RESEARCH ARTICLE OPEN ACCESS

Onset of Convection in a Nanofluid Saturated Porous Layer with Temperature Dependent Viscosity

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ABSTRACT
The effect of nanofluid viscosity varying exponentially with temperature on the onset of convection in a layer of nanofluid saturated Darcy porous medium is investigated. The nanoparticle flux is zero condition on the boundaries is invoked to account for physically realistic situation. The resulting eigenvalue problem is solved numerically using the Galerkin method. It is observed that the instability sets in only as stationary convection and the occurrence of oscillatory convection is ruled out. The effect of viscosity parameter on the characteristics of stability is found to be significant and dual in nature. The onset of convection is hastened and the size of convection cells is enlarged with an increase in the value of modified diffusivity ratio, concentration Darcy-Rayleigh number, the modified particle density increment parameter and the Lewis number.
Keywords: nanofluid, porous medium, variable viscosity, Galerkin method, thermal convection

I. Introduction

Nanofluids are colloidal suspensions of nanometer-sized particles in base fluids that show great potential applications in many fields. The developments in this field are reviewed by several authors and well documented in the literature (Kakac and Pramuanjaroenkij [1], Yu and Xie [2], Goharshadi et al. [3]). Nanofluids are found to have high thermal conductivity at very low nanoparticles concentration and considerable enhancement of convective heat transfer. As a result, buoyancy driven convection in a nanofluid layer has been studied extensively (Kim et al. [4] and references therein). Its counterpart in a nanofluid saturated horizontal layer of porous medium has also attracted equal importance in the literature because of its importance in many fields of modern science, engineering and technology, chemical and nuclear industries and bio-mechanics. The problem was first considered by Nield and Kuznetsov [5] and following this formalism several studies have been undertaken subsequently to investigate various additional effects on the problem by the same authors and others. The details can be found in the monograph of Nield and Bejan [6].

It is imperative to note that thermal convective instability in nanofluids is affected significantly by their properties and specifically by nanofluid viscosity and thermal conductivity. Copious literature available on thermal convection in nanofluids is mainly concerned with the heat transfer properties such as heat transfer enhancement, thermal conductivity measurement, improvement and enhancement, estimation of thermal conductivity and so on [7-11]. Nonetheless, the viscosity is an important property to be considered for all thermal applications involving nanofluids as it describes the internal resistance of a fluid to the flow. Moreover, the pumping power and convective heat transfer coefficient are influenced by viscosity. Hence, viscosity is as important as thermal conductivity in engineering systems involving nanofluid flow. Experiments have been conducted extensively in the past to measure the effect of temperature on nanofluid viscosity and it is observed that the viscosity decreases with increasing temperature. In particular, it has been shown that viscosity of CuO/EG-water nanofluid, CuO/PG-water nanofluid, Al2O3/EG-water nanofluid and SiO2/EG-water nanofluid decreases exponentially with the increasing temperature in different temperature ranges [12-14]. Mahbubul et al. [15] have thoroughly compiled and reviewed different characteristics of viscosity of nanofluids.

Under the circumstances, considering variation in viscosity of the nanofluid with temperature becomes equally important in order to predict the correct contribution of nanoparticles on thermal convective instability in a nanofluid saturated porous layer. Besides, all the studies on the onset of convection in nanofluid porous layer are based on Buongiorno [7] model and it is assumed that the value of the nanoparticle fraction at the boundaries can be prescribed in the same way as the temperature.

But the assumption of boundary conditions controlling the nanoparticle volume fraction on the boundaries may be difficult in practice and they are to be amended. Recently, Nield and Kuznetsov [16] have commented on the authenticity of these boundary conditions and argued that vanishing of nanoparticle flux condition on the boundaries is more appropriate to account for physically realistic situation.

The intent of the present study is, therefore, to investigate the onset of convection in a nanofluid saturated layer of porous medium by considering nanofluid viscosity depending exponentially on temperature and imposing physically realistic boundary conditions on nanoparticle volume fraction, that is nanoparticle flux is zero on the boundaries. The eigenvalue problem is solved numerically using higher order Galerkin method to obtain more accurate results and also to account for the influence of all the parameters on the onset of convection. A detailed parametric study is under taken and observed that oscillatory convection is not a preferred mode of instability.

II. Mathematical formulation

We consider a nanofluid saturated horizontal porous layer of thickness d as shown in Fig. 1. The lower and upper surfaces are held at constant but different temperatures T_0 and T_1 ($<T_0$), respectively and the model considered incorporates the effects of both Brownian motion and thermophoresis. A Cartesian coordinate system (x,y,z) is chosen such that the origin is at the bottom of the porous layer and the z-axis vertically upward in the presence of gravitational field.

The viscosity of the nanofluid μ is assumed to vary exponentially with temperature in the form

$$\mu=\mu_0 \exp\left[-\eta\left(T-T_a\right)\right] \tag{1}$$

where, μ_0, η are positive constants and $T_a=(T_0+T_1)/2$ is the average temperature.

The governing equations are:

$$\nabla\cdot\vec{q}=0 \tag{2}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial\vec{q}}{\partial t}=-\nabla p+\rho\vec{g}-\frac{\mu}{K}\vec{q} \tag{3}$$

$$\begin{aligned}(\rho c)_m\frac{\partial T}{\partial t}+(\rho c)_f\left(\vec{q}\cdot\nabla\right)T=k\nabla^2 T\\ +\varepsilon(\rho c)_p\left[D_B\nabla\phi\cdot\nabla T+\frac{D_T}{T_a}\nabla T\cdot\nabla T\right]\end{aligned} \tag{4}$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}\cdot\nabla\right)\phi=D_B\nabla^2\phi+\frac{D_T}{T_a}\nabla^2 T \tag{5}$$

$$\rho=\phi\rho_p+(1-\phi)\rho_f\left\{1-\beta\left(T-T_a\right)\right\} \tag{6}$$

where, $\vec{q}=(u,v,w)$ the velocity vector, p the pressure, ρ the overall density of the nanofluid, ρ_p the density of nanoparticles, ρ_f the density of base fluid, $\vec{g}$ the gravitational acceleration, ϕ the nanoparticle volume fraction, K the permeability of the porous medium, ε the porosity of the porous medium, T the temperature of the nanofluid, β the coefficient of thermal expansion, c the specific heat, k the thermal conductivity, D_B the Brownian diffusion coefficient and D_T the thermophoretic diffusion coefficient.

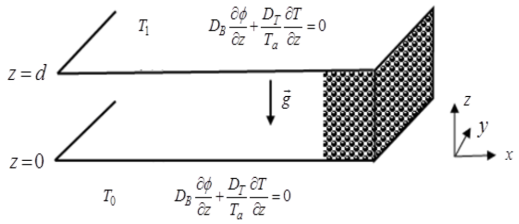

Fig. 1: Physical configuration

The basic state is quiescent and in the dimensionless form it is found to be

$$T_b=1-2z\,,\ \phi_b=\phi_0+N_A z \tag{7}$$

where, $N_A=D_T\left(T_0-T_1\right)/\phi_0 D_B T_a$ is the modified diffusivity ratio and ϕ_0 is the reference value of nanoparticle volume fraction. The pressure is of no consequence here as it will be eliminated subsequently.

To study the stability of the system, the basic state is perturbed in the form $\vec{q}=\vec{q}\,'$, $p=p(z)+p'$, $T=T_b+T'$ and $\phi=\phi_b+\phi'$, where $\vec{q}\,'$, p', T' and ϕ' are assumed to be small compared to their equilibrium counterparts. Following the standard linear stability analysis procedure as outlined in [16], the dimensionless stability equations can then be shown to be

$$\begin{aligned}\frac{\omega}{Pr}\left(D^2-a^2\right)W=\\ -f(z)\left[\Gamma D+\left(D^2-a^2\right)\right]W\\ -(1/2)R_t\,a^2\,\Theta+R_n\,a^2\,\Phi\end{aligned} \tag{8}$$

$$M\omega\Theta=2W+\left(D^2-a^2\right)\Theta-\frac{2N_B}{Le}D\Phi \tag{9}$$

$$\omega\Phi = -N_A W + \frac{N_A}{2Le}\left(D^2 - a^2\right)\Theta + \frac{1}{Le}\left(D^2 - a^2\right)\Phi \tag{10}$$

where, $D = d/dz$, $\omega(=\omega_r + i\,\omega_i)$ is the growth rate, $a = \sqrt{l^2 + m^2}$ is the overall horizontal wave number, W is the amplitude of the perturbed vertical velocity, Θ is the amplitude of perturbed temperature, Φ is the amplitude of perturbed nanoparticle volume fraction and

$$f(z) = \exp\left[-\Gamma\left(1/2 - z\right)\right] \tag{11}$$

In the above equations, $R_t = dK\beta\rho_f\,(T_0 - T_1)\,g/\mu_0\kappa$ is the thermal Darcy - Rayleigh number, $R_n = Kd\left(\rho_p - \rho_f\right)g\phi_0/\mu_0\kappa$ is the concentration Darcy - Rayleigh number, $Le = \kappa/\varepsilon D_B$ is the Lewis number, $N_B = (\rho c)_p\,\phi_0/(\rho c)_f$ is the modified particle density increment, $\Gamma = \eta\Delta T$ is the viscosity parameter, $Pr = \mu_0\varepsilon^2 d^2/\rho_f\kappa K$ is the Darcy-Prandtl number and $M = (\sigma/\varepsilon)$ is the heat capacity ratio.
The isothermal boundaries are impermeable and the nanoparticle flux is zero on the boundaries. The appropriate boundary conditions are:

$$W = 0,\quad \Theta = 0,\quad D\Phi + N_A D\Theta = 0 \quad \text{at } z = 0,1. \tag{12}$$

III. Numerical solution

Equations (8) - (10) constitute an eigenvalue problem and solved numerically by the Galerkin method. The variables are written in a series of basis functions as

$$W(z) = \sum_{i=1}^{N} A_i W_i(z),\quad \Theta(z) = \sum_{i=1}^{N} B_i\Theta_i(z),\quad \Phi(z) = \sum_{i=1}^{N} C_i\Phi_i(z) \tag{13}$$

where, A_i, B_i and C_i are unknown coefficients. The basis functions are represented by the power series satisfying the respective boundary conditions

$$W_i(z) = \Theta_i(z) = z^i(1-z),\ i = 1,2,3,4...$$
$$\Phi_1(z) = N_A\left(z^2 - z\right), \tag{14}$$
$$\Phi_i(z) = N_A(z^i/i) \text{ for } i = 2,3,4...$$

Multiplying Eq.(8) by $W_j(z)$, Eq.(9) by $\Theta_j(z)$ and Eq.(10) by $\Phi_j(z)$; performing the integration by parts with respect to z between $z=0$ and 1, and using the boundary conditions, we obtain the following system of algebraic equations:

$$\begin{aligned} A_i C_{ji} + B_i D_{ji} + C_i E_{ji} &= \omega A_i F_{ji} \\ A_i G_{ji} + B_i H_{ji} + C_i I_{ji} &= \omega B_i J_{ji} \\ A_i K_{ji} + B_i L_{ji} + C_i M_{ji} &= \omega C_i N_{ji}. \end{aligned} \tag{15}$$

The coefficients $C_{ji} - N_{ji}$ involve the inner products of the basis functions and are given by

$$C_{ji} = -\left\langle f(z)W_j D^2 W_i\right\rangle + a^2\left\langle f(z)W_j W_i\right\rangle - \Gamma\left\langle f(z)W_j DW_i\right\rangle$$
$$D_{ji} = -(1/2)a^2 Rt\left\langle W_j\Theta_i\right\rangle,\ E_{ji} = a^2 Rn\left\langle W_j\Phi_i\right\rangle$$
$$F_{ji} = -\frac{1}{\Pr}\left\{\left\langle DW_j DW_i\right\rangle + a^2\left\langle W_j W_i\right\rangle\right\},$$
$$G_{ji} = 2\left\langle\Theta_j W_i\right\rangle,\ H_{ji} = -\left\langle D\Theta_j D\Theta_i\right\rangle - a^2\left\langle\Theta_j\Theta_i\right\rangle,$$
$$I_{ji} = -\frac{2N_B}{Le}\left\langle\Theta_j D\Phi_i\right\rangle,\ J_{ji} = M\left\langle\Theta_j\Theta_i\right\rangle,$$
$$K_{ji} = -N_A\left\langle\Phi_j W_i\right\rangle,$$
$$L_{ji} = -\frac{N_A}{2Le}\left\{\left\langle D\Phi_j D\Theta_i\right\rangle + a^2\left\langle\Phi_j\Theta_i\right\rangle\right\},$$
$$M_{ji} = -\frac{1}{Le}\left\{\left\langle D\Phi_j D\Phi_i\right\rangle + a^2\left\langle\Phi_j\Phi_i\right\rangle\right\},$$
$$N_{ji} = \left\langle\Phi_j\Phi_i\right\rangle.$$

The system of equations given by Eq. (15) is a generalized eigenvalue problem which can be written in the form

$$\Delta_1 X = \omega\Delta_2 X \tag{16}$$

where, Δ_1 and Δ_2 are real matrices of order $N \times N$ and X is the eigenvector. By using the subroutine GVLRG of the IMSL library, the complex eigenvalue ω is determined when the other parameters are specified. Then one of the parameters, say R_t, is varied until the real part of ω vanishes. The zero crossing of real part of ω is achieved by Newton's method for fixed point determination. The corresponding value of R_t and a are the critical conditions for neutral stability. Then the critical Rayleigh number with respect to the wave number is calculated using the golden section search method.

The imaginary part of ω indicates whether the instability onsets into steady convection or into growing oscillations.

IV. Results and discussion

The effect of exponential variation in nanofluid viscosity with temperature on the onset of convection in a nanofluid saturated Darcy porous medium is investigated by considering physically more realistic boundary conditions for the nanoparticle fraction. The resulting eigenvalue problem is solved numerically by employing Galerkin method. The parametric values vary with the base fluid and nanoparticles chosen. The ratio of density of the nanoparticles to that of a base fluid for Cu (copper) and Ag (silver) is 8.96 and 10.5, respectively. The ratio of heat capacity based on the volume fraction of nanoparticles to that of a base fluid is 0.83 for Cu and 0.59 for Ag. For such nanofluids, R_n turns out to be of the order $1\sim10$ and N_B of the order $10^{-3}\sim10^{-1}$. The value of modified diffusivity ratio is not more than 10 and Le is taken in the order of $1\sim 10$.

Table 1: Comparison of R_{tc} for different orders of approximations in the Galerkin expansion for $N_A=2,\ R_n=2,\ N_B=0.059$ and $Le=1$

N	R_{tc}			
	$\Gamma=0$	$\Gamma=3$	$\Gamma=5$	$\Gamma=8$
1	32.0000	50.2214	91.8729	267.3621
2	30.1340	30.1562	33.8914	53.1522
3	29.7876	28.2585	23.6465	19.2704
4	29.7004	28.1578	22.3523	11.0813
5	29.6998	28.0979	22.2734	9.1550
6	29.6998	28.0914	22.2237	8.9022
7		28.0912	22.2104	8.8728
8		28.0912	22.2092	8.8448
9			22.2092	8.8343
10				8.8343

It is observed that oscillatory convection is not a preferred mode of instability for all the parametric values chosen. To have a check on the accuracy of the numerical procedure adopted, it is instructive to look at the results for various levels of the Galerkin approximation to know the process of convergence and also the accuracy of the results. Table 1 shows the numerically computed values of critical Darcy-Rayleigh number R_{tc} for various values of viscosity parameter Γ and for different orders of approximations in the Galerkin expansion when $N_A=2$, $R_n=2$, $Le=1$ and $N_B=0.059$. From the tabulated values, it is clear that the convergence of the results crucially depends on the value of Γ. More number of terms in the Galerkin expansion is required to get the convergent results with increasing Γ. When $\Gamma=0=N_A$, the well-known value of the critical Darcy-Rayleigh number 39.4784 for ordinary fluids is recovered.

Figures 2 (a, b, c, d) show the neutral stability curves in the (R_t, a)-plane for different values of N_A (Fig. 2a), R_n (Fig. 2b), Le (Fig. 2c) and Γ (Fig. 2d). The neutral curves are similar to those observed in the Darcy-Bénard problem for ordinary viscous fluids and exhibit single minimum with respect to the wave number. It is observed that an increase in the value of N_A, R_n, Le and Γ is to decrease the region of stability and hence their effect is to reinforce instability on the system. In Fig. 2(a), the result for $N_A=0$ corresponds to that of regular fluid.

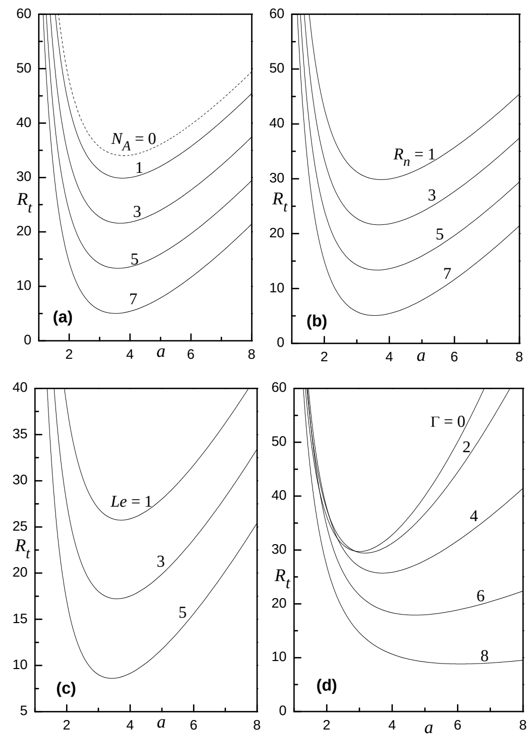

Fig. 2: Neutral curves for different values of (a) N_A with $R_n=2,\ N_B=0.059,\ Le=1$ and $\Gamma=4$ (b) R_n with $N_A=2,\ N_B=0.059,\ Le=1,\ \Gamma=4$ (c) Le with $R_n=2,\ N_A=2,\ N_B=0.059,\ \Gamma=4$ (d) Γ with $Le=1$, $R_n=1$, $N_A=4,\ N_B=0.059$

The influence of various physical parameters on the onset of convection is exhibited in Figs. 3-6. The variations of critical Darcy-Rayleigh number R_{tc} and the corresponding critical wave number a_c as a function of Γ for different values of N_A are shown in Figs. 3(a) and (b), respectively for $Le = 1$, $N_B = 0.059$ and $R_n = 2$. From the figures it is evident that increase in the value of N_A is to decrease R_{tc} and increase a_c.

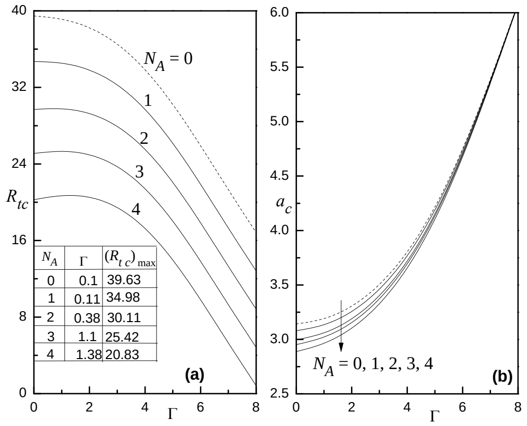

Fig. 3: Variation of (a) R_{tc}, (b) a_c with Γ for different values of N_A when $Le = 1$, $N_B = 0.059$ and $R_n = 2$

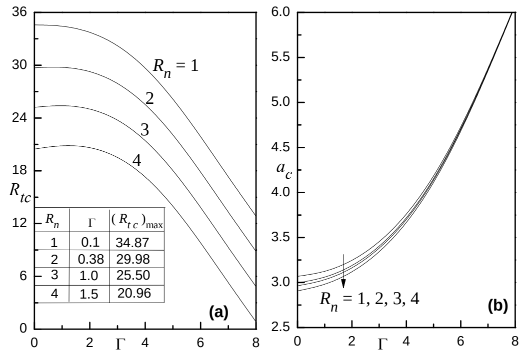

Fig. 4: Variation of (a) R_{tc} (b) a_c with Γ for different values of R_n when $Le = 1$, $N_A = 2$ and $N_B = 0.059$.

Hence, the effect of increasing N_A is to reinforce instability on the system and also to enlarge the size of convection cells. The result for $N_A = 0$ corresponds to regular fluids and it is observed that higher heating is required to instil instability in this case indicating the presence of nanoparticles is to augment heat transfer and to hasten the onset of convection due to diffusion of nanoparticles by thermophoresis and Brownian motion. Although thermophoresis and Brownian motion are responsible for the motion of nanoparticles in the base fluid, it is observed that thermophoresis is more dominating effect to initiate the diffusion of nanoparticles and hence increase in the value of N_A is to hasten the onset of convection. The results obtained for different values of concentration Darcy-Rayleigh number R_n for $Le = 1$, $N_A = 2$ and $N_B = 0.059$ exhibit similar behaviour on the stability characteristics of the system and the same is evident from Figs. 4(a) and (b). That is, increase in the nanoparticle concentration in the base fluid is to hasten the onset of convection.

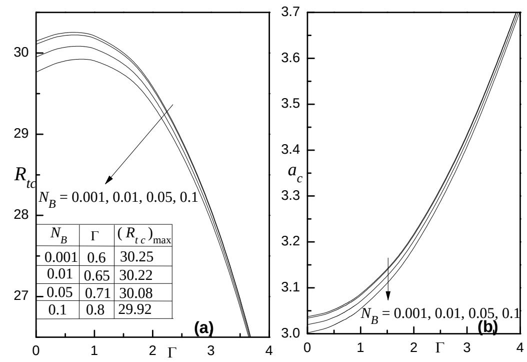

Fig. 5: Variation of (a) R_{tc} (b) a_c with Γ for different values of N_B when $N_A = 2$, $R_n = 2$ and $Le = 1$

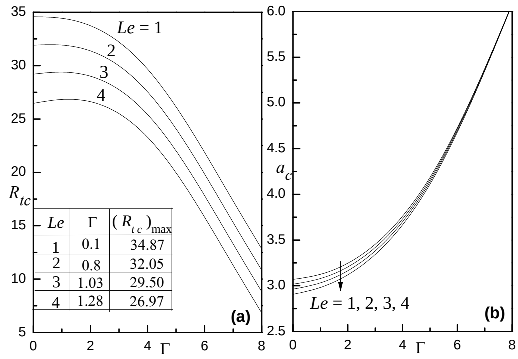

Fig. 6: Variation of (a) R_{tc} (b) a_c with Γ for different values of Le when $N_B = 0.059$, $N_A = 2$ and $R_n = 1$

The modified particle density increment parameter N_B does influence the stability characteristics of the system with higher number of terms in the Galerkin expansion. Its effect is found to be missing if single term is considered in the Galerkin expansion. This suggests the limitation of single term Galerkin method in studying the problem. The effect of increasing N_B is to hasten the onset of convection and also to increase the size of convection cells and this is evident from Figs. 5(a) and (b), respectively. The results shown in these figures are for $N_A = 2$, $R_n = 2$ and $Le = 1$. Figures 6(a) and (b) represent the variation of R_{tc} and a_c as a function of Γ for different values of *Le*, respectively when $N_B = 0.059$, $N_A = 2$ and $R_n = 1$. From the figures it is noted that the effect of increasing Le is to reinforce instability on the system and also to enlarge the size of convection cells.

A closer inspection of the above figures also reveals that the viscosity parameter Γ has a strong influence on the onset of convection in a nanofluid saturated porous layer. For different values of N_A (Fig. 3a), R_n (Fig. 4a), N_B (Fig. 5a) and Le (Fig. 6a), two regions are distinguished. With increasing Γ, R_{tc} increases initially up to a certain peak value, depending on the parametric values, and then decreases rapidly. The values of Γ at which R_{tc} attains its maximum value $(R_{tc})_{\max}$ are computed numerically and shown in the respective figures for the parametric values considered. It is seen that $(R_{tc})_{\max}$ decreases while Γ increases with increasing N_A, R_n, N_B and Le. At the maximum value of R_{tc}, a sublayer starts to form and once the viscosity parameter is further increased the thickness of the sublayer as well as R_{tc} are reduced. Thus Γ displays a dual role on the onset of convection.

V. Conclusions

The influence of nanofluid viscosity varying exponentially with temperature on the onset of convection is investigated. The eigenvalue problem is solved numerically using the Galerkin method by considering vanishing of nanoparticle flux boundary conditions at the boundaries. The onset of convection is found to occur only via stationary mode and the characteristics of stability of the system are strongly dependent on the viscosity parameter Γ. The effect of increasing concentration Darcy - Rayleigh number R_n, modified diffusivity ratio N_A, modified particle-density increment N_B and the Lewis number Le is to hasten the onset of convection and also to enlarge the size of convection cells. The viscosity parameter Γ exhibits a dual effect on the onset of convection depending on the values of N_A, R_n, N_B and Le. It shows stabilizing effect on the system initially but displays a reverse trend once Γ exceeds a threshold value depending on the other parametric values.

VI. Acknowledgement

One of the authors (M.D) wishes to thank the Head of the Department and the Management of his college for their encouragement and support.

References

[1] S. Kakac and A. Pramuanjaroenkij, Review of convective heat transfer enhancement with nanofluids. *Int. J. Heat Mass Transf.* 52, 2009, 3187-3196.

[2] W. Yu and H. Xie, A Review on Nanofluids: Preparation, Stability Mechanisms, and Applications. *J. Nanomaterials.* 2012, 1-17.

[3] E. K. Goharshadi, H. Ahmadzadeh, S. Samiee and M. Hadadian, Nanofluids for heat transfer enhancement-a review, *Phys. Chem. Res.* 1, 2013, 1-33.

[4] J. Kim, Y. T. Kang and C. K. Choi, Analysis of convective instability and heat transfer characteristics of nanofluids, *Phys. Fluids*16, 2004, 2395-2401.

[5] D. A. Nield and A. V. Kuznetsov, Thermal instability in a porous medium layer saturated by a nanofluid, *Int. J. Heat Mass Trans.*52, 2009, 5796-5801.

[6] D. A. Nield and A. Bejan, *Convection in porous media* (Springer. 4th ed., 2013).

[7] J. Buongiorno, Convective transport in nanofluids, *ASME J. Heat Transf.*128, 2006, 240-250.

[8] P. Vadasz, Heat conduction in nanofluid suspensions, *ASME J. Heat Transf.*128, 2006, 465-477.

[9] D. Y. Tzou, Thermal instability of nanofluids in natural convection, *Int. J. Heat Mass Transf.*51, 2008, 2967-2979.

[10] K. Khanafer and K. Vafai, A critical synthesis of thermophysical characteristics of nanofluids, *Int. J. Heat Mass Transf.*54, 2011, 4410-4428.

[11] K. B. Anoop, T. Sundararajan and Sarit K. Das, Effect of particle size on the convective heat transfer in nanofluid in the developing region, *Int. J. Heat Mass Transf.* 52, 2009, 2189-2195.

[12] P. Namburu, D. Kulkarni, D. Misra and D. Das, Viscosity of copper oxide nanoparticles dispersed in ethylene glycol and water mixture, *Exp. Therm.Fluid Sci.* 32 (2), 2007, 397-402.

[13] D. P. Kulkarni, D. K. Das and R. S. Vajjha, Application of nanofluids in heating buildings and reducing pollution, *Appl. Energy* 86 (12), 2009, 2566-2573.

[14] M. T. Naik, G. R. Janardhana, K. V. K. Reddy and B. S. Reddy, Experimental investigation into rheological property of copper oxide nanoparticles suspended in propylene glycol- water based fluids, *ARPN J. Eng. Appl. Sci.*5(6), 2010, 29-34.

[15] I. M. Mahbubul, R. Saidur and M. A. Amalina, Latest developments on the viscosity of nanofluids, *Int. J. Heat Mass Trans.* 55, 2012, 874-885.

[16] D. A. Nield and A.V. Kuznetsov, Thermal instability in a porous medium layer saturated by a nanofluid: A revised model, *Int. J. Heat Mass Trans.*68, 2014, 211-21.