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Fixed Point Results In Fuzzy Menger Space With Common Property (E.A.)

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Abstract: This paper presents some common fixed point theorems for weakly compatible mappings via an implicit relation in Fuzzy Menger spaces satisfying the common property (E.A)
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I. Introduction

The study of fixed point theorems in probabilistic metric spaces is an active area of research. The theory of probabilistic metric spaces was introduced by Menger [1, 2] introduced the notion of Probabilistic Metric spaces (in short PM spaces) as a generalization of metric spaces. In fact the study of such spaces received an impetus with the pioneering work of Schweizer and Sklar [3]. The theory of Probabilistic Metric spaces is of fundamental importance in Probabilistic Functional Analysis especially due to its extensive applications in random differential as well as random integral equations. Fixed point theory is one of the most fruitful and effective tools in mathematics which has enormous applications within as well as outside mathematics. By now, several authors have already established numerous fixed point and common fixed point theorems in PM spaces. For an idea of this kind of the literature, one can consult the results contained in [3–14].

In 1986, Jungck [13] introduced the notion of compatible mappings and utilized the same (as a tool) to improve commutativity conditions in common fixed point theorems. This concept has been frequently employed to prove existence theorems on common fixed points. However, the study of common fixed points of non-compatible mappings is also equally interesting which was initiated by Pant [14]. Recently, Aamri and Moutawakil [15] and Liu et al. [16] respectively, defined the property (E.A) and the common property (E.A) and proved some common fixed point theorems in metric spaces. Imdad et al. [17] extended the results of Aamri and Moutawakil [15] to semi metric spaces. And, Kubiaczyk and Sharma [18] defined the property (E.A) in PM spaces and used it to prove results on common fixed points wherein authors claim to prove their results for strict contractions which are merely valid up to contractions. Most recently Rajesh Shrivastav, Vivek Patel and Vanita Ben Dhagat[23] have given the definition of fuzzy probabilistic metric space and proved theorems for Fuzzy Menger Space. In this paper, we prove the fixed point theorems for weakly compatible mappings via an implicit relation in Fuzzy Menger spaces satisfying the common property (E.A). Our results substantially improve the corresponding theorems contained in [20,21,22] along with some other relevant results in Fuzzy Menger as well as metric spaces.

II. Preliminaries and Definitions:

Let us define and recall some definitions:

Definition 2.1 A fuzzy probabilistic metric space (FPM space) is an ordered pair (X, F_α) consisting of a nonempty set X and a mapping F_α from XxX into the collections of all fuzzy distribution functions $F_\alpha \in R$ for all $\alpha. \in [0,1]$. For $x, y \in X$ we denote the fuzzy distribution function F_α (x,y) by $F_{\alpha(x,y)}$ and $F_{\alpha(x,y)}$ (u) is the value of$F_{\alpha(x,y)}$ at u in R.

The functions $F_{\alpha(x,y)}$ for all $\alpha. \in [0,1]$ assumed to satisfy the following conditions:

(a) $F_{\alpha(x,y)}(u) = 1 \ \forall\, u > 0 \ iff\ x = y,$

(b) $F_{\alpha(x,y)}(0) = 0 \ \forall\, x, y$ in X,

(c) $F_{\alpha(x,y)} = F_{\alpha(y,x)} \ \forall\, x, y$ in X,

(d) If $F_{\alpha(x,y)}(u) = 1$ and $F_{\alpha(y,z)}(v) = 1 \Rightarrow F_{\alpha(x,z)}(u+v) = 1 \ \forall\, x, y, z \in X$ and $u, v > 0$.

Definition 2.2 A commutative, associative and non-decreasing mapping $t: [0,1] \times [0,1] \rightarrow [0,1]$ is a t-norm if and only if $t(a, 1) = a \ \forall a \in [0,1]$, $t(0,0) = 0$ and $t(c, d) \geq t(a, b)$ for $c \geq a,\ d \geq b$.

Definition 2.3 A Fuzzy Menger space is a triplet (X, F_α, t), where (X, F_α) is a FPM-space, t is a t-norm and the generalized triangle inequality
$F_{\alpha(x,z)}(u+v) \geq t\,(F_{\alpha(x,z)}(u), F_{\alpha(y,z)}(v))$

holds for all x, y, z in X $u, v > 0$ *and* $\alpha \in [0,1]$.
The concept of neigh borhoods in Fuzzy Menger space is introduced as

Definition2.4 Let (X, F_α, t) be a Fuzzy Menger space. If $x \in X, \varepsilon > 0$ and $\lambda \in (0,1)$, then $(\varepsilon, \lambda) -$ neighborhood of x, called U_x (ε, λ), is defined by

$$U_x\ (\varepsilon, \lambda) = \{y \in X : F_{\alpha(x,y)}(\varepsilon) > (1 - \lambda)\}.$$

An $(\varepsilon, \lambda) -$ topology in X is the topology induced by the family $\{U_x\ (\varepsilon, \lambda) : x \in X, \varepsilon > 0, \alpha \in [0,1] \text{ and } \lambda \in (0,1)\}$ of neighborhood.

Remark: If t is continuous, then Fuzzy Menger space (X, F_α, t) is a Housdroff space in $(\varepsilon, \lambda) -$ topology.
Let (X, F_α, t) be a complete Fuzzy Menger space and $A \subset X$. Then A is called a bounded set if $\lim_{u\to\infty} \inf_{x,y\in A} F_{\alpha(x,y)}\ (u) = 1$

Definition 2.5 A sequence $\{x_n\}$ in (X, F_α, t) is said to be convergent to a point x in X if for every $\varepsilon > 0$ *and* $\lambda > 0$, there exists an integer $N = N(\varepsilon, \lambda)$ such that $x_n \in U_x(\varepsilon, \lambda) \forall\ n \geq N$ or equivalently $F_\alpha\ (x_n, x;\ \varepsilon) > 1 - \lambda$ for all $n \geq N$ and $\alpha \in [0,1]$.

Definition 2.6 A sequence $\{x_n\}$ in (X, F_α, t) is said to be cauchy sequence if for every $\varepsilon > 0$ *and* $\lambda > 0$, *t*here exists an integer $N = N(\varepsilon, \lambda)$ such that for all $\alpha \in [0,1]$ $F\alpha(x_n, x_m;\ \varepsilon) > 1 - \lambda\ \forall\ n, m \geq N$.

Definition 2.7 A Fuzzy Menger space (X, F_α, t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all $\alpha \in [0,1]$.

Definition 2.8 Let (X, F_α, t) be a Fuzzy Menger space. Two mappings $f, g : X \to X$ are said to be weakly comptable if they commute at coincidence point for all $\alpha \in [0,1]$.

Lemma 1 Let $\{x_n\}$ be a sequence in a Fuzzy Menger space (X, F_α, t) , where t is continuous and $t(p, p) \geq p$ for all $p \in [0,1]$, if there exists a constant $k \in (0,1)$ such that for all $p > 0$ *and* $n \in N$ $F_\alpha\ (x_n, x_{n+1};\ kp) \geq F\alpha(x_{n-1}, x_n;\ p)$, for all $\alpha \in [0,1]$ then $\{x_n\}$ is cauchy sequence.

Lemma 2 If (X, d) is a metric space, then the metric d induces, a mapping $F_\alpha : XxX \to L$ defined by $F_\alpha(p, q) = H_\alpha(x - d(p, q)), p, q \in R$ for all $\alpha \in [0,1]$. Further if $t: [0,1] \times [0,1] \to [0,1]$ is defined by $t(a, b) = \min\{a, b\}$, then (X, F_α, t) is a Fuzzy Menger space. It is complete if (X, d) is complete.

Definition 2.9: Let (X, F_α, t) be a Fuzzy Menger space. Two self-mappings $f, g: X \to X$ are said to be compatible if and only if $F_{\alpha(fgx_n, gfx_n)}(t) \to 1$ for all $t > 0$ whenever $\{x_n\}$ in X such that $fx_n, gx_n \to z$ for some $z \in X$.

Definition 2.10: Let (X, F_α, t) be a Fuzzy Menger space. Two self-mappings $f, g: X \to X$ are said to satisfy the property (E.A) if there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$$

Definition 2.11: Two pairs $\{f, g\}$ and $\{p, q\}$ of self-mappings of a Fuzzy Menger space(X, F_α, t) are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X and some z in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} qy_n = z$$

Example 2.12. Let (X, F_α, t) be a Fuzzy Menger space with $X = [-1,1]$ and

$$F_{\alpha(x,y)}(t) = \begin{cases} e^{\frac{-|x-y|}{t}} & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases}$$

for all $x, y \in X$. Define self-mappings f, g, p and q on X as $fx = \frac{x}{2}$, ,$gx = \frac{-x}{2}$,$px = \frac{x}{4}$, and $qx = \frac{-x}{4}$ for all $x \in X$. Then with sequences $\{x_n\} = \frac{1}{n}$ and $\{y_n\} = \frac{-1}{n}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} qy_n = 0$$

This shows that the pairs $\{f, g\}$ and $\{p, q\}$ share the common property (E.A).

Definition 2.13: Two self mappings f and g of a Fuzzy Menger space (X, F_α, t) are said to be weakly compatible if the mappings commute at their coincidence points i.e $fx = gx$ for some $x \in X$ implies $fgx = gfx$ We shall call $w = fx = gx$ a point of coincidence of f and g.

III. Definition 2.14: Implicit Relation

Let $\boldsymbol{\Phi}$ be the set of all real continuous functions $\Phi(\mathbb{R}^5): [0,1] \to \mathbb{R}$, non-decreasing in the argument satisfying the following conditions:

(a) For $u, v \geq 0, \phi(u, v, u, v, 1) \geq 0$ implies that $u \geq v$.

(b) $\phi(u, 1, 1, u, 1) \geq 0$ or $\phi(u, u, 1, 1, u) \geq 0$ or $\phi(u, 1, u, 1, u) \geq 0$ implies that $u \geq 1$

Example2.15: Let's consider $\phi(t_1, t_2, t_3, t_4, t_5) \geq 40t_1 - 18t_2 + 12t_3 - 14t_4 - t_5 + 1$, then $\phi \in \boldsymbol{\Phi}$.

IV. Main Result

We begin with the following observation.

Lemma 3.1. Let p, q, f and g be self-mappings of a Fuzzy Menger space (X, F_α, t) satisfying the following:
(i) the pair (p, f) (or (q, g) satisfies the property (E.A);
(ii) for any $p, q \in X, \phi \in \mathbf{\Phi}$ for all $t > 0$,

$$\phi\left(F_{\alpha(px,qy)}(t), F_{\alpha(fx,gy)}(t), F_{\alpha(fx,px)}(t), F_{\alpha(gy,qy)}(t), F_{\alpha(gy,px)}(t)\right) \geq 0 \ldots\ldots \quad (3.1)$$

(iii) $p(X) \subset g(X)$ (or $q(X) \subset f(X)$),
Then the pair (p, f) and (q, g)share the common property (E. A).

Proof. First we assume that the pair (p, f) owns the property (E.A), and we show that the pair (p, f) (or (q, g) share common property (E.A). Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = x$ for some $x \in X$.
Since $p(X) \subset g(X)$, hence for each $\{x_n\}$ there exists $\{y_n\}$ in X such that $px_n = fy_n$.
Therefore, $\lim_{n\to\infty} px_n = \lim_{n\to\infty} gy_n = x$
Thus in all, we have $px_n \to x$, $fx_n \to x$ and $gy_n \to x$. Now we prove that $qy_n \to x$.
On contrary let $qy_n \nrightarrow x$ as $n \to \infty$, then from equation (3.1), we obtain

$$\phi\left(F_{\alpha(px_n,qy_n)}(t), F_{\alpha(fx_n,gy_n)}(t), F_{\alpha(fx_n,px_n)}(t), F_{\alpha(gy_n,qy_n)}(t), F_{\alpha(gy_n,px_n)}(t)\right) \geq 0$$

letting $n \to \infty$, we have

$$\phi\left(F_{\alpha(x,qy_n)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,qy_n)}(t), F_{\alpha(x,x)}(t)\right) \geq 0$$

This is, $\phi\left(F_{\alpha(x,qy_n)}(t), 1, 1, F_{\alpha(x,qy_n)}(t), 1\right) \geq 0$
Using implicit function definition, we have $F_{\alpha(x,qy_n)}(t) \geq 0$ for all $t > 0$.
$F_{\alpha(x,qy_n)}(t) = 1$, thus $qy_n \to x$.
Hence the pairs (p, f) and (q, g) share the common property (E.A).

Remark 3.2. The converse of Lemma 3.1 is not true in general. For a counter example, one can see Example 3.9 (presented in the end).

Theorem 3.3. Let p, q, f and g be self-mappings on a Menger PM space (X, F_α, t) satisfying the following :
(i) the pair (p, f) (or (q, g)) satisfies the property (E.A);
(ii) for any $p, q \in X, \phi \in \mathbf{\Phi}$ for all $t > 0$,

$$\phi\left(F_{\alpha(px,qy)}(t), F_{\alpha(fx,gy)}(t), F_{\alpha(fx,px)}(t), F_{\alpha(gy,qy)}(t), F_{\alpha(gy,px)}(t)\right) \geq 0 \ldots\ldots\ldots (3.1)$$

(iii) $p(X) \subset g(X)$ (or $q(X) \subset f(X)$),
(iv) f(X) (or g(X)) is a closed subset of X.
(v) the pairs (p, f) and (q, g) are weakly compatible.
Then the pairs (p, f) and (q, g) have a point of coincidence each. Moreover, p, q, f and g have a unique common fixed point.
Proof. In the view of lemma 3.1 pairs (p, f) and (q, g) shares the common property (E.A), that is, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} qy_n = \lim_{n\to\infty} gy_n = x$ for some $x \in X$.
Suppose that f(X) is a closed subset of X, then $x = fu$ for some $u \in X$. If $x \neq pu$, then from given inequality, we obtain

$$\phi\left(F_{\alpha(pu,qy_n)}(t), F_{\alpha(fu,gy_n)}(t), F_{\alpha(fu,pu)}(t), F_{\alpha(gy_n,qy_n)}(t), F_{\alpha(gy_n,pu)}(t)\right) \geq 0$$

taking $n \to \infty$, we get

$$\phi\left(F_{\alpha(pu,x)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,pu)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,pu)}(t)\right) \geq 0$$

or, $\phi\left(F_{\alpha(pu,x)}(t), 1, F_{\alpha(x,pu)}(t), 1, F_{\alpha(x,pu)}(t)\right) \geq 0$
Using implicit function definition, we get
$F_{\alpha(x,pu)}(t) \geq 1$ for all $t > 0$.
This gives $F_{\alpha(pu,x)}(t) = 1$.
Thus, $pu = x$.
Hence $fu = pu = x$.
Since $p(X) \subset g(X)$, there exists $v \in X$ such that $x = pu = gv$.
If $x \neq qv$, then using inequality (3.1),we have $\phi\left(F_{\alpha(pu,qv)}(t), F_{\alpha(fu,gv)}(t), F_{\alpha(fu,pu)}(t), F_{\alpha(gv,qv)}(t), F_{\alpha(gv,pu)}(t)\right) \geq 0$
Or $\phi\left(F_{\alpha(x,qv)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,qv)}(t), F_{\alpha(x,x)}(t)\right) \geq 0$

Or $\phi\big(F_{\alpha(x,qv)}(t), 1,1, F_{\alpha(x,qv)}(t), 1\big) \geq 0$
Using implicit function definition, we get
$F_{\alpha(x,qu)}(t) \geq 1$ for all $t > 0$.
This gives $F_{\alpha(x,qv)}(t) = 1$.
Thus, $x = qv$.
Hence $gv = qv = x$. Thus $fu = pu = gv = qv = x$.
Since the pairs(p, f) and (q, g) are weakly compatible and $fu = pu, gv = qv$,
Therefore $fx = fpu = pfu = px$, and $gx = gqv = qgv = qx$.
If $px \neq x$, then using inequality (3.1), we have

$$\phi\big(F_{\alpha(px,qv)}(t), F_{\alpha(fx,gv)}(t), F_{\alpha(fx,px)}(t), F_{\alpha(gv,qv)}(t), F_{\alpha(gv,px)}(t)\big) \geq 0$$

Or, $\phi\big(F_{\alpha(px,x)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,px)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,px)}(t)\big) \geq 0$
Or, $\phi\big(F_{\alpha(px,x)}(t),1, F_{\alpha(x,px)}(t),1, F_{\alpha(x,px)}(t)\big) \geq 0$
Using implicit function definition, we get
$F_{\alpha(px,x)}(t) \geq 1$ for all $t > 0$.
This gives $F_{\alpha(px,x)}(t) = 1$.
Hence $px = x$. Therefore $px = fx = x$.
Similarly, If $qx \neq x$, then using inequality (3.1), we have

$$\phi\big(F_{\alpha(pu,qx)}(t), F_{\alpha(fu,gx)}(t), F_{\alpha(fu,pu)}(t), F_{\alpha(gx,qx)}(t), F_{\alpha(gx,pu)}(t)\big) \geq 0$$

Or, $\phi\big(F_{\alpha(x,qx)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,qx)}(t), F_{\alpha(x,x)}(t)\big) \geq 0$
Or, $\phi\big(F_{\alpha(x,qx)}(t), 1,1, F_{\alpha(x,qx)}(t), 1\big) \geq 0$
Using implicit function definition, we get
$F_{\alpha(x,qx)}(t) \geq 1$ for all $t > 0$.
This gives $F_{\alpha(x,qx)}(t) = 1$.
Hence $x = qx$. Therefore $qx = gx = x$.
Hence $x = px = fx = qx = gx$, and x is a common fixed point of p, q, f and g.

Uniqueness: Let w and z be two common fixed points of maps p, q, f and g. Put $x = z$ and $y = w$ in the inequality (3.1), we get

$$\phi\big(F_{\alpha(pz,qw)}(t), F_{\alpha(fz,gw)}(t), F_{\alpha(fz,pz)}(t), F_{\alpha(gw,qw)}(t), F_{\alpha(gw,pz)}(t)\big) \geq 0$$

Or

$$\phi\big(F_{\alpha(z,w)}(t), F_{\alpha(z,w)}(t), F_{\alpha(z,z)}(t), F_{\alpha(w,w)}(t), F_{\alpha(w,z)}(t)\big) \geq 0$$

Or

$$\phi\big(F_{\alpha(z,w)}(t), F_{\alpha(z,w)}(t),1,1, F_{\alpha(w,z)}(t)\big) \geq 0$$

Using implicit function definition, we get
$F_{\alpha(z,w)}(t) \geq 0$ for all $t > 0$
This gives $F_{\alpha(z,w)}(t) = 0$. Thus $z = w$.
So, z is the unique common fixed point of p, q, f and g.

Corollary 3.4. Let p and f be self-mappings on a Fuzzy Menger space (X, F_α, t) such that
(i) the pair (p, f) satisfies the common property (E.A),
(ii) for any $p, q \in X, \phi \in \boldsymbol{\Phi}$ for all $t > 0$ $\phi\big(F_{\alpha(px,py)}(t), F_{\alpha(fx,fy)}(t), F_{\alpha(fx,px)}(t), F_{\alpha(fy,py)}(t), F_{\alpha(fy,px)}(t)\big) \geq 0$
(iii) f(X) is a closed subset of X.
(iv) the pairs (p, f) are weakly compatible
Then p and f have a coincidence point. Moreover, if the pair (p, f) is weakly compatible, then p and f have a unique common fixed point.

Theorem 3.5. Let p, q, f and g be self-mappings on a Menger PM space (X, F_α, t) satisfying the following :
(i) the pair (p, f) and (q, g) shares the common property (E.A);
(ii) for any $p, q \in X, \phi \in \boldsymbol{\Phi}$ for all $t > 0$,

$$\phi\big(F_{\alpha(px,qy)}(t), F_{\alpha(fx,gy)}(t), F_{\alpha(fx,px)}(t), F_{\alpha(gy,qy)}(t), F_{\alpha(gy,px)}(t)\big) \geq 0 \ldots\ldots\ldots (3.1)$$

(iii) f(X) and g(X) are closed subsets of X.
(iv) the pairs (p, f) and (q, g) are weakly compatible.
Then p, q, f and g have a unique common fixed point in X.

Proof. Let the pairs (p,f) and (q,g) shares the common property (E.A), then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} qy_n = \lim_{n\to\infty} gy_n = x$ for some $x \in X$.
Since $p(X)$and $f(X)$ is a closed subset of X, then $x = fu = gv$ for some $u, v \in X$. If $x \neq pu$, then from given inequality, we obtain

$$\phi\left(F_{\alpha(pu,qy_n)}(t), F_{\alpha(fu,gy_n)}(t), F_{\alpha(fu,pu)}(t), F_{\alpha(gy_n,qy_n)}(t), F_{\alpha(gy_n,pu)}(t)\right) \geq 0$$

Now taking $n \to \infty$, we get

$$\phi\left(F_{\alpha(pu,x)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,pu)}(t), F_{\alpha(x,x)}(t), F_{\alpha(x,pu)}(t)\right) \geq 0$$

or, $\phi\left(F_{\alpha(pu,x)}(t), 1, F_{\alpha(x,pu)}(t), 1, F_{\alpha(x,pu)}(t)\right) \geq 0$
Using implicit function definition, we get
$F_{\alpha(x,pu)}(t) \geq 1$ for all $t > 0$.
This gives $F_{\alpha(pu,x)}(t) = 1$.
Thus, $pu = x$.
Hence $fu = gv = pu = x$.

Since the pair(p,f) and $(q,g)$are weakly compatible and both the pair have point of coincidence u and v respectively. Following the lines of proof of Theorem 3.3, one can easily prove the existence of unique common fixed point of mappings p, q, f and g.

Theorem 3.7. Let p, q, f and g be self-mappings on a Menger PM space (X, F_α, t) satisfying the following :
(i) the pair (p,f) and (q,g) shares the common property (E.A);
(ii) for any $p, q \in X, \phi \in \boldsymbol{\Phi}$ for all $t > 0$,

$$\phi\left(F_{\alpha(px,qy)}(t), F_{\alpha(fx,gy)}(t), F_{\alpha(fx,px)}(t), F_{\alpha(gy,qy)}(t), F_{\alpha(gy,px)}(t)\right) \geq 0 \ldots\ldots\ldots (3.1)$$

(iii) $\overline{p(X)} \subset g(X)$ and $\overline{q(X)} \subset f(X)$.
(iv) the pairs (p,f) and (q,g) are weakly compatible.
Then p, q, f and g have a unique common fixed point in X.

Corollary 3.8. Let p, q, f and g be self-mappings on a Menger PM space (X, F_α, t) satisfying the following :
(i) the pair (p,f) and (q,g) shares the common property (E.A);
(ii) for any $p, q \in X, \phi \in \boldsymbol{\Phi}$ for all $t > 0$,

$$\phi\left(F_{\alpha(px,qy)}(t), F_{\alpha(fx,gy)}(t), F_{\alpha(fx,px)}(t), F_{\alpha(gy,qy)}(t), F_{\alpha(gy,px)}(t)\right) \geq 0 \ldots\ldots\ldots (3.1)$$

(iii) $p(X)$ and $q(X)$are closed subsets of X whereas $p(X) \subset g(X)$ and $q(X) \subset f(X)$.
(iv) the pairs (p,f) and (q,g) are weakly compatible.
Then p, q, f and g have a unique common fixed point in X.

Example 3.9. Consider $X = [-1,1]$ and define $F_{\alpha(x,y)} = H(t - |x - y|)$ for all$x, y \in X$. Then (X, F_α, t) is the fuzzy menger space with $\Delta(a,b) = \min\{a,b\}$. Define self mappings p, q, f and g on X as

$$p(X) = \begin{cases} \frac{3}{5}, & \text{if } x \in \{-1,1\} \\ \frac{x}{4}, & \text{if } x \in (-1,1) \end{cases} \qquad q(X) = \begin{cases} \frac{3}{5}, & \text{if } x \in \{-1,1\} \\ \frac{-x}{4}, & \text{if } x \in (-1,1) \end{cases}$$

$$f(X) = \begin{cases} \frac{1}{2}, & \text{if } x = -1 \\ \frac{x}{2}, & \text{if } x \in (-1,1) \\ \frac{-1}{2}, & \text{if } x = 1 \end{cases} \qquad g(X) = \begin{cases} \frac{-1}{2}, & \text{if } x = -1 \\ \frac{-x}{2}, & \text{if } x \in (-1,1) \\ \frac{1}{2}, & \text{if } x = 1 \end{cases}$$

Then with the sequences $\{x_n = \frac{1}{n}\}$ and $\{y_n = \frac{-1}{n}\}$ in X, we have
$\lim_{n\to\infty} px_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} qy_n = \lim_{n\to\infty} gy_n = 0$ which shows that pair (p,f) and (q,g) share the common property$(E.A)$.
Also $p(X) = q(X) = \left\{\frac{3}{5}\right\} \cup \left(-\frac{1}{4}, \frac{1}{4}\right) \not\subset \left[-\frac{1}{2}, \frac{1}{2}\right] = f(X) = g(X)$.
Thus all the conditions of Theorem 3.1 are satisfied and 0 is the unique common fixed point of the pair (p,f) and (q,g)which is their coincidence point as well.

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