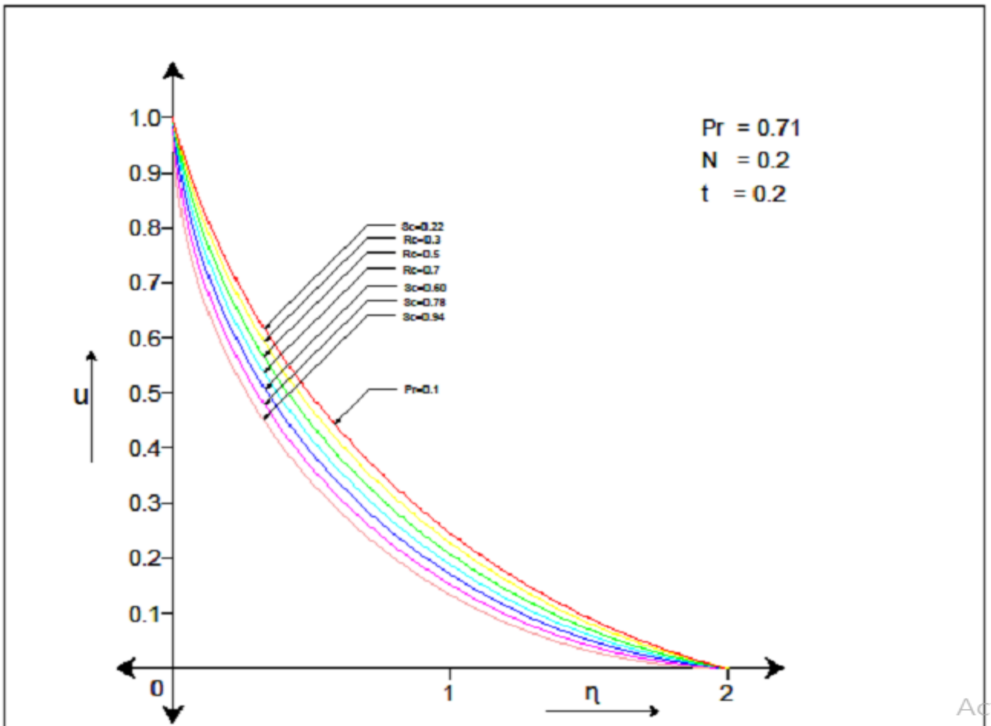


FIG.2 Axial Velocity Profile

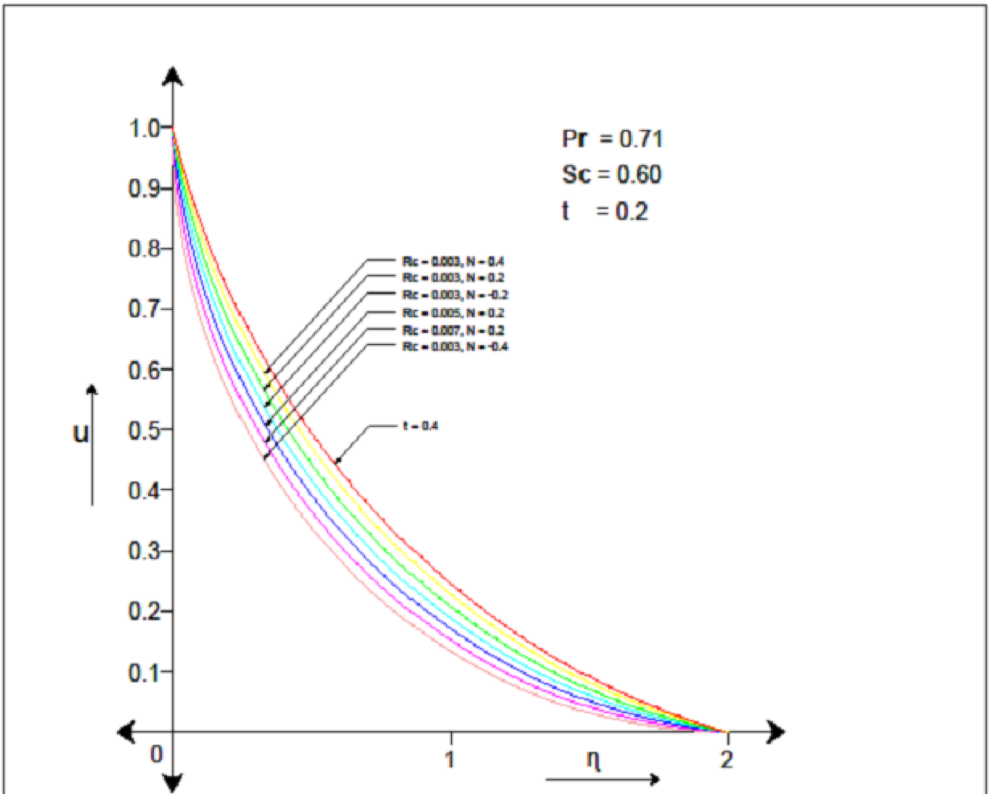


FIG.3 Axial Velocity Profile

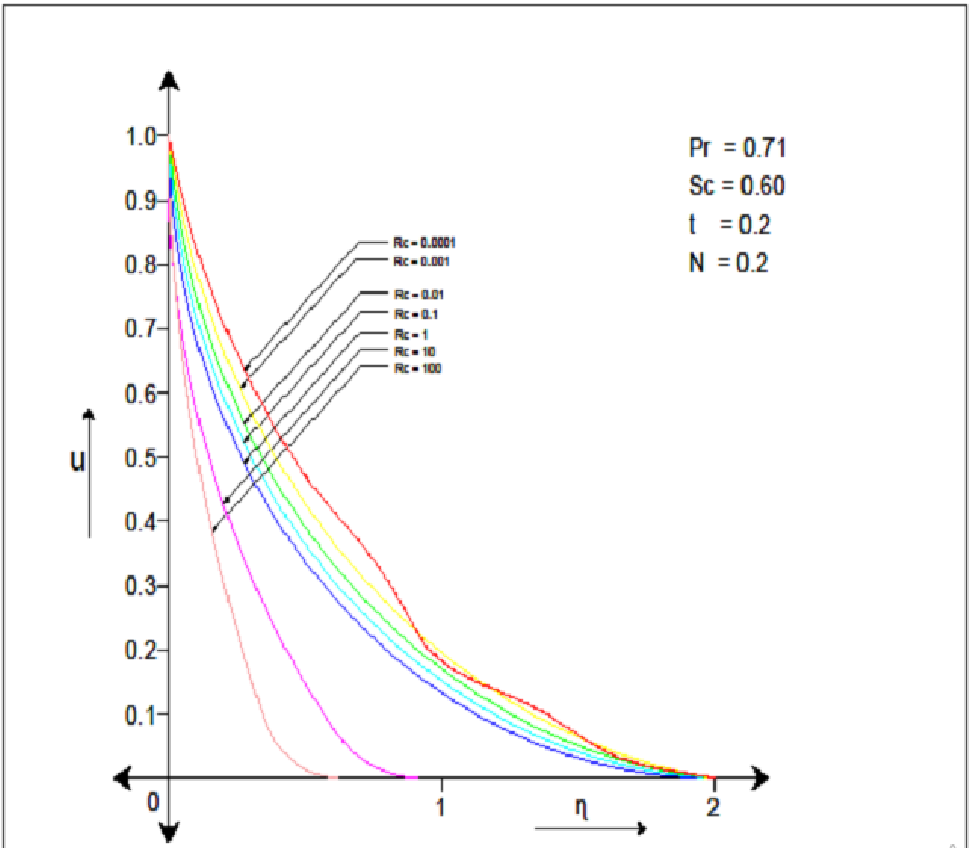


FIG.4 Axial Velocity Profile

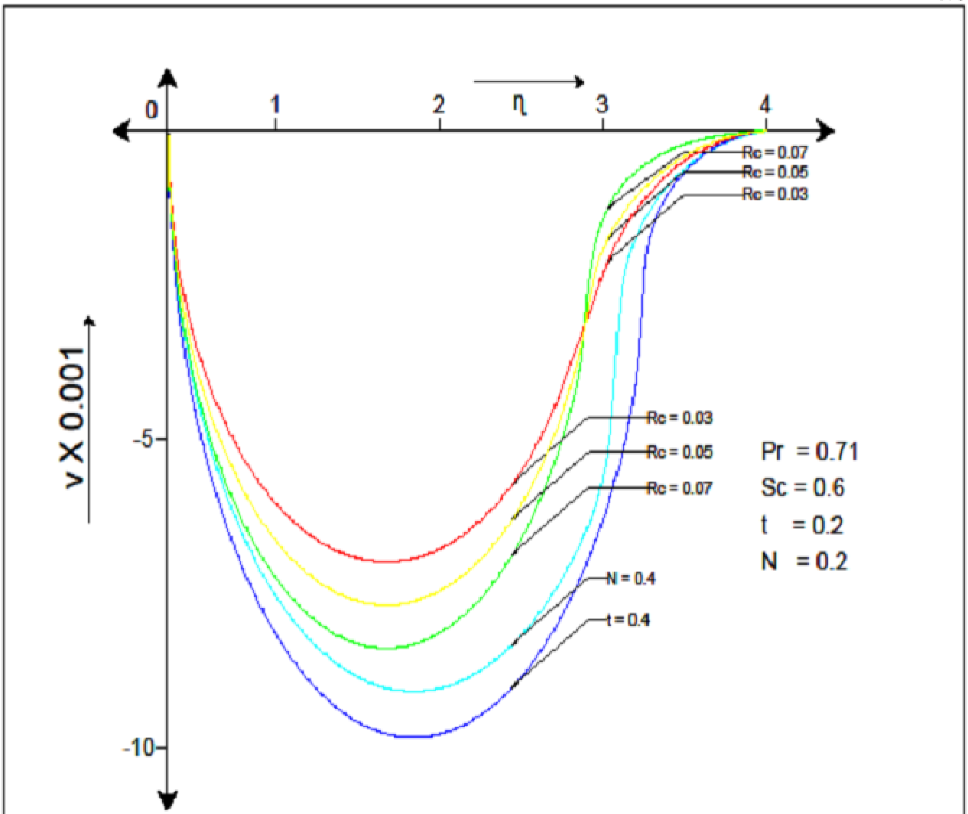


FIG.5 Transverse Velocity Profile

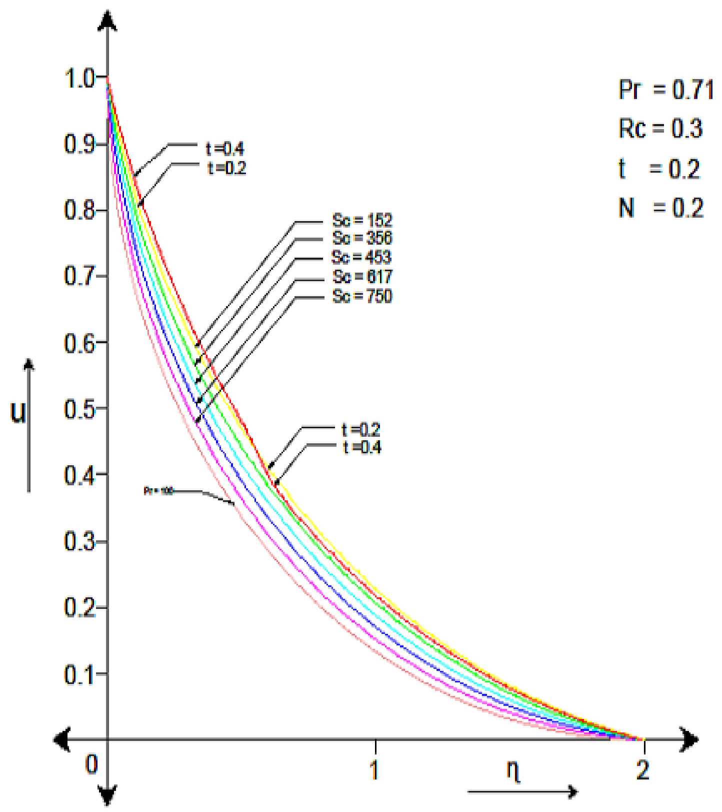
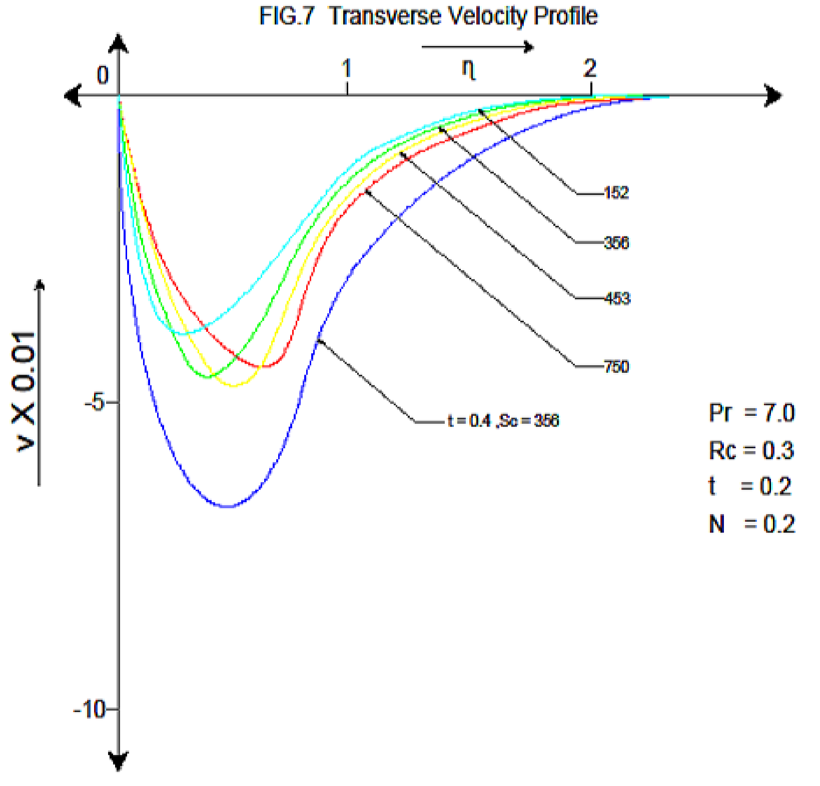


FIG.6 Axial Velocity Profile

FIG.7 Transverse Velocity Profile



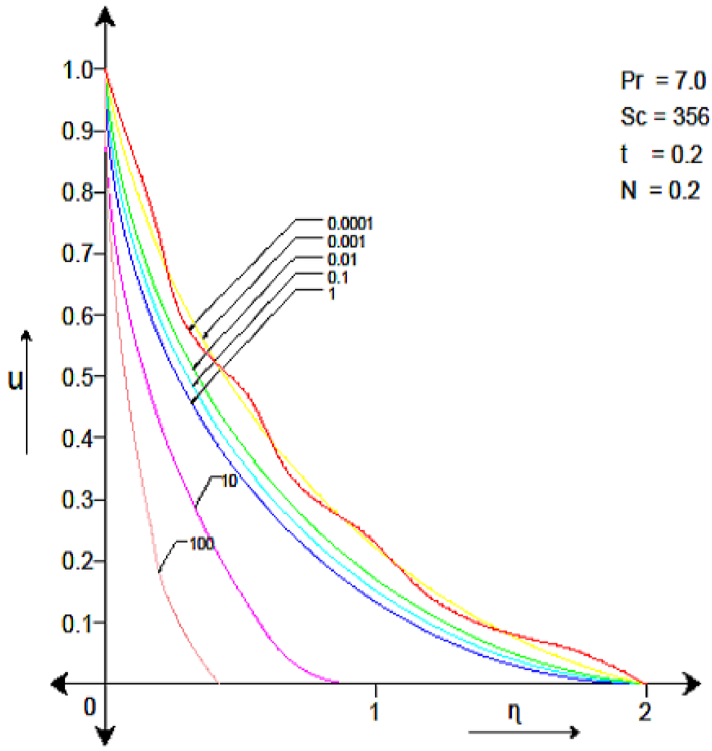


FIG.8 Axial Velocity Profile

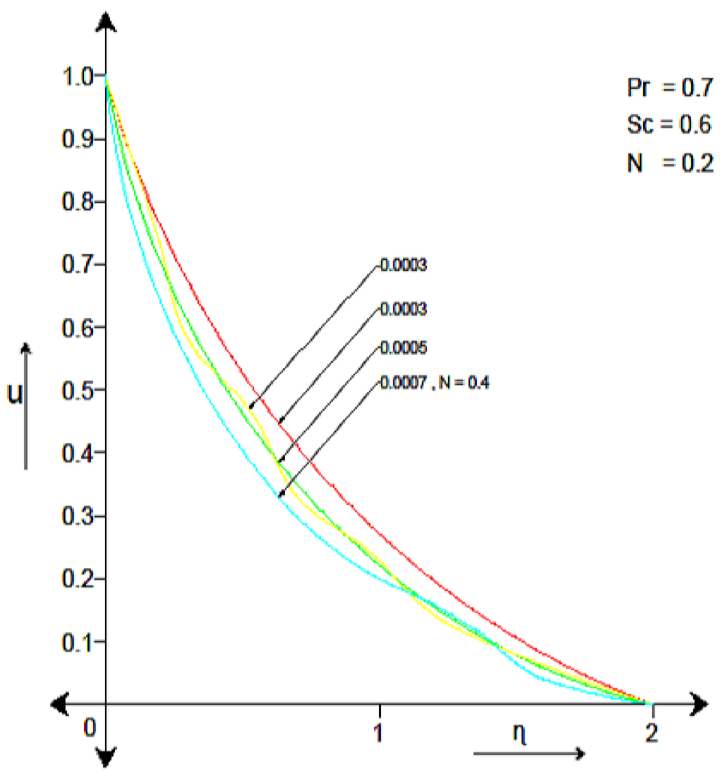


FIG.9 Axial Velocity Profile