

## A Comparative Study of Image Denoising Methods Using Wavelet Thresholding Techniques

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### ABSTRACT

This paper compares image denoising methods using threshold that exploits the dependency of the sub-band and wavelet transformation to give estimates of signal variance by using local coefficients. There are many methods and each method may remove a different coefficient which leads to poor image quality. This paper compares Visu shrink, Bayes shrink, SURE shrink and minmax using Mean Squared Error and Peak Signal to Noise Ratio at various noise.

**Index Terms:** Wavelet transform, Bayes Shrink, Visu Shrink, Sure Shrink, Minimax, MSE, PSNR.

### I. INTRODUCTION

A digital image is usually corrupted by noise signals during its acquisition process, because of this introduction of noises in images some of the data of image is lost which results in poor image quality. In the recent years, there is increase in the research of image denoising techniques using wavelet thresholding [1, 2], this process requires finding a requisite value for thresholding as the removal of noise from image requires a specific amount of thresholding to get clean image with losing less data as possible. The VisuShrink, SureShrink, and BayesShrink methods are commonly used threshold selection methods. A wavelet is like a form of oscillation having an amplitude with a mean point at zero, the value of this wavelet oscillated back and forth this zero point. The wavelets are created purposefully with certain properties which are required in processing of signals. These are modeled in a mathematical environment, these can be used using various functions, and in image processing the wavelets are combined using convolution technique. The denoising of data is an important part of data processing, the thresholding is a powerful tool that have been researched in a very wide area wherever

work on large data is required such as communication systems. Wavelet shrinkage technique is based on finding a certain value for which the smaller coefficients than this threshold are removed or rejected with losing minimum useful data as possible while maintaining the visual quality of image.

The denoising using wavelet threshold consists of three steps:

- 1) Applying Wavelet transform [3, 4] to image data and calculate the wavelet coefficients.
- 2) Finding the optimum value for threshold and applying hard or soft thresholding method [2].
- 3) Calculating the denoised signal and reconstructing the image.

In this paper four of the most commonly used thresholding techniques i.e. visu shrink, bayes shrink, SURE shrink and minimax estimation are compared. A comparative analysis of these methods are carried which is interpreting the performance using graphical and visual demonstration

### II. METHODOLOGY

The image denoising method involves the following steps depicted in the figure 1.

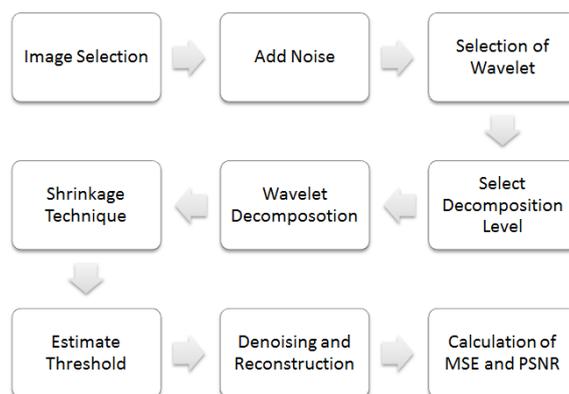


Figure:1 Image denoising using wavelet thresholding.

STEP 1: The image is selected having some detailed information so that the changes occurred during the denoising process it is visually detectable to judge the effectiveness of algorithm.

STEP 2: The selected image is added with a noise, the noises used in comparison are Gaussian noise, salt & pepper noise.

STEP 3: The wavelet model is considered such as haar or daubechies.

STEP 4: Decomposition level [5, 6] is selected according to the thresholding technique to be used. Such as for visu shrink level is taken not more than 3 whereas for bayes and SURE estimation level 1 to 5 can be used.

STEP 5: The decomposition of noisy image takes place with specified levels.

STEP 6: The wavelet shrinkage technique [7] is applied on our input data.

STEP 7: The shrinkage method gives an estimate of the thresholding value

STEP 8: The noisy image is denoised using the estimated threshold value and then reconstructed into image.

### III. THRESHOLD METHODS FOR WAVELET BASED DENOISING

The performance of a denoising algorithm is dependent on its estimated value of threshold. Good denoising of data requires an optimum amount of threshold value. The nonlinear threshold functions are of two types i.e. fixed threshold and adaptive threshold. Fixed threshold methods use same threshold value for hard/soft threshold for the complete set of wavelet coefficients. The ranges of the magnitudes of wavelet sub-bands are not similar. Hence, the fixed threshold method is likely to over-smooth the image details, failing to preserve image detailed data. On the other hand sub-band and scale adaptive threshold methods were proposed to handle the problem.

#### A. VisuShrink

Visushrink method employs universal thresholding method as proposed by Dohono and Johnstone [1, 2], The threshold for visushrink is calculated by equation (1):

$$T_v = \sigma \sqrt{2 \log K} \quad (1)$$

Where  $\sigma$  is noise variance and  $K$  is number of pixels of the image. The maximum value of any  $K$  will be in the range  $N(0, 2\sigma)$ . This will always tend to be less than the universal threshold having high probability, and the probability approaches 1 as  $K$  increases. Therefore when probability is high enough, the pure noise signal can be taken as approximately zero in value.

The very nature of universal threshold when derived for high probability requires the estimate to be as

smooth as the signal. Hence this method gives an overly smooth estimate. This leads to poor adaptation of the signal and its discontinuities as many useful coefficients are killed in the process.

#### B. SURE Shrink

In this method, the entire band is divided into smaller sub-bands according to Stein's unbiased estimator for risk [8, 9]. The sure is a method to estimate the loss in an unbiased manner. Equations (2-4)

$$SURE(t;X) = d - 2 \sum_{i=1}^d \min(|X_i|, t) + \sum_{i=1}^d \min(|X_i|, t)^2 \quad (2)$$

Where  $\{X_i : i=1, \dots, d\}$ ,  $\hat{X}$  is the estimation for soft threshold

$$\hat{X}_i = \eta_i(X_i) \quad (3)$$

we apply Stein's result to get an unbiased estimate of the risk

we could find the threshold  $t_s$  that minimizes  $SURE(t;x)$ ,

$$T_s = \arg \min SURE(t;X) \quad (4)$$

#### C. BayesShrink

Bayes estimation [10, 11, 12] employs different threshold for each sub-band and the noise distribution is taken to be Gaussian by default. The relationship between wavelet coefficients of the degraded image, the Gaussian noise and uncorrupted image coefficients is given by equation (5).

$$Y = V + X. \quad (5)$$

Where Gaussian distribution is  $N(0, \sigma^2)$  ( $Y$ ,  $V$  and  $X$  respectively). These mutually independent factors satisfy the equation (6).

$$\sigma_y^2 = \sigma_x^2 + \sigma_v^2 \quad (6)$$

Where variance is given in equation (7)

$$\hat{\sigma}_y^2 = \frac{1}{J} \sum_{j=1}^J W_j^2 \quad (7)$$

Where  $W_j$  are the wavelet coefficients in each scale 'j' and 'J' is the total number of wavelet coefficients. The threshold value using Bayes shrink thresholding is given by equation (8), where  $\hat{\sigma}_x$  is calculated using equation (9).

$$T_b = \begin{cases} \hat{\sigma}_v^2 & \text{if } \hat{\sigma}_v^2 < \hat{\sigma}_x^2 \\ \hat{\sigma}_x & \text{otherwise} \end{cases} \quad (8)$$

$$\hat{\sigma}_x = \sqrt{\max(\hat{\sigma}_y^2 - \hat{\sigma}_v^2, 0)} \quad (9)$$

The equation (8) can be used to improve corrupted images by Gaussian noise. But its sensitivity is low in the case of noise around the edges. It is completely capable in flat regions of the image.

**D. Minimax Shrink**

The minimax principle [13, 14] is a borrowed technique from the design estimators used in statistics. This technique realizes the minimum from a given set of functions of the maximum mean square error and denoised signal is assimilated to the estimator function of the unknown regression functions. It gives optimal performance by minimizing the constant terms in the upper bound of the risks involved in the estimation. The technique uses DLP (Diagonal Linear Projection) and DLS (Diagonal Linear Shrink), these are calculated using equations (10, 11).

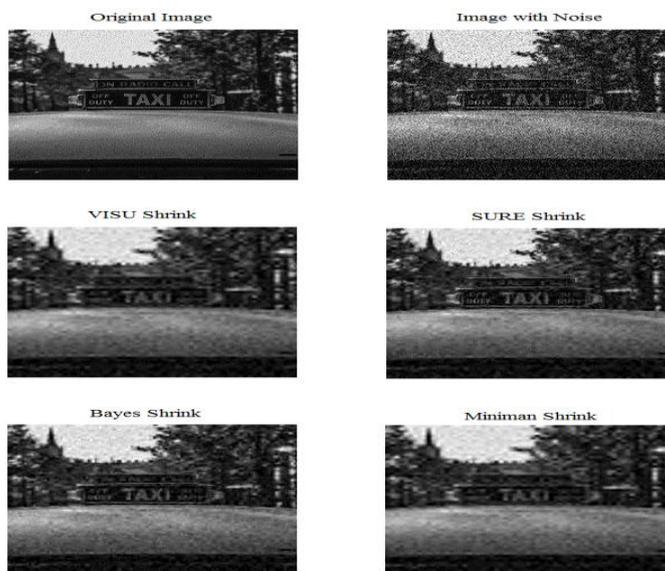
$$Risk_{DLP}(k) = \min(d^2, 1) \quad (10)$$

$$Risk_{DLS}(k) = \frac{d^2}{1 + d^2} \quad (11)$$

The DLP informs whether to accept or discard any wavelet coefficient whereas the DLS estimates the shrinking to be applied to each wavelet coefficient.

**IV. EXPERIMENTAL RESULTS**

The study was carried out using MATLAB R2015a 8.5.0.197613. We analyzed the performance of methods, we take three test images: Tree, Taxi, and Plant each of size 256×256 pixels. The images were contaminated with Gaussian noise with noise variance: 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8. When the MSE is lower the denoised image is said to be very much closer to the original image. We have compared the results of the shrinking methods of the SureShrink, VisuShrink, BayesShrink, and Minimax methods in terms of MSE and PSNR. The resulting images obtained by applying different denoising algorithms are presented in figure 2. This figure shows the visual representation of denoised images for comparison of performance of the algorithms. The MSE and PSNR for these images at different noise levels are presented in table 1 for MSE and table 2 for PSNR. Figure 3 shows graphical representation of MSE with noise variance on x-axis and MSE on y-axis, similarly figure 4 shows graphical representation of PSNR at different noise levels with noise variance on x-axis and PSNR value on y-axis.



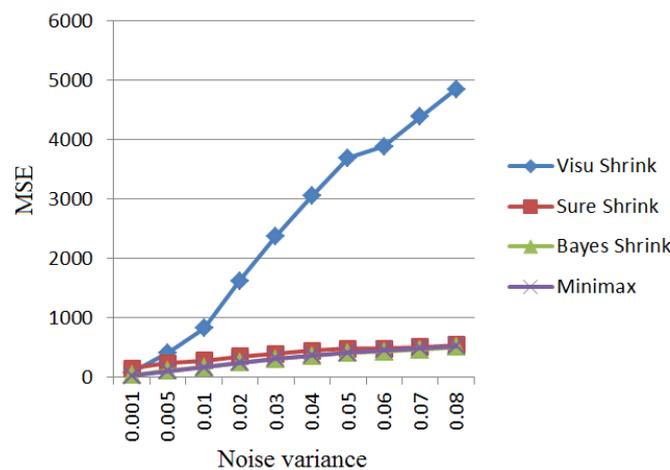
**Figure:2** Denoised images (Gaussian noise sigma=30, wave name db4)

**Table:1** Comparison Of Mse At Different Noise Variance

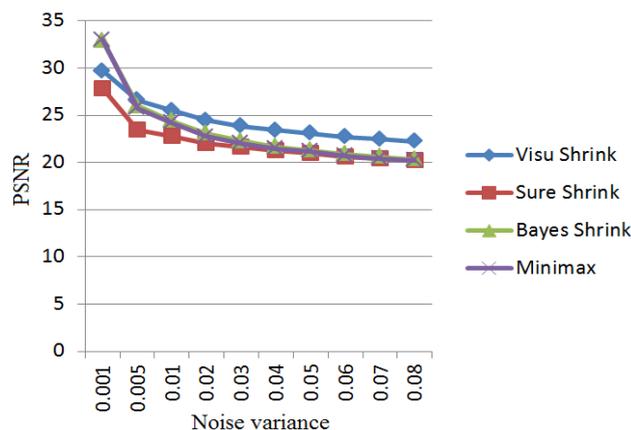
| Noise Variance | Visu Shrink | Sure Shrink | Bayes Shrink   | Minimax        |
|----------------|-------------|-------------|----------------|----------------|
| 0.001          | 83.8162     | 153.303     | 36.1155        | <b>31.8566</b> |
| 0.005          | 419.134     | 239.383     | 115.354        | <b>111.404</b> |
| 0.01           | 831.363     | 282.005     | 173.997        | <b>166.963</b> |
| 0.02           | 1625.45     | 350.899     | <b>251.083</b> | 252.053        |
| 0.03           | 2374.27     | 399.682     | <b>313.661</b> | 314.858        |
| 0.04           | 3053.83     | 447.208     | <b>365.580</b> | 375.657        |
| 0.05           | 3694.41     | 482.801     | <b>417.610</b> | 418.882        |
| 0.06           | 3883.48     | 485.376     | <b>435.033</b> | 457.730        |
| 0.07           | 4383.51     | 513.615     | <b>472.244</b> | 490.614        |
| 0.08           | 4851.07     | 546.812     | <b>511.231</b> | 525.694        |

**Table:2** comparison Of Psnr At Different Noise Variance

| Noise Variance | Visu Shrink | Sure Shrink | Bayes Shrink | Minimax  |
|----------------|-------------|-------------|--------------|----------|
| .001           | 29.66449    | 27.85235    | 32.85191     | 32.92002 |
| 0.005          | 26.59635    | 23.4753     | 25.9615      | 25.75942 |
| 0.01           | 25.48252    | 22.78507    | 24.38173     | 24.2062  |
| 0.02           | 24.43786    | 22.04486    | 23.01827     | 22.72293 |
| 0.03           | 23.84963    | 21.59998    | 22.25721     | 21.99238 |
| 0.04           | 23.39152    | 21.21388    | 21.66141     | 21.40355 |
| 0.05           | 23.05656    | 20.9497     | 21.28858     | 21.11765 |
| 0.06           | 22.68082    | 20.60141    | 20.83883     | 20.66966 |
| 0.07           | 22.44567    | 20.40516    | 20.56552     | 20.43789 |
| 0.08           | 22.21067    | 20.18738    | 20.29987     | 20.20612 |



**Figure:2** Comparison of MSE for denoising algorithms



**Figure:3** Comparison of PSNR for denoising algorithms

## V. Conclusion

This paper presents a comparative study of wavelet based thresholding techniques for image denoising. Wavelet transform finds the optimum threshold value, it is used to determine the efficiency of the denoising algorithm and is identified as an efficient tool for image denoising. The result shows that the visu shrink produce an over smooth image while the bayes shrink gives smoother and visually

more appealing image, its performance is better in terms of MSE and PSNR for both detailed and smooth images among the tested shrinking methods. SURE shrink improves the visual quality of the image considerably. Due to limited directional selectivity of wavelets, the images denoised by wavelet based denoising are prone to artifacts formation. Minimax has an advantage of providing with predictive performance but does not give good visual quality.

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