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The Influence Of Infinite Impedance Flanges On The Electromagnetic Field Of A Plane Waveguide

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ABSTRACT
The problem of analysis of the electromagnetic field behaviour from the open end of the parallel-plate waveguide with infinite impedance flanges is theoretically investigated. The case with the absence of impedance flanges is also considered. Furthermore, we take into account particular features of the waveguide edges. The effects of the impedance flanges and the edge features on the electromagnetic field and the radiation patterns of a plane waveguide with flanges are demonstrated. The method of moments (MoM) technique which was used to solve the integral equations is presented along with the numerical results.
Keywords - Impedance, method of moment, radiation pattern, waveguide.

I. INTRODUCTION

To change the characteristics of radar location of military equipment, radar absorbing materials and coverings are effective when applied to mobile installations, as well as to those on which antennas systems are placed [1]. From the point of view of electrodynamics, the usage of radar absorbing materials and coverings amounts to changing the distribution of surface currents, by means of the management of surface wave amplitudes formed on the object by the attenuating structures. However, the redistribution of fields on the surface of the object leads to variations not only in the scattering characteristics of the object but also in its radiation characteristics, in relation to antennas located nearby.

Methods are known for decreasing the interaction between antennas using an alteration of amplitude and phase distributions on certain surfaces. Among the accepted measures used to decrease the coupling between antennas are mutual shielding of antennas and placement of additional screens across the interface. In the case of near-omnidirectional antennas, two groups of additional measures are applied: radio-wave absorbing materials, and surface decoupling devices. Since the waveguide as a radiating element finds much usage in antenna techniques, the analysis of the radiation characteristics of such a device represents a field of significant scientific interest [2]. Furthermore, the analysis of the impedance influence on the electromagnetic field behaviour across the open-end of the plane waveguide is one of the major applied problems of electrodynamics. It is also known [3] that varying the reactive resistance allows one to change the phase field re-radiated by the structure, which consequently facilitates managing the re-radiated field.

For an approximate solution to the analysis of electromagnetic wave radiation from a semi-infinite waveguide with ideally conducting flanges, the method of integral equations is often used [1]. It is necessary to generalize this familiar approach when considering a radiator with impedance flanges not equal to zero [4].

The purpose of this paper is to analyse the influence of an impedance on the electromagnetic field behaviour of a radiating plane waveguide. In particular, we investigate the behaviour of the tangential field components on a waveguide surface with infinite impedance flanges, taking into account the specificities of the edges and the radiation pattern.

The paper is organized as follows: in section 2, we consider the solution to the problem of the electromagnetic field (EMF) radiation from the open end of a parallel-plate waveguide with infinite impedance flanges; numerical results are analyzed in Section 3, and section 4 is devoted to the conclusion.

II. RADIATION OF A PARALLEL-PLATE WAVEGUIDE WITH INFINITE IMPEDANCE FLANGES

2.1. Statement of the problem

First, we consider the solution to the problem of EMF radiation from the open end of a parallel-plate waveguide with infinite impedance flanges in the following setting (Figure 1). Let a parallel-plate waveguide be excited by a wave characterized by $\vec{E}^i, \vec{H}^i$, with components,

$$H_z^i = H_0 e^{-iky},\ E_x^i = -WH_0 e^{-iky},\ y<0, \quad (1)$$

where $W = 120\pi\ \Omega$ is the characteristic resistance of free space and H_o is the amplitude of the incident wave. On the infinite flanges $\left(y=0;\ x\in(-\infty,0]\ \text{and}\ [a,\infty)\right)$, the boundary impedance conditions of Shukin-Leontovich are fulfilled:

$$\vec{n}\times\vec{E} = -Z\vec{n}\times(\vec{n}\times\vec{H}), \quad (2)$$

$$E_x = ZH_z,$$

where $\vec{n} = -\vec{i}_y$ is the unit normal to the $y=0$ plane, and $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields, respectively. Z is the surface impedance, so that $Z=0$ specifies a perfect conductor.

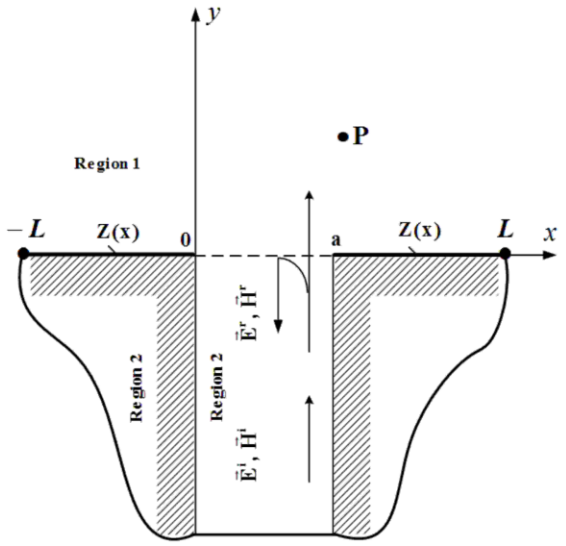

Figure 1. The open end of a parallel-plate waveguide with infinite impedance flanges.

It is necessary to find the EMF in both regions: inside the waveguide ($y \le 0$ and $0 \le x \le a$, region 2) and the upper half-space ($y \ge 0$, region 1).

The required EMF should satisfy Maxwell's equations applied to radiation and the boundary conditions on the flanges, as well as the conditions relevant to the infinite character of tangential components in the opening. We consider this problem two-dimensionally.

2.2. Solution of the problem

For the solution of the proposed problem, we use the Lorentz lemma in integral form for regions V_1 and V_2 as in [4-6], and we choose a thread in-phase with the magnetic current parallel to the z - axis: $\vec{j}^{\,e.ex.} = 0$ and $\vec{j}^{\,m.ex.} = \vec{i}_z J_0^m \delta(p,q)$,

where $\delta(p,q)$ is a two-dimensional delta-function, p is the point of observation, q is the point of integration and, J_0^m is the current amplitude.

As a result, the integral correlation for each region becomes:

$$H_{z1}(x) = \int_{-\infty}^{\infty} E_{x1}(x') H_{z1}^m(x,x')dx', \quad (3)$$

$$H_{z2}(x) = 2H_0 - \int_0^a E_{x2}(x') H_{z2}^m(x,x')dx' \quad (4)$$

where $H_{z_1}^m(x,x')$ and $H_{z2}^m(x,x')$ are the subsidiary magnetic fields in regions V_1 and V_2, respectively (defined below), and a is the dimension of the open end of the waveguide.

2.3. Integral equations relative to E_x.

Correlations (3) and (4) compose a system of integral equations on the $y=0$ plane. Taking into account the equality of the tangential field components, $H_{z1} = H_{z2} = H_z$,and $E_{x1} = E_{x2} = E_x$ in the opening $(y=0, x\in[0,a])$, and the boundary condition of equation (2) $(E_x = ZH_z)$ on the parts of the flanges with finite impedance ($(x\in[-L,0])$ and $(x\in[a,L])$), we can obtain a system of integral equations relative to the unknown tangential component of the electric field $E_x(x)$ on the $y=0$ plane:

$$\begin{cases} E_x(x) - Z(x)\int_{-L}^{L} E_x(x')H_{z1}^m(x,x')dx' = 0 & x\in[-L,0] \\ \int_{-L}^{0} E_x(x')H_{z1}^m(x,x')dx' + \int_0^a E_x(x')[H_{z1}^m(x,x') + H_{z2}^m(x,x')]dx' + & \\ +\int_a^L E_x(x')H_{z1}^m(x,x')dx' = 2H_0 & x\in[0,a] \\ E_x(x) - Z(x)\int_{-L}^{L} E_x(x')H_{z1}^m(x,x')dx' = 0 & x\in[a,L] \end{cases} \quad (5)$$

where the subsidiary magnetic fields,

$$H_{z_1}^m(x,x') = -\left(kJ_0^m/2W\right)\cdot H_0^{(2)}(kR)$$

and

$$H_{z2}^m(x,x') = -(k/Wa)\sum_{n=0}^{\infty}(\varepsilon_n/k_n)\cos\gamma_n x\cos\gamma_n x',$$

are solutions of the non-uniform Helmholtz equation for the complex amplitudes of the vector potential in regions V_1 and V_2, respectively. This equation can be solved using the standard method of separation of variables [7]. Here, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, $R = \sqrt{(x-x')^2 + (y-y')^2}$ is the distance from the point of observation to the point of integration, $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind, $k_n = \sqrt{k^2 - \gamma_n^2}$ is the propagation constant of the wave $\left(\gamma_n = \dfrac{n\pi}{a}; n = 0,1,2\ldots\right)$, and $\varepsilon_n = \begin{cases} 1 & n = 0, \\ 2 & n > 0. \end{cases}$ is the dielectric permittivity of layer n.

2.4. Effect of the waveguide edges

At the points $x = 0$ and $x = a$, the sidewalls of the waveguide V_2 and the flanges converge at right angles to form edges. Because of the presence of these edges with zero curvature (see Figure 1), the surface density of the electric charge becomes locally infinite. As a result, the vector $\vec{E}$ at these points must be given special consideration [8]:

$$E_x \Rightarrow \rho^{-1/3},$$

where ρ is the distance from an edge. In order to properly take this edge into account in the solution, we formulate the required value on the interval $[0,a]$ as follows:

$$\frac{E_x(x')}{W} = \frac{X(x')}{((a-x')x')^{\alpha}},$$

where $\alpha = 1/3$, and the new unknown value $X(x')$ is not specified. Equation (5) can then be rewritten as follows:

$$\begin{cases} E_x(x) - Z(x)\int\limits_{-L}^{0} E_x(x')H_{z1}^{m}(x,x')dx' - Z(x)\int\limits_{0}^{a} E_x(x')H_{z1}^{m}(x,x')dx' - \\ - Z(x)\int\limits_{a}^{L} E_x(x')H_{z1}^{m}(x,x')dx' = 0 & x \in [-L,0] \\ \int\limits_{-L}^{0} E_x(x')H_{z1}^{m}(x,x')dx' + \int\limits_{0}^{a} \dfrac{X(x')}{((a-x')x')^{\alpha}}[H_{z1}^{m}(x,x') + H_{z2}^{m}(x,x')]dx' + \\ + \int\limits_{a}^{L} E_x(x')H_{z1}^{m}(x,x')dx' = 2H_0 & x \in [0,a] \\ E_x(x) - Z(x)\int\limits_{-L}^{0} E_x(x')H_{z1}^{m}(x,x')dx' - Z(x)\int\limits_{0}^{a} \dfrac{X(x')}{((a-x')x')^{\alpha}}H_{z1}^{m}(x,x')dx' - \\ - Z(x)\int\limits_{a}^{L} E_x(x')H_{z1}^{m}(x,x')dx' = 0 & x \in [a,L]. \end{cases} \tag{6}$$

In the case of ideal conducting flanges ($Z(x) = 0$, which makes $E_x = 0$ on the flanges), equation (6) turns into a single integral equation:

$$\int\limits_{0}^{a} \frac{X(x')}{((a-x')x')^{\alpha}}[H_{z1}^{m}(x,x') + H_{z2}^{m}(x,x')]dx' = 2H_0, \quad x \in [0,a]. \tag{7}$$

The solution of equations (6) or equation (7) is conveniently solved with the use of the Krylov-Bogolyubov method [9], which is a piece-wise continuous approximation to the required value. In this case, the integral equations (6) transform into a system of linear algebraic equations (SLAE):

$$\sum_{m=1}^{M} X_m C_{n,m} = 2H_0, \quad n = 1,2,\ldots M, \tag{8}$$

where

$$C_{n,m} = \int\limits_{x_m-\delta}^{x_m+\delta} \frac{1}{((a-x')x')^{\alpha}}[H_{z1}^{m}(x_n,x') + H_{z2}^{m}(x_n,x')]dx',$$

$$\delta = 0.5\Delta x, \quad \Delta x = (x_{n+1} - x_n),$$

and x_n $(n = 1,2,\ldots M)$ are the lattice points.

II. Results and discussion

Figure 2*a* shows the results of the solution to the integral equation (7) for $X(x)$ (dashed line) and $E_x(x) = X(x)(x(a-x))^{-\alpha}$ (solid line) for a waveguide antenna with dimensions, $a = 0.4\lambda$, with ideal conducting flanges, and with the number of lattice points $M = 60$ ($\Delta x = 0.01\lambda$). Analogous results are presented in Figure 2*b* for a waveguide with an opening dimension of $a = 0.8\lambda$. The discretization step, Δx, in both cases is the same as in Figure 2*a*. Research shows that the required precision in solving the integral equation is reached with a number of points across the opening equal to 8 or more.

Figure 3 shows the modulus of the reflection coefficient (solid line) inside the waveguide, the argument of this coefficient (dotted curve) and the modulus of the complete field inside (dashed curve), which consists of the decaying fields and the reflected fields. Numerical analysis of the curves shows that the reflection coefficient is equal to 0.6 and its argument represents a travelling wave inside the waveguide in the form of a saw-tooth curve. The complete field represents a travelling wave in sinusoidal form.

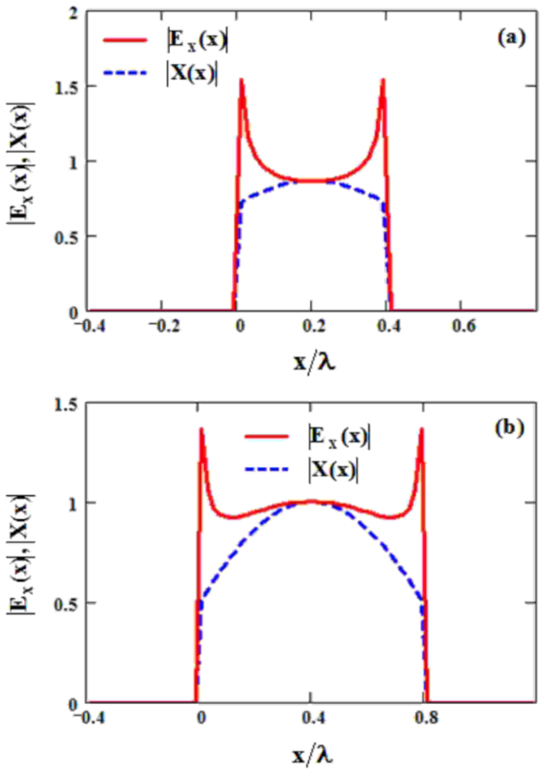

Figure 2. Solution of the integral equation for a waveguide antenna weith ideal conducting flanges. The dimension of the open end of the antenna is (a) $a = 0.4\lambda$ **and (b)** $a = 0.8\lambda$ **. The width** $L = 0.4\lambda$ **.**

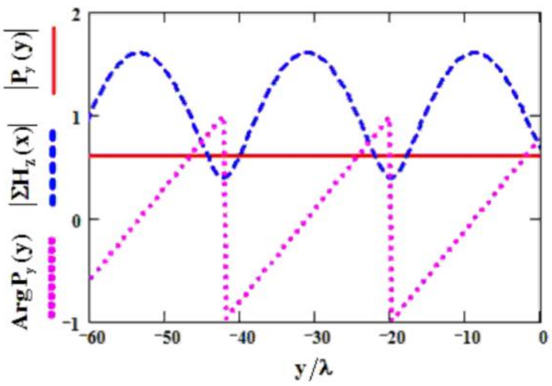

Figure 3. Modulus of the reflection coefficient (solid curve), the argument of this coefficient (dotted curve) and the modulus of the complete field (dashed curve) inside the waveguide for the radiated plane waveguide. Parameters used are $a = 0.4\lambda$ **and** $L = 0.4\lambda$ **.**

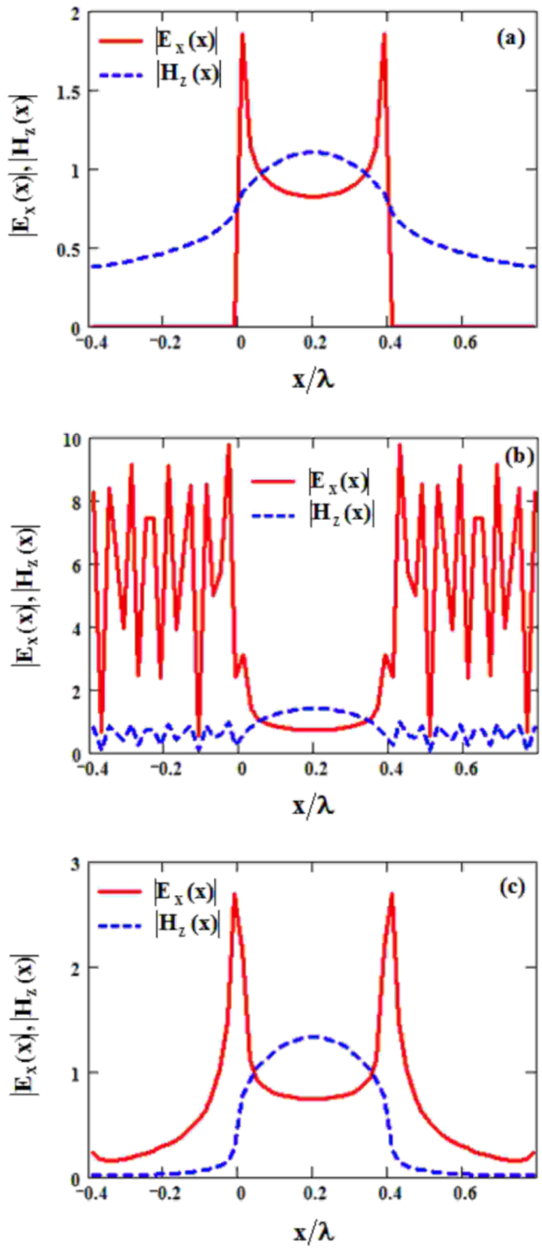

Figure 4. Behavior of vectors $\vec{E}$ **(solid red curve) and** $\vec{H}$ **(dashed blue curve) on the open end of the waveguide (a) in the absence of impedance flanges,** $Z = 0$ **, and (b-c) in the presence of the flanges (b)** $Z = 10i$ **, and (c)** $Z = -10i$ **. Parameters used are** $a = 0.4\lambda$ **and** $L = 0.4\lambda$ **in all cases.**

Figures 4(a-c) show the behaviour of vectors $\vec{E}$ and $\vec{H}$ on the open end of the waveguide in the absence of impedance flanges (Figure 4*a*), and in the presence of the flanges (Figures 4b and c). The parameters used are $a = 0.4\lambda$ and $L = 0.4\lambda$ in all cases. As shown, vector $\vec{E}$ in all cases has the form of a parabola over the open end of the waveguide. This form is inverted for the vector $\vec{H}$ in all cases. In the presence of impedance flanges, we note a large intensity of the electric field vector on the flanges for the case when $Z=10i$, while for the case with $Z=-10i$, the field exponentially decreases away from the waveguide opening. Furthermore, the level of the electric field in the open end and on the flanges is higher for $Z=10i$. Numerical calculations show that these vectors are calculated with an error margin not higher than 5 %.

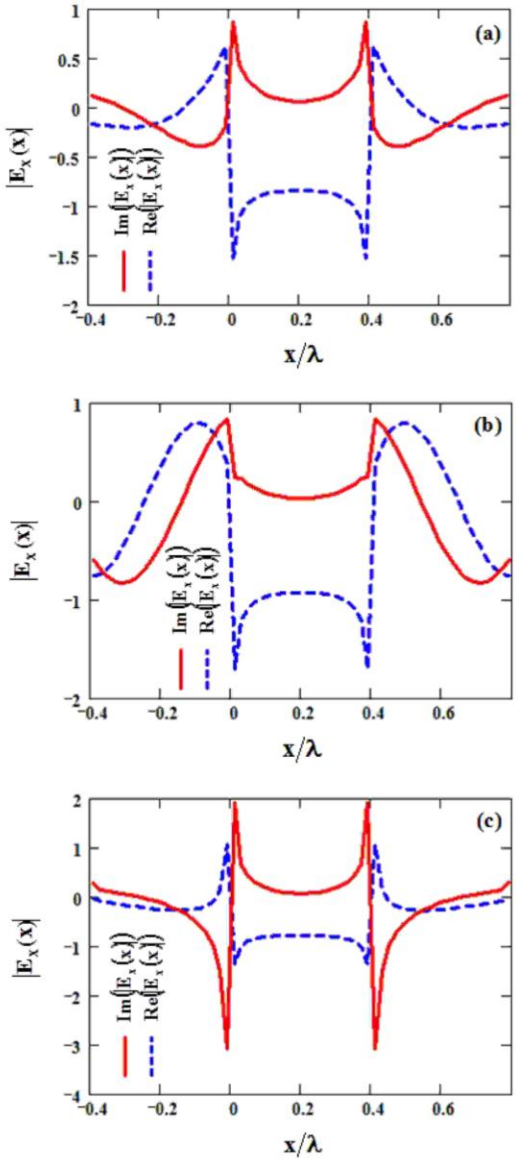

Figure 5. Solution of the integral equation giving the real and imaginary parts of $|E_x(x)|$ for (a) $Z=-10i$, (b) $Z=10i$, and (c) $Z=1W$, respectively.

Let us consider the behavior of the tangential vector components of the field on the $y=0$ surface, for a waveguide with impedance flanges. Figure 5*a* presents the variation of the tangential component of vector $\vec{E}$ $(E_x(x))$ for an antenna with capacitive impedance, $Z=-10i$, over the regions $[-L,0]$ and $[a,L+a]$ $(a=L=0.4\lambda)$ of the flanges (the impedance is normalized to $W=120\pi\ \Omega$). Corresponding variations are presented in Figures 5*b* and 5*c*, for inductive $(Z=10i)$ and resistive $(Z=1)$ impedance cases, respectively, for the same dimensions as in Figure 5*a* ($Re(Z)$, dashed line and $Im(Z)$, solid line). As we can see, the presence of an inductive impedance leads to the appearance of an electric field anomaly at the edges of the waveguide antenna on the sides of the impedance flanges. In addition, the intensity of the electric field vector located on different sides of the waveguide edges has opposite phases. The presence of this specific feature at the edges of the waveguide at $x=0$ and $x=a$ can be taken into account by introducing the required functional form in the following way:

$$E_x(x)=X(x)\begin{cases}(|x|(a-x))^{-\xi} & x\in[-L,0]\\ (x(a-x))^{-\alpha} & x\in[0,a],\\ (x(x-a))^{-\xi} & x\in[a,a+L]\end{cases} \tag{9}$$

where $X(x)$ is a new unknown function without any specification, and α and ξ define the impedance of the flanges with dimension L.

In this case, the system of integral equations (6), in accordance with the Krylov-Bogolyubov method, becomes a SLAE relative to $X_n=X(x_n)$:

$$\begin{cases} X_n f_n^{\langle 1\rangle}-\sum\limits_{m=1}^{M_{12}} X_m U_{n,m}=0 & n=1\cdots M_1\\ \sum\limits_{m=1}^{M_{12}} X_m T_{n,m}=2H_0 & n=M_1+1\cdots M_{01}\\ X_n f_n^{\langle 2\rangle}-\sum\limits_{m=1}^{M_{12}} X_m U_{n,m}=0 & n=M_{01}+1\cdots M_{12}\end{cases}, \tag{10}$$

$Z_n=Z(x_n)$; M_1, M, M_2 are the numbers of lattice points in the regions $[-L,0]$, $[0,a]$, $[a,a+L_2]$, respectively; $M_{01}=M+M_1$; $M_{12}=M_{01}+M_1$; $f_n^{\langle 1\rangle}=[|x_n|(a-x_n)]^{-\xi}$; $f_n^{\langle 2\rangle}=[x_n(x_n-a)]^{-\xi}$;

$$U_{n,m}=Z_n\begin{cases} D_{n,m}^{\langle 1\rangle} & m\in[1,M_1]\\ G_{n,m} & m\in[M_1+1,M_{01}]\\ D_{n,m}^{\langle 2\rangle} & m\in[M_{01}+1,M_{12}]\end{cases};$$

$$T_{n,m}=\begin{cases} D_{n,m}^{\langle 1\rangle} & m\in[1,M_1]\\ C_{n,m} & m\in[M_1+1,M_{01}]\\ D_{n,m}^{\langle 2\rangle} & m\in[M_{01}+1,M_{12}],\end{cases}$$

and $D^{\langle 1\rangle}_{n,m} = \int\limits_{x_m-\delta}^{x_m+\delta} \frac{H^m_{z1}(x_n, x')}{\left((a-x')|x|\right)^{\xi}} dx' \quad x_n \in [-L, 0]$

$$D^{\langle 2\rangle}_{n,m} = \int\limits_{x_m-\delta}^{x_m+\delta} \frac{H^m_{z1}(x_n, x')}{\left((x'-a)x\right)^{\xi}} dx' \quad x_n \in [a,\ a+L];$$

$$G_{n,m} = \int\limits_{x_m-\delta}^{x_m+\delta} \frac{H^m_{z1}(x_n, x')}{\left((a-x')x'\right)^{\alpha}} dx', \ x_n \notin [0, a];$$

$$C_{n,m} = \int\limits_{x_m-\delta}^{x_m+\delta} \frac{H^m_{z1}(x_n, x') + H^m_{z2}(x_n, x')}{\left((a-x')x'\right)^{\alpha}} dx', \ x_n \in [0, a]$$

Numerical research shows that the smoothest variation of the required function $X(x)$ is obtained with the following functional forms of the values of α and ξ:

$\alpha = 0.33 + 0.17Q^{5,5}$ and $\xi = 0.21 + 0.44Q^{4,3}$,

where $Q = \frac{|Z|}{1+|Z|}$.

As an example, in Figures 6(a-c), we present the variation of $X(x)$ (dashed line) and the tangential component of vector $\vec{E}$ $(E_x(x))$ (solid line) of an antenna with $a = 0.4\lambda$, $-L = L = 0.4\lambda$, and with the impedance flanges of $Z = -1i$ (Figure 6*a*), $Z = -10i$ (Figure 6*b*) and $Z = -100i$ (Figure 6*c*), respectively. Research shows that with $Im Z \le -10$, the anomaly (seen in $X(x)$) at edge of the impedance flange is higher than in the waveguide opening. In the reverse case, it is vice versa. Thus, taking into account the anomaly of the electric field $E_x(x)$ at the edges allows us to reduce the solution of the boundary problem to the search for a gradually changing function $X(x)$ and consequently, to a significant reduction in equation (10) of the order of the solved SLAE.

The radiation patterns of the single waveguide antenna are shown in Figure 7 for the capacitive (solid line), inductive (dashed line), and the ideal conducting (dotted line) flanges. The impact on the radiation pattern caused by including impedance regions on the flanges ($a = 0.4\lambda$, $L = 0.4\lambda$, $Z = 0$, $Z = -10i$, and $Z = 10i$), is seen as the difference between the solid and dashed lines. As we can see, the capacitive impedance significantly distorts the wave by increasing the directivity of the antenna but, the presence of inductive impedance leads to significant distortion of the radiation pattern and to a significant growth of lateral lobes along the impedance structure, as well as the deviation of the main lobe, which is due to the appearance of travelling surface waves along the structure.

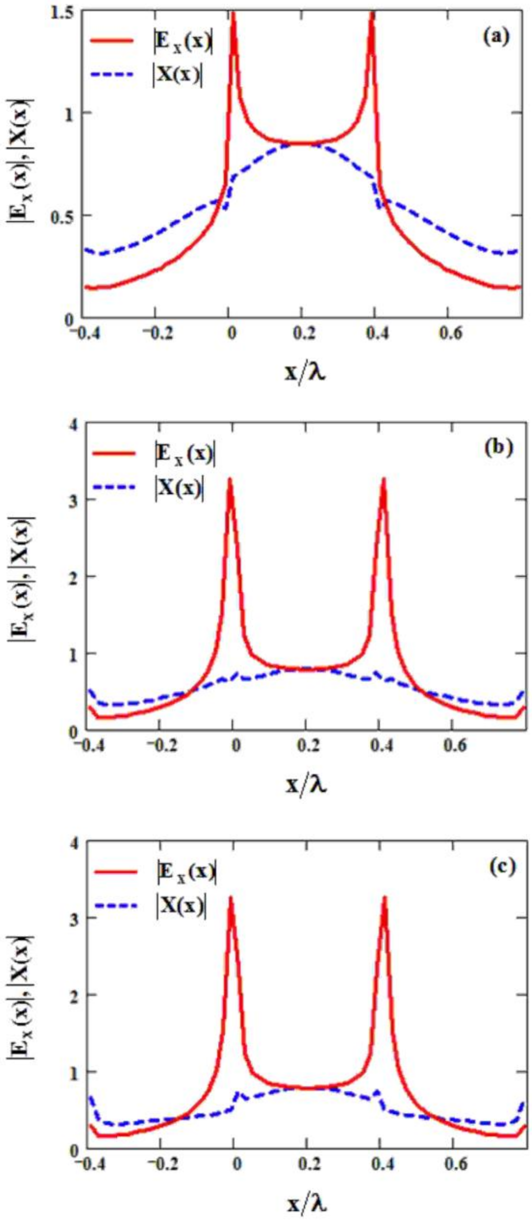

Figure 6. Solution of the integral equation, giving $|E_x(x)|$ (solid red curve) and $|X(x)|$ (dashed blue curve) for (a) $Z = -1i$, (b) $Z = -10i$, and (c) $Z = -100i$.

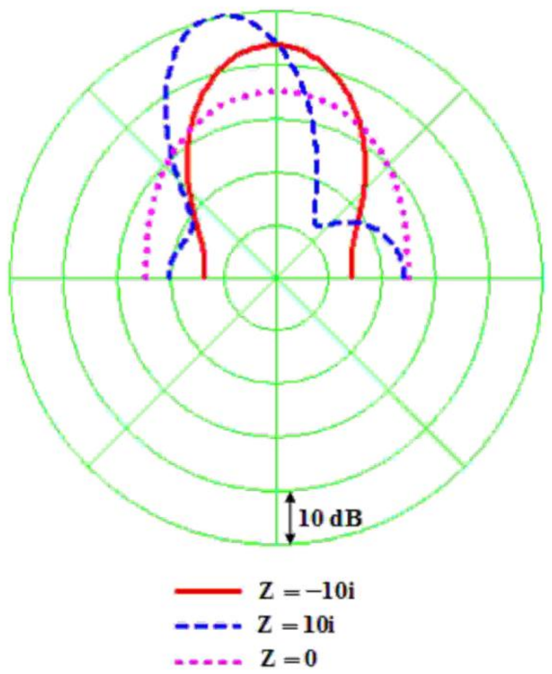

Figure 7. Radiation patterns of a single waveguide antenna for ideal conducting ($Z = 0$, dotted line), capacitive ($Z = -10i$, solid line), and inductive ($Z = 10i$, dashed line) flanges, respectively. The impact of the impedance part of the flanges on the radiation patterns is demonstrated with the solid and dashed lines. Parameters used are $a = 0.4\lambda$, $L = 0.4\lambda$.

III. Conclusion

In summary, we have studied the radiation of a parallel-plate waveguide antenna. We have calculated a strict solution to the problem of analysis of the electromagnetic field behaviour of a single antenna in the shape of an open end of a parallel-plate waveguide with an infinite flat impedance flanges. We have shown that to obtain a higher precision when solving the integral equation we need to use a large number of points across the opening of the waveguide, and also to take into account the particular features of the waveguide edges.

The electromagnetic field behaviour was investigated by using the impedance approach and the Krylov-Bogolyubov method, which is an iterative approximation to the required value. The choice of this method is explained by the fact that it is the most straightforward method and it allows us to obtain in analytical form the matrix of coefficients of the system of linear algebraic equations, thereby reducing the computational time.

Finally, we have also shown that a capacitive impedance distorts the wave by increasing the directivity of the antenna while, the inductive impedance leads to significant distortion of the radiation pattern and to a significant growth of lobes along the impedance structure. We also note that the study of the impedance influence on the electromagnetic field behaviour of a parallel-plate waveguide antenna is necessary for solving the problem of coupling between apertures antennas located on a common impedance surface.

References

[1] M.Yu. Zvezdina, Influence of impedance properties located on a mobile construction on antennas parameters, *Radio Electronics, (3)*, 2004, 25-29.

[2] Yu.V. Pimenov, V.I. Volman, and A.D. Muravtsov, *Technical Electrodynamics*, (Moscow: Radio and Communication Press, 2002).

[3] B.A. Michoustin, Radiation from the open end of waveguide cylinder with infinite flanges, *Transactions of Higher Education Institutions, Radio Physics*, Gorky (Russia), *8(6)*, 1965, 1178-1186.

[4] Y.S. Joe, J.-F. D. Essiben, and E.M. Cooney, Radiation Characteristics of Waveguide Antennas Located on the Same Impedance Plane, *Journal of Physics D.: Applied Physics*, *41(12)*, 2008, (125503) 1-11.

[5] J.-F. D. Essiben, E. R. Hedin, and Y. S. Joe**,** Radiation Characteristics of Antennas on the Reactive Impedance Surface of a Circular Cylinder Providing Reduced Coupling, *Journal of Electromagnetic Analysis and Applications, 2(4)*, 2010, 195-204.

[6] J.-F. D. Essiben, E. R. Hedin, Y.D. Kim, and Y.S. Joe, Electromagnetic Compatibility of Aperture Antennas Using Electromagnetic Band Gap Structure, *IET Microwaves, Antennas and Propagation, doi: 10.1049/iet-map.2012.0100, 6(9)*, 2012, 982-989.

[7] T. Cwik, Coupling into and scattering from cylindrical structures covered periodically with metallic patches, *IEEE Transactions on Antennas and Propagation, 38(2)*, 1990, 220-226.

[8] G.F. Zargano, L.V. Ptyalin, V.S. Mikhalevskiy, Y.M. Sinelnikov, G.P. Sinyavskiy, and I.M. Chekrygin, *Waveguides of Complicated Cuts*, (Moscow: Radio and Communication Press, 1986).

[9] O.N. Tereshin, V.M. Sedov, and A.F. Chaplin, *Synthesis of Antennas on Decelerating Structures*, (Moscow: Radio and Communication Press, 1980).