Design of Non-Uniform Cosine Modulated Filter Banks Using Windows

K. Sumanth¹, S Nagakishore Bhavanam², B. Bhaskara Rao³
¹PG Student, ²³Assistant Professor, Department Of ECE, University College Of Engineering And Technology
Acharya Nagarjuna University, Guntur, Andhra Pradesh, India

ABSTRACT
A technique for the design of non-uniform cosine modulated filter bank (NUCMFB) is presented in this paper. Non-uniform cosine modulated filter banks are derived by merging the adjacent filters of uniform cosine modulated filter banks. So, the design process of NUCFB reduces to the design of a single prototype filter. Hence the design of uniform cosine modulated filter bank (CMFB) has gained prime importance. The prototype filter can be designed with the aid of different adjustable window functions such as tukey window, dolph-cheneybyshev window and also Interpolated Finite Impulse Response (IFIR) design technique can be used. In this method either cut off frequencies or passband edge frequency is varied in order to adjust filter coefficients so that reconstruction error could be optimized/minimized to zero. Attributes such as peak reconstruction error (PRE), aliasing distortion (AD),computational time (CPU) and number of iterations (NOI) can be evaluated and performance can be measured. Finally the NUCFB designed by the proposed method can be exploited for the sub band coding of ECG signal.

Keywords: CMFB, NUCMFB, IFIR, PRE, AD, CPU, NOI.

I. INTRODUCTION
The research in multirate filter banks have gained importance because multirate filter banks find wide applications in many areas of digital signal processing such as sub band coding, transmultiplexer, image, video, and audio compression, adaptive signal processing and bio signal processing. Filter banks decompose a digital signal into different frequency bands. On the basis of time frequency resolution, filter banks can be classified into two categories i.e., uniform and non-uniform filter banks. Uniform filter bank provides fixed and uniform time decomposition [38]. However in some applications like audio analysis, broad band array signal processing non-uniform and variable time frequency resolution may lead to better performance and reduced arithmetic complexity, which is provided by non-uniform filter banks [17,40,41]. Over the years, a number of design methods have been proposed by different authors. In 1993, vaidyamathan carried out the research work on QMF banks. Later in 2002 Jovanovic-Dolesek extended the work for the design of multichannel uniform banks. Among the different classes of multi-channel uniform filter banks, cosine modulated filter banks are the most frequently used filter banks due to their simpler design, where analysis and synthesis filter banks are derived by cosine modulation of the low-pass prototype filter. Thus the design of whole filter bank reduces to that of a single low pass prototype filter. To better exploit the signal characteristics

Non-uniform frequency partitioning may be employed in applications such as antenna systems, biomedical signal processing, subband adaptive filtering digital audio industry and communication [16,27,33,39,43]. In ECG signal processing especially for heart beat detection, filter banks with fast switching resolution, adjustable stopband attenuation and non-uniform frequency partitioning is required [2,3,18]. The non-uniform filter banks have less quantization error and low computational complexity.

The research has also been done towards the theory and design of the non-uniform filter banks [22]. The non filter banks are most commonly designed using nonlinear optimization with considerable number of parameters where a direct structure is adopted. In indirect method of design certain channels of uniform filter bank are merged giving rise to near-PR (perfect reconstruction) recombination. In [6] a new class of non-uniform filter bank based on cosine-modulated FBs (CMFBs) have been proposed. In [31] authors have proposed a time domain approach for designing non-uniform filter banks (NUFB). In designing CMNUFBs with near perfect reconstruction Several methods were present in the literature. In most of the methods convergence to optimal solutions depends on initial guess and these methods are unable to find a global optimum solution. These approaches are not suitable for larger filter due to high degree of nonlinearity. In [42] iterative methods have been proposed to overcome the above drawbacks. It has
been further modified in [32]. Though the above approaches perform better in terms of reconstruction error they give better solution in large number of iterations. Hence an algorithm that can minimize the reconstruction error, number of iterations and computational time has been proposed.

This paper proposes a technique for the design of non uniform cosine modulated filter banks. The design of NUFB depends on the uniform CMFB. Hence primarily the design of uniform CMFB has been discussed and then the discussion has been extended to the design of non-uniform CMFB.

II. FILTER BANKS

M- channel maximally decimated filter banks are widely used in different applications. Filter banks decompose a digital signal into different frequency bands. The basic requirement is the frequency selectivity of the individual filters which is characterized by small passband ripple, narrow transition bandwidth and high stop band attenuation.

2.1 Uniform cosine Modulated Filter Bank:

In uniform cosine modulated filter bank all the analysis and synthesis section are obtained by cosine modulation of single linear phase prototype low pass filter which normally has linear phase and a finite length impulse response.

A maximally decimated filter bank structure is shown in figure 1

2.2 Non-Uniform Cosine Modulated Filter Banks:

The non uniform cosine modulated filter bank. The adjacent filters of uniform cosine modulated filter bank are merged to obtain the non uniform filter bank [30], let the analysis filter of the non uniform filter bank obtained after merging l_d(≥1) adjacent filters of F_k(Z) i.e., from k=n through k=n+l_d-1 of uniform filter bank be

\[ H_k(Z) = \frac{1}{M} \sum_{k=0}^{M-1} G_k(Z) H_k(Z) \]

CM \( H_k(Z) = \sum_{k=n}^{n+l_d-1} H_k(Z) \)

let \( \tilde{F}_k(Z) \) for i=0,1, . . . , M-1 be the synthesis filters of NUFB, then

\[ \tilde{F}_k(Z) = 1/ \sum_{k=0}^{n+l_d-1} F_k(Z) \]

The above \( \tilde{H}_k(Z) \), \( \tilde{F}_k(Z) \) represents the analysis and synthesis filters of the M-derived non CMFB. The minimum value of \( n \) for i=0 is zero and its maximum value is \( n_M = M \) also.
\( l_0 + l_1 + l_2 + \ldots + l_{M-1} = M. \) The decimation factor for individual channel of the derived non uniform CMFB is estimated \( M_i = M/l_i \) for \( i = 0,1,2, \ldots ,M-1. \) where \( M_i \) gives the decimation ratio for the \( i^{th} \) channel. The resultant non uniform CMFB is shown as

![Figure 2: \( M^2 \) band derived nonuniform cosine modulated filter bank](image)

As the design of NUCMFB is based upon the uniform CMFB, the design of uniform CMFB has gained primary importance. And the NUCMFB design reduces to the design of uniform CMFB. Therefore indirectly the NUCMFB reduces to the design of a single prototype filter.

In multichannel CMFB, Perfect reconstruction (PR) is possible if the following equations are satisfied

\[
[H_0(e^{j\omega}))^2 + (H_0(e^{j\omega[j=1])^2}] = 1
\]

for \( 0 \leq \omega \leq \pi / m; \) At \( \omega = \pi / [2M] \) then above equation reduces to

\[
[H_0(e^{j\omega[j=1])^2 + (H_0(e^{j\omega[j=1])^2}] = 1
\]

\[ H_0(0) = 0.707. \]

### III. PROTOTYPE FILTER DESIGN TECHNIQUES

As the prototype filter design has gained primary importance, it is to be designed first. There are various prototype design techniques available, and are classified into five categories.

#### 3.1 Non-linear optimization techniques:

There are two types under this category. They are unconstrained optimization and constrained optimization.

i. **Unconstrained optimization**: The unconstrained optimization of NPR CMFBs are given by the objective functions. The objective function reduces the amplitude distortion and improves the stopband attenuation.

ii. **Constrained optimization**: In this a set of quadratic constraints in the impulse response coefficients of the prototype filter relates the \( 2M^{th} \) band condition and the stopband energy is given as objective function.

#### 3.2 Spectral Factorization Approach:

Spectral factorization is used as an alternative to the non-linear optimization in the NPR CMFB design. The prototype filter is obtained as a spectral factor of \( 2M^{th} \) bandfilter. The prototype filter is obtained as the spectral factor using spectral factorization algorithms like the inverse LPC based spectral factorization technique. To design the filter bank with high stopband attenuation using spectral factorization algorithms is complex.

#### 3.3 Linear search techniques:

i. **Park Mccellan Algorithm**: In this algorithm the filter length, relative error weighting and stopband edge are fixed, then the passband edge is varied in small steps to reduce the objective function. For each iteration the Parks M.clellan algorithm is used and the filter is designed.

ii. **Window method**: The prototype filter design using the Kaiser window is made by Lin and vaidyanathan. Different windows such as Blackman, saramaki, Ultra spherical and hybrid windows are used to design the prototype filter. And spline functions are also used to design the prototype filter.

#### 3.4 Interpolated Finite Impulse Response(IFIR) Approach:

when the number of channels of a filter bank is very high then this technique is applicable. IFIR technique is used to obtain narrow bandwidth FIR filter with sharp transition width and less complexity. IFIR filter consists of a model filter and an image suppressor filter.

**Frequency Response Masking (FRM)**

Approach: FRM FIR filter consists of designing three filters. They are the bandedge shaping filter, masking filter and complementary masking filter. This method uses the concept that interpolation by a factor \( L \) reduces the transition width by factor \( L. \)
IV. PROPOSED PROTOTYPE FILTER DESIGN TECHNIQUE

In this paper the prototype filter is designed by using different window functions and constrained equiripple FIR approach. Three parameters such as order of the filter (N), window shape parameter and $W_c$ are used for the filter design using window technique.

The desired stop band attenuation ($A_s$) is obtained if the order of the prototype filter (N) is computed by

$$N = \frac{(\Delta f - 7.5)}{14.85 \Delta f}$$

Where $\Delta f$ is the transition band given by

$$\Delta f = \frac{(W_s - W_p)}{2}$$

$W_p$ is the pass band edge frequency and $W_s$ is the stop band edge frequency

The cut off frequency is estimated using the given equation

$$W_c = \frac{1}{2}(W_p + W_s)$$

The stop band attenuation is used to estimate the window shape parameter. In this paper adjustable windows such as tukey, dolph chebyshev are used.

The expressions for the windows used are given as,

**Tukey Window:** The Tukey window also known as the tapered cosine window, can be regarded as a cosine lobe of width $\alpha N/2$ that is convolved with a rectangular window of width $(1-\alpha/2) N$.

$$W(n) = \left\{\begin{array}{ll}
\frac{1}{2} \left[ 1 + \cos \left\{ \pi \left( \frac{2n}{\alpha(N-1)} - 1 \right) \right\} \right] & \text{if } 0 \leq n < \frac{N}{2} \\
\frac{1}{2} \left[ 1 + \cos \left\{ \pi \left( \frac{2n}{\alpha(N-1)} - 2 \right) \right\} + 1 \right] & \text{if } \frac{N}{2} \leq n < N
\end{array}\right.$$  

**Dolph-chebyshev Window:** The zero-phase Dolph-chebyshev window function $w_o(n)$ is usually defined in terms of its real-valued discrete Fourier transform, $W_o(k)$:

$$W_o(k) = \frac{\cos \left[ \beta \cos^{-1} \left( \frac{1}{2} \right) \right]}{\cosh \left[ \frac{1}{2} \cosh^{-1}(\frac{1}{2}) \right]}$$

$$\beta = \cosh \left[ \frac{1}{2} \cosh^{-1}(\frac{1}{2}) \right]$$

$\alpha$ is the window shape parameter, which controls the ripple ratio.

The sequential steps required for designing the prototype filter for NUCMFB using window technique are:

- **step 1:** Specify the design specifications such as $A_s$.
- **step 2:** Assume initial values of error(\(\varepsilon\)) counter (count) and step size ($\Delta$).
- **step 3:** Compute $w_c$ and $N$ by using respective equations and from the given design specifications.
- **step 4:** Evaluate the prototype filter coefficients using different window functions with $N$ and $w_c$. Compute the magnitude response of designed filter (MRD) at $M$ and error or deviation from ideal condition

$$\text{Error} = 0.707 - \text{MRD}$$

- **step 5:** Check whether ‘error’ is comparable to error(\(\varepsilon\)).

If yes, then derive other analysis and synthesis filters of uniform CM filter bank using this prototype filter, and finally derive their non-uniform counterparts.

If no, follow the next step.
- **step 6:** Check whether $\text{MRD} > 0.707$. If yes, decrease the cut-off frequency by step size and follow the next step. If no, increase $W_c$ by step size and also follow next step.

- **step 7:** Increase counter by one and $\Delta = \frac{\Delta}{2}$.

- **step 8:** Re-evaluate the prototype filter coefficients using window functions with the same $N$ but with a new cut-off frequency. Also, compute the error or deviation from ideal condition. Then go to step 5.

V. PROTOTYPE FILTER DESIGN USING IFIR APPROACH

IFIR technique is used to obtain narrow bandwidth FIR filter with sharp transition width and less complexity. The prototype filter design using this method can be applicable to these CMFB’s when the number of channels is every high i.e., with small passband and small transition band. IFIR filter consists of a model filter and an image suppressor filter.

![Interpolated FIR filter](image)

For the given cut-off frequencies $\Omega_p$ and $\Omega_s$, the steps for synthesizing then filter $H(Z)$ are follows.
Step 1: Determine a suitable interpolation factor \( L \) so that first image frequency response can be separated from the base band frequency using a filter \( I(Z) \).

Step 2: Design the prototype \( G(Z) \) with the pass band cut off frequencies \( \Omega_{p,G} = L \cdot \Omega_p \) and the stop band cut off frequencies \( \Omega_{s,G} = L \cdot \Omega_s \).

Step 3: Choose passband cut off frequency \( \Omega_{p,1} \) of the anti-imaging filter \( I(Z) \) as \( \Omega_p \).

The stopband cut off frequencies \( \Omega_{s,1} \) of \( I(Z) \) must be \( \Omega_{s,1} = \frac{2\pi}{L} - \Omega_s \).

The required passband ripple of the filter \( H(Z) \) must be distributed among the passband ripple of the filter \( G(Z) \) and \( I(Z) \) and is given by

\[
(1+ \delta_{p,G} \cdot (1+ \delta_{s,F})) = 1 + \delta_p
\]

For small value of the ripple \( \delta_p \) can use the approximation

\[
(\delta_{p,G}) + (\delta_{p,I}) \approx \delta_p
\]

If the stopband ripple \( \delta_s \) of \( H(Z) \) is known the stopband ripple of the filter \( G(Z) \) and \( I(Z) \) are given by

\[
\delta_{s,G} \cdot (1 + \delta_{p,G}) = \delta_s, \quad \Omega s \leq \Omega \leq \Omega_{s,1}
\]

and

\[
\delta_{s,I} \cdot (1 + \delta_{p,I}) = \delta_s, \quad \Omega_{s,F} \leq \Omega \leq \pi
\]

provided \( \delta_p \) is small. And also it is possible to make further approximation.

\[
\delta_{s,G} = \delta_{s,I} = \delta_s
\]

VI. ATTRIBUTES EVALUATED:

The following attributes can be evaluated for describing the performance of the designed filter banks.

6.1.1 Peak reconstruction error (PRE):

The peak reconstruction can be evaluated by using the expression given by:

\[
\text{PRE} = \max \left( \sum_{n=-\infty}^{\infty} |h(n) \cdot e^{-j2\pi \omega_n/2} \right) - \min \left( \sum_{n=-\infty}^{\infty} |h(n) \cdot e^{-j2\pi \omega_n/2} \right)
\]

6.1.2 Number of iterations (NOI):

It gives the specified number of times a set of instructions can be repeated until a specific result is achieved.

6.1.3 Computational time (CPU time):

It gives the amount of time to execute instructions and is measured in seconds.

6.1.4 Aliasing Distortion (AD):

It is an effect that causes different signals to become indistinguishable when sampled. It also refers to the distortion that results when the signal reconstructed from samples is different from the continuous signal.

VII. CONCLUSION

The NUCMFB design that depends on the design of prototype filter is proposed in this paper. The prototype filter design using different adjustable windows and also IFIR approach is proposed in this paper. The application of the NUCMFB thus designed can be extended to the speech signals and ECG signals.

REFERENCES


[22] M Harirha, S Nagakishore Bhavanam and Vasujadevi M “A PIC compatible RISC CPU core implementation for FPGA based Configurable SOC platform for Embedded Applications”. International Journal on Electrical and Power Engineering (IJEPE), Vol.5, No.2 Aug 2011 ISSN 2158-7574 (Online)ISSN 2158-7566 (Print) organized by USA, pp.11-15.


[27] S Nagakishore Bhavanam, Vasujadevi M and M. Madana Gopal “Implementation of FPGA based LOW - COST Logic Signal Analyzer (LSA)”. International Journal on Recent Trends in Engineering and Technology (IJRTET), Vol.4, No.4, ISSN 2158-5563 (Online); ISSN 2158-5555 (Print), November 2010, published jointly by USA, ACEEE, AMAE, ACEE, Florida, pp.170-174.


[34] S. Nagakishore Bhavanam and Vasujadevi M “Video Improvement Technique for Vibrating Video signals in Surveillance Applications”. International Journal on Recent Trends in Engineering and Technology (IJRTE), Vol.5, No.2, ISSN 2158-5563 (Online); ISSN 2158-5555 (Print), March 2011, Published jointly by USA, ACEEE, AMAE, ACEE, Florida. pp.116-121.


