A Mathematical Modeling Approach of the Failure Analysis for the Real-Time Mexican Satellite Space Launch Center

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ABSTRACT
In this paper, a simulation of the Mathematical Model for Real-Time Satellite Launch Platform approach in Mexico is presented. Mexico holds the fourth best place in the world for building a platform to launch space satellites, since its geographic location is optimal for its construction. It is essential to have the Probabilistic Failure Analysis in Space Systems Engineering from its design, in order to minimize risks and avoid any possible catastrophe. The mathematical approach of Failure Analysis presented throughout this paper, is complementary to the simulation results, previously obtained with Windchill Quality Software. The final results were performed with the Failure Analysis through fault trees (FTA) by means of a probabilistic approach Quantitative Mathematical Model. This is the first step to propose and build the first Satellite Launch Platform in Mexico.

Keywords - Fault Tree Analysis (FTA), Real Time, Platform to Launch Space Satellites, Space Rockets, External factor, Quantitative data, Mathematical Approach.

I. INTRODUCTION
The Approximation of the Mathematical Model in Real-Time for the Satellite Launcher Platform in Mexico, and its real-time design, presents an opportunity to collaborate and create multinational technology that will benefit mankind in various fields such as telecommunications, earth observation, and ecosystems by preventing disasters caused both by global warming and the human actions. Few countries in the world have Platforms for Rocket Launching, and there is an opportunity for Mexico to participate in the space community with high-impact projects intend for launching geostationary satellite - originated and designed in Mexico- into orbit, as it was presented along the consultation forums for creating the Mexican Space Agency in 2010 and 2011. For over ten years in Europe, Russia and the United States, satellites have been involved in the exploration of space. Space exploration brings indirect benefits that positively affect the economy of those countries involved in the industry, and it is also a great source of innovation [23]. Satellites have several objectives, including telecommunications, and the Earth observation, as well as the absence of gravity that allows processing, manipulating and investigating different raw materials and pharmaceuticals, which are unattainable under normal gravity conditions on the Earth.

The proposed Mexican Spaceport consists of the Architecture and Engineering of the Launch Center Buildings, and it is based on the Master Plan for the construction of the Mexican Space Launch Platform [28]. The Spaceport includes the Space Center, the Satellite Launch Center, the Meteorological and Telemetry Complex, the Final Assembling Tower, the Takeoff Integration, the Propulsion Integration, the Rocket Launcher Platform, the Testing Center and the Dust Accelerator, the Liquid Hydrogen Production Plant, the Propellant Factory, the Restriction Zone, the takeoff Area, the Landing Airstrip, and the Future Technology Development Center [28]. A typical Space Launch Centre has services and supply lines, for example; the service structure provides access to the platform to analyze the launch, before the actual launching, and most structures can be rotated within a safe distance. The supply lines provide fuel, gas, energy and communication with the launcher, which is located on the launching pad, provided with a reflective structure, in order to avoid the rocket flames and to endure the intense heat and energy generated by the engines during Rocket launching [28].

The rockets are launched from a steel platform and concrete structures used to assemble all the components of the rocket. The Space Launching Platform is an assembly of concrete structures, where
the rocket is prepared with one or more metal leaning towers equipped with elevators and stories, which provide access to all components of the rocket in order to prepare the launching. After completing the launching state, the metal tower is removed to prevent damages by the gas spilled out of the rocket or the risk of producing an accidental explosion at the site.

These platforms are complex, sophisticated engineering structures, and very expensive to build. Planned missions are extremely precise and fragile, including the preparation of the mission, because there is a risk, while hazardous substances -necessary for a launch- are handled. The difficulty of managing these systems, involves the transfer of some low boiling point fluids, such as hydrogen and oxygen. Moreover, it is necessary to control the heat produced by the rocket before and after a launching.

II. GENERAL SYSTEM DESIGN

The proposed architecture for building concrete structures and buildings is original and was proposed along the Forums for creating the Mexican Space Agency [28] (2010-2011). The Architecture and the Full Design of hardware and software systems involved in the Satellite Launch Platform was presented at the 7th IAASS Conference in Friedrichshafen, Germany [30]. This work included the handling and monitoring of sensors, which provide Real-time valuable information to the operators, and that trigger alarms in case of an impending disaster or a dysfunctional behavior of the systems involved. In this paper, a detailed FTA failure analysis, with a quantitative probabilistic mathematical and analytical approach is presented.

2.1 General System

The Real-Time Design for the Satellite Launch Platform in Mexico [30], was designed using the design methodologies of Structured Analysis in Real Time (SA-RT) [3], and the design of the Software with LACATRE [4, 5]. The components of the Space Launch Platform are described in figure 1. Data and bits within the data communication module, will be sent to the Monitoring Center of Space Launch Systems, then, this information will be transmitted through a network that will work as an interface with the system in Real Time; which in turn, will function to monitor incoming events. Information managed from the launch pad is bidirectional, and the Monitoring Center of Space Launch Systems, receives information from those sensors located on target components and systems of the Rocket, and the Launch Platform, in order to monitor the physical variables of the systems and components, and thus, be able to prevent a catastrophe. Real-Time Monitoring systems of the physical variables on the monitored elements can be supported by mobile devices.

2.2 Context Diagram

The Context Diagram (figure 2), is designed with the Real-Time Structured Analysis (SA-RT) [3] and describes the overall system context. The complete systems Engineering is described in Mexican Real-Time Space Satellite Launch Center Failure Model, System Design and Failure Analysis [30]. The Mexican Space Launch Center wirelessly transmits data to the Data Transmission System which in turn, will wirelessly send signals to the Monitoring Center of the Space Launch System. While the system is working, all the information will be shown on a screen, and should there be an anomalous event or a hazardous activity recorded by the Critical Parameters, an alarm will be activated. The information of the signals is sent from the components being monitored, for example; following the components of the Soyuz rocket architecture: Stage 1, Stage 2, Stage 3, and the Escape Tower [26], as critical components of the Rocket, although they can be adapted to any type of rocket architecture. Platform Components and the rocket components will receive and send information bidirectional in Real Time, and for this reason, all systems are monitored by sensors. The platform includes components such as an Orbital system, Power System, the Service Module, and the Orbit Calibration System, which is responsible for monitoring the orbital launch in the proper position, in order to avoid launching errors caused by launch failures and to adjust the positioning of the satellite to the correct orbit after launch. Some launching failures are currently observed in some rockets, such as the Galileo, where it was positioned in the wrong orbit, despite having been launched successfully [10, 11].
III. RISK ANALYSIS


3.1 Fault Model proposed: FTA Fault Tree

The Fault Analysis for critical systems is performed by the mathematical Fault Trees method, and it describes the behavior of possible faults on the Mexican Satellite Launch Platform. The Fault Tree method constructs diagrams logically interconnected by means of AND & OR gates, to find combinations of components failures, including the minimal cut sets, describing the minimum combinations of component failures for the TOP catastrophic event occurs. This is a deductive method and; in addition to the TOP event, it has intermediate events, and initial base events, where system faults begin [12, 16].

The full Fault Tree proposed, has the TOP event as the highest risk, dangerous event, describing the catastrophic event; the simulation of probabilistic failures is carried out with the Windchill Quality Solutions Software [19], and is represented in sub-trees, seeking to find the basic probability of the initial event, considering the failure probability of all the components in the Mexican Launching Pad for Space Satellites.

Complete Fault Tree, determines the following components faults which in turn are divided into sub-trees, all with their respective sub-systems in Real-Time, are shown below:

**TOP event:** Explosion, Take Off and Trajectory.

**The Rocket failure:** The scape tower failure, the stage 1, stage 2, stage 3 failures, the Oxygen Tank and Kerosene Tank Failure, the Propellant F1, F2, F3 failure, the Liquid Oxygen Tank failure, the Radiator Panel failure, the Launch Engine failure, the Brake Chute failure and the Sensor Adjustment Control failure.

**The platform Failure:** the Power System failure, the Service Module failure, the Orbital System failure and the Real-Time Calibration System Failure (to measure and calibrate an erroneous orbit).

**The External Factor failure:** the Human Factor failure and the Natural Factor failure.

The Top equation of this general Fault Tree for Mexican Space Satellite Launching Platform, based on initial events. Subsequently, the simulation and simplification of the mathematical expression is developed. It is represented by (1):

\[
TOP = \{(A*B*(C*D*E)) * (F*(G*H)*(I*J) * (K*(L*M*N)*O*(P*Q*))) * ((R*S)*T)\} * [(U*V)*(W*X)*(Y*Z)*(A'*B') \]

\[
[C' + (D'*F'*G')] \]

\[ \text{(1)} \]

The initial events of the Rocket Sub-Fault Tree, are as follows: \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T\}; The initial events for the Platform Sub-Tree events are: \{U, V, W, X, Y, Z, A’, B’\} and the initial events of the Human Factor Sub-Tree, are as follows: \{C’, D’, F’, G\}. All events represent the Complete system and its Fault Simulation Model.

IV. FTA SIMULATION RESULTS

The final model has several states, but the semantics of the Mexican System Platform for Space Satellite Launch Center is respected. The failure of any initial system event is not allowed, as all the systems must function properly at the same time. If there were a failure on any initial event, the
The whole system is divided into three Fault Trees, with a different probability of occurrence for each component. This means, for example, that the probability of failure is different for the Rocket, for the platform, and for the external factor.

For example, the Rocket has a 0.4 probability of occurrence of a fault, the launch pad has 0.4 probability of having a Critical failure, and the external factor, as well as the human factor, including an event related to failures of the Environment or unsuitable conditions has another 0.2. So, if there is a fault in any of these items it will be a cause to abort the mission.

The Fault Tree 1, the Fault Tree 2 and the Fault Tree 3 of Figures 4, 6, 8, and 10 are individual elements of the complete Fault Tree to obtain the TOP event, and it is divided in order to understand the proper functioning of the overall system. If the mission will succeed or not depend on the Platform Systems, the environment, the components that function by means of the Real-time monitoring system via sensors, and by the Orbital positioning System.

The following data are the result of the simulation of the first two branches of the Rocket Fault Tree. Both figure 3 and figure 5 represent the same Rocket Fault Tree.

The information in figure 6, is the result of the second part of the Fault Tree Rocket.

The Different Fault states of the General Fault Tree have the following initial events:


They all represent the initial event of faults with different distribution and probability of occurrence of events. It is more likely that the Rocket or the Space Launching Pad have failures, triggered by the malfunction of any component, than the probability of occurrence. For example, a fault related to the external factor or a failure of natural hazard.

The probability of occurrence of an external factor such as the weather, or a human error are theoretically lower; however, they must be taken into account as part of the General System. Similarly, satellites must be positioned in the correct orbit; otherwise, the mission will have failed although the launch has been successful. The Fault Tree Platform is shown in Figure 7.
The simulation results from the fault analysis of the Launch Pad are displayed with data in figure 8.

The system must interact with the hardware systems; such as the sensors, and of the software; such as Real-Time Threads. For this reason, the Orbit System has a portion of the monitoring Hardware (including infrared sensors of Real-time monitoring) and Software developed in order to detect whether any orbital position is at risk of failure. If the system state changes, the Software must be able to trigger an Orbital Positioning fault, or else, to have the failure visible on the Monitoring system of the Mission in progress.

The simulation results for this part of the tree, and of all the FTA analysis is particularly important, because it includes the Human Factor, and the Environment Factor that have produced some of the most important accidents in Space history such as that of the Challenger, where official reports [21] suggest that climate and human interaction contributed greatly to the failure of the mission. The Fault Tree for the External Factor (figure 10) and the simulation results are presented in figure 9.

The probability of occurrence for each initial event is shown in figure 10. This is the result obtained for each FTA Sub-Tree analysis using Windchill Quality Software [19].

The Rocket initial events and their probability are represented in blue color; the platform is displayed in green; and the external factor is represented in yellow (figure 11). Note that the probability of occurrence $p(C') = 0.1$ represents the Human Factor failure of the (OR gate) so, it is critical to minimize the Human Risk to reduce the historical disasters previously described [21].
If the Human Factor is determined, as part of the system (as though it is another physical component of the system with an AND gate) then you have the chance of Failure, including the Rocket (R) \( p(R) = 6.69142E-36 \). The platform (P) \( p(P) = 3.90625E-11 \) and the External Factor (EF) \( p(EF) = 3.6926E-06 \), based on the initial events probabilistic of figure 12, and from the results of the Fault Tree simulation. Then, the Final complete probability of the Mexican Satellites Launching Center (SLCF) which is obtained by multiplying the failure probabilities of the rocket, the Platform, and the External Factor: \( p(SLCF) = p(R) \cdot p(P) \cdot p(EF) \) where \( p(SLCF) = p(6.69142E-36) \cdot p(3.90625E-11) \cdot p(3.6926E-06) = 9.65186E-52 \). The mathematical approach is proposed and demonstrated in section V.

V. MATHEMATICAL MODEL APPROACH

The approach of a launch model with Fault Analysis for Real-Time positioning of satellites has the TOP event of the Equation (1). However, this expression can be simplified to the following equations (2) and (3) of TOP and Reliability respectively:

\[
\text{TOP} = \text{Rocket} \cdot \text{P (platform)} \cdot \text{P (external factor)} = [f(c_1) \cdot f(c_2)] \cdot f(p) \cdot f(fe) = 1 \quad (2)
\]

\[
R(S) = 1 - U \quad (3)
\]

Given that the main event is expressed as the union of minimal cut sets, the probability of the TOP event can be approximated as the multiplication and addition of a cut that establishes the individual probabilities, provided that these probabilities are small [31].

The odds of all basic events are:

\[
P(A)=0.0333, \quad p(B)=0.0333, \quad p(C)=0.0111,
\]

\[
p(D)=0.0111, \quad p(E)=0.0111, \quad p(F)=0.0333,
\]

\[
p(G)=0.0166, \quad p(H)=0.0166, \quad p(I)=0.0166,
\]

\[
p(J)=0.0166, \quad p(K)=0.025, \quad p(L)=0.0083,
\]

\[
p(M)=0.0083, \quad p(N)=0.0083, \quad p(O)=0.025,
\]

\[
p(P)=0.0125, \quad p(Q)=0.0125, \quad p(R)= 0.025, \quad p(S)=0.025,
\]

\[
p(T)=0.05, \quad p(U)=0.05, \quad p(V)=0.05, \quad p(W)=0.05,
\]

\[
p(X)=0.05, \quad p(Y)=0.05, \quad p(Z)=0.05, \quad p(A')=0.05,
\]

\[
p(B')=0.05, \quad p(C')=0.01, \quad p(D')=0.0333,
\]

\[
p(E')=0.0333, \quad p(F')=0.0333.
\]

The development of the equation, by applying the associative law is as follows:

Rocket (part 1)

\[
F (c_1) = AB*(C*D*E)*(F*GH*IJ)
\]

\[
F (c_1) = (ABC*ABD*ABE)*(FGH*FIJ)
\]

\[
F (c_1) = ABCFGH*ABCFIJ*ABDFGH * ABDFIJ * ABEFGH*ABEFJI
\]

Rocket (part 2)

\[
F (c_2) = (K*L*M*N*O*P*Q*O*(R*S)*T)
\]

\[
F (c_2) = KL*KM*KN*(OPR*OPS*OQR*OQS)*T
\]

\[
F (c_2) = KL*KM*KN*(TOPR*TOPS*TOQR*TOQS)\]

\[
F (c_2) = KLTOPR* KLTOPS* KLTOQR* KLTOQS* KLOPTPR* KLOPTQS* KLOPTQR* KLOPTQS
\]

\[
F (p) = (U*V)*(W*X)*(Y*Z)*(A*B)
\]

\[
F (p) = UV*(W*X)*(Y*Z)*(A*B)
\]

\[
F (p) = UVW*UVX*(Y*Z)*(A*B)
\]

\[
F (p) = UVWY*UVWZ*UVXY*UVXZ*(A*B)
\]

\[
F (p) = (UVWY*UVWZ*UVXY*UVXZ*UVWA)* UVWB*UVX*UVZA* UVWB*UVXYB*UVXZB*
\]

\[
F (p) = (C'D'*C'E'*C'F')=0.00100111.
\]

\[
F (p) = (D'*E'*F')=C'D'*C'E'*C'F'=0.00100111.
\]

The original equation (1) is modified; because the variables corresponding to each sub-system are applied, in addition, the associative logic law [32] is used.

\[
\text{TOP}= (ABC*ABD*ABE)*(FGH*FIJ)* KL*KM* KN*(TOPR*TOPS*TOQR*TOQS)* UVW*UVX*(Y*Z)*(A'B)*(C'*(D'*E'*F'))
\]

(4)

Each product represents a minimum cut, established as follows:

\[
\text{TOP}= ABCFGH * ABCFIJ * ABDGFH * ABDFIJ * ABDFIJ * ABEFGH * ABEFIJ * KLOPTR * KLOPTPS * KLOPTQR * KLOPTQS * KLOPTPR * KLOPTQS * KLOPTQR * KLOPTQS * KLOPTQR * KLOPTQS
\]

(5)

System reliability is calculated using equation (3), conducting the operations of all minimum cut established.

\[
R (S) = 1 - \text{equation (5)}
\]

(6)

When solving equation (5) it becomes:

\[
\text{TOP}= 1.12946E-10 * 1.12946E-10 * 1.12946E-10 * 1.12946E-10 * 1.12946E-10 * 1.12946E-10 * 1.12946E-10
\]

(7)

5.234

(8)

0.00333 * 0.00333

(9)

0.00333 * 0.00333

(9)

0.00333 * 0.00333

(9)

0.00333 * 0.00333

(9)

So the end result is as follows:

\[
\text{TOP}=1.7941E-234
\]

(8)

Applying the mathematical expression (3) we obtain the following reliability:

\[
R_S=1-1.7941E-234
\]

(9)

\[
R_S= 1
\]

(10)

For obtaining the result of system reliability, it is multiplied by 100, for presenting the result on this scale, we have the following:
RS = (value obtained reliability) * (scale to be presented)  \hspace{1cm} (11) 
RS=1*100  \hspace{1cm} (12) 
RS = 100 %  \hspace{1cm} (13)

Figure 12, graphically shows the difference of probability of reliability, so you can observe that the numerical difference is very small (0.00036926%), it should be noted that on Launch Systems, such a value can mean a catastrophic malfunction, which generates countless losses, significantly affecting the mission to be develop.

![Graph showing probability of reliability](image)

Figure 12. Probability of percentage of reliability

While our country does not have its own Space Launch Center, the commitment of Universities, Research Centers and Agencies (public and private) to generate contributions that provide solutions to the problems related to the space sector is great. Through this work, the first research approach of a model based on the Fault trees of Space Systems is presented. This model is presented in a general way, allowing its application to any type of space mission, for example; the recently occurred event to the Geospatial Navigation System Galileo (22 August 2014), which, by having been positioned in the wrong orbit, caused the failure of the mission; and the explosion of the Cygnus spacecraft after takeoff, in the NASA facilities (October 29, 2014) [33].

Via this the design of a Fault System approach, which integrates the human factor; the reliability of the launching system is much higher. Based on the events and / or disasters occurred recently, you can recognize the importance of including the external factors within the events likely to occur, which can be considered to have a minimal probability of occurrence.

VI. CONCLUSION

The Risk Analysis, including the approach of the Mathematical Model and the Risk Analysis with FTA, the results of simulation, the system design, and the related work is the first step to perform the Full Risk Analysis intended to avoid failures and catastrophes on the future Mexican Platform to Launch Space Geostationary Satellites. The mathematical model shows that it is essential to have all the possible elements for a proper failure analysis, and not allow history, through human or environmental errors, be the cause of disasters which could have been avoided. This is the first step towards the actual construction of this major project.

This article presents the first approximation of the Mathematical Model of Faults, where all the Space designs and Systems designs, as well as of the Architecture of the complex previously submitted, must work together in order to accomplish this project, thus, obtaining the benefits of those countries that possess Space Launch Centers, same that have an important activity in the World Space Community by launching geostationary satellites for benefiting mankind.

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