Cost Optimization for Series-Parallel Petroleum Transportation Pape-Lines under Reliability Constraints

M. Amara, R. Meziane, A. Zeblah

Abstract
This paper uses an ant colony meta-heuristic optimization method to solve the cost-optimization problem in petroleum industry. This problem is known as total investment-cost minimization of series-parallel transportation pape lines. Redundant Electro-Pumpe coupled to the papes lines are included to achieve a desired level of availability. System availability is represented by a multi-state availability function. The Electro-pumpe (pape lines) are characterized by their capacity, availability and cost. These electro-pumpe are chosen among a list of products available on the market. The proposed meta-heuristic seeks to find the best minimal cost of petrol transportation system configuration with desired availability. To estimate the series-parallel pape lines availability, a fast method based on universal moment generating function (UMGF) is suggested. The ant colony approach is used as an optimization technique. An example of petrol transportation system is presented.

Index Terms—Ant colony; Electro-pumpe; Redundancy optimization; Multi-state systems; Universal generating function (UMGF), petrol transportation

1. INTRODUCTION
One of the most important problems in petrole industry is the redundancy optimization problem. This latter is well known combinatorial optimization problem where the design goal is achieved by discrete choices made from papes available on the market. The natural objective function is to find the minimal cost configuration of a series-parallel petrole transportation pape lines system under availability constraints. The system is considered to have a range of performance levels. In this case the system is called a multi-state system (MSS). Let consider a multi-state pape lines system containing n subsystems \( C_i \) \( (i = 1, 2, \ldots, n) \) in series arrangement. For each subsystem \( C_i \) there are various versions, which are proposed by the suppliers on the market. Electro-pumpe (pape lines) are characterized by their cost, capacity and availability according to their version. For example, these electro-pumpe can represent Electro-pumpe coupled to the pape lines in petrole station system to accomplish a task on fluide in our case they represent the chain of electro-pumpe and papes cring systems (Electro-pumpe station, transportation pape lines, etc..). Each subsystem \( C_i \) contains a number of electro-pumpe coupled to the pape lines connected in parallel, then a second station put in series ect... Different versions of electro-pumpe may be chosen for any given subsystem. Each subsystem can contain electro-pumpe of different versions as sketched in figure 1.

1.1-Previous Work
The vast majority of classical reliability or availability analysis and optimization assume that electro-pumpe and system are in either of two states (i.e., complete working state and total failure state). However, in many real life situations we are actually able to distinguish among various levels of performance (capacity) for both system and electro-pumpe. For such situation, the existing dichotomous model is a gross oversimplification and so models assuming multi-state (degradable) systems and electro-pumpe are preferable since they are closer to reality. Recently much works treat the more sophisticated and more realistic models in which systems and electro-pumpe may assume many states ranging from perfect functioning to complete failure. In this case, it is important to develop MSS reliability theory. In this paper, an MSS reliability theory will be used, where the binary state system theory is extending to the multi-state case. As is addresses in recent review of the literature for example in [1] or [2]. Generally, the methods of MSS reliability assessment are based on four different approaches:
(i)-The structure function approach, (ii)-The stochastic process, (iii)-The Monte-Carlo simulation technique, (iv)-The universal moment generating function (UMGF) approach. In reference [1], a comparison between these four approaches highlights that the UMGF approach is fast enough to be used in the optimization problems where the search space is sizeable.

The problem of total investment-cost minimization, subject to reliability constraints, is well known as the redundancy optimization problem (ROP). The ROP is studied in many different forms as summarized in [3], and more recently in [4]. The ROP for the multi-state reliability was introduced in [5]. In [6] and [7], genetic algorithms were used to find the optimal or nearly optimal transformation system structure. This work uses an ant colony optimization approach to solve the ROP for multi-state petroleum transport system. The idea of employing a colony of cooperating agents to solve combinatorial optimization problems was recently proposed in [8]. The ant colony approach has been successfully applied to the classical travelling salesman problem in [9], and to the quadratic assignment problem in [10]. Ant colony shows very good results in each applied area. It has been recently adapted for the reliability design of binary state systems in [11]. The ant colony has also been adapted with success to other combinatorial optimization problems such as the vehicle routing problem in [12]. The ant colony method has not yet been used for the redundancy optimization of multi-state systems.

1.2- Approach and Aoutlines

The problem formulated in this paper lead to a complicated combinatorial optimization problem. The total number of different solution to be examined is very large, even for rather small problems. An exhaustive examination of all possible solutions is not feasible given reasonable time limitations. Because of this, the ant colony optimization (or simply ACO) approach is adapted to find optimal or nearly optimal solutions to be obtained in a short time. The newer developed meta-heuristic method has the advantage to solve the ROP for MSS without the limitation on the diversity of versions of electro-pumpes in parallel.

During the optimization process, artificial ants will have to evaluate the availability of a given selected structure of the series-parallel petroleum transport system. To do this, a fast procedure of availability estimation is developed. This procedure is based on a modern mathematical technique: the z-transform or UMGF which was introduced in [13]. It was proven to be very effective for high dimension combinatorial problems: see e.g., in [2]. The universal moment generating function is an extension of the ordinary moment generating function (UGF) in [14]. The method developed in this paper allows the availability function of reparable series-parallel MSS to be obtained using a straightforward numerical procedure.

The ROP is studied in many different forms as summarized in [3], and more recently in [4]. The method developed in this paper allows the availability function of reparable series-parallel MSS to be obtained using a straightforward numerical procedure.

The rest of this paper is outlined as follows. We start in section 2 with the formulation of the optimization problem of petroleum transportation. In section 3, we develop the reliability estimation of a series-parallel multi-state petroleum system method. In section 4, we describe the ant colony optimization approach to solve the redundancy optimization problem of petroleum transportation industry. In section 5, illustrative examples and numerical results are presented in which the optimal choice of pipe-lines in a system is found. Conclusions are drawn in section 6.

II. FORMULATION OF THE OPTIMIZATION PROBLEM OF PETROLEUM TRANSPORTATION SYSTEM

Let consider a series-parallel petroleum transportation system containing n subsystems \( C_i (i = 1, 2, \ldots, n) \) in series arrangement as represented in figure 1. Every subsystem \( C_i \) contains a number of different Electro-pumpes (pipe-lines) connected in parallel. For each subsystem \( i \), there are a number of Electro-pumpes versions available in the market. For any given system Electro-pumpe, different versions and number of Electro-pumpes may be chosen. For each subsystem \( i \), Electro-pumpes are characterized according to their version \( v \) by their cost \((C_{iv})\), availability \((A_{iv})\) and capacity or (Debit) \((\Sigma_{iv})\).

The structure of subsystem \( i \) can be defined by the numbers of parallel Electro-pumpes (of each version) \( k_{iv} \), for \( 1 \leq v \leq V_i \), where \( V_i \) is a number of versions available for Electro-pumpes of type \( i \). Figure 1 illustrates these notations for a given subsystem \( i \). The entire system structure is defined by the vectors \( k_i = [k_{iv}] \) (1 \( i \leq n, 1 \leq v \leq V_i \)). For a given set of vectors \( k_1, k_2, \ldots, k_n \) the total cost of the system can be calculated as:

\[
C = \sum_{i=1}^{n} \sum_{v=1}^{V_i} k_{iv} C_{iv}
\]  

2.1-Reliability of reparable multi-states petroleum system

The series-parallel petroleum transport system is composed of a number of failure prone Electro-pumpes, such that the failure of some Electro-pumpes leads only to a degradation of the system performance. This system is considered to have a range of performance levels from perfect working to complete failure. In fact, the system failure can lead to decreased capability to accomplish a given task,
but not to complete failure. An important MSS measure is related to the ability of the system to satisfy a given demand.

For instance in electric power systems, reliability is considered as a measure of the ability of the system to meet the load demand \(D\), i.e., to provide an adequate supply of electrical energy \(\Sigma\). This definition of the reliability index is widely used in power systems: see e.g., in [14,15,16] and in [6-7]. The Loss of Load Probability index \((LOLP)\) is usually used to estimate the reliability index in [17]. This index is the overall probability that the load demand will not be met. Thus, we can write \(R = \text{Probab}(\Sigma \geq D)\) or \(R = 1-\text{LOLP} = \text{Probab}(\Sigma < D)\). This reliability index depends on consumer demand \(D\). For reparable MSS, a multi-state steady-state availability \(E\) is used as \(\text{Probab}(\Sigma \geq D)\) after enough time has passed for this probability to become constant in [16]. In the steady-state the distribution of states probabilities is given by equation (2), while the multi-state stationary availability is formulated by equation (3):

\[
P_j = \lim_{t \to \infty} \text{Probab}(\Sigma(t) = \sum j) \quad (2)
\]

\[
E = \sum_{\sum \geq D} P_j \quad (3)
\]

If the operation period \(T\) is divided into \(M\) intervals (with duration's \(T_1, T_2, \ldots, T_M\) and each interval has a required demand level \((D_1, D_2, \ldots, D_M)\), respectively), then the generalized MSS availability index \(A\) is:

\[
A = \frac{1}{M} \sum_{j=1}^{M} \text{Probab}(\Sigma \geq D_j) \quad T_j \quad (4)
\]

We denote by \(D\) and \(T\) the vectors \([D_j]\) and \([T_j]\) \((1 \leq j \leq M)\), respectively. As the availability \(A\) is a function of \(k_1, k_2, \ldots, k_n, D\) and \(T\), it will be written \(A(k_1, k_2, \ldots, k_n, D, T)\). In the case of a power system, the vectors \(D\) and \(T\) define the cumulative load curve (consumer demand). In reality the load curves varies randomly; an approximation is used from random curve to discrete curve see in [18]. In general, this curve is known for every power system.

### 2.2- Optimal design optimisation

The multi-state petroleum transport system redundancy optimization problem can be formulated as follows: find the minimal cost system configuration \(k_1, k_2, \ldots, k_n\) such that the corresponding availability exceeds or equal the specified availability \(A_0\). That is, Minimize

\[
C = \sum_{i=1}^{n} \sum_{v=1}^{V_i} k_{iv} C_{iv} \quad (5)
\]

**Subject to**

\[
A(k_1, k_2, \ldots, k_n, D, T) \geq A_0 \quad (6)
\]

The input of this problem is the specified availability and the outputs are the minimal investment-cost and the corresponding transport petroleum configuration. To solve this combinatorial optimization problem, it is important to have an effective and fast procedure to evaluate the availability index for a series-parallel plastic transport petroleum system. Thus, a method is developed in the next section to estimate the value of \(A(k_1, k_2, \ldots, k_n, D, T)\).

### III. Multi-States Availability Estimation

The procedure used in this paper is based on the universal \(z\)-transform, which is a modern mathematical technique introduced in [13]. This method, convenient for numerical implementation, is proved to be very effective for high dimension combinatorial problems. In the literature, the universal \(z\)-transform is also called universal moment generating function (UMGF) or simply \(u\)-function or \(\alpha\)-transform. In this paper, we mainly use the acronym UMGF. The UMGF extends the widely known ordinary moment generating function in [14].

The UMGF of a discrete random variable \(\Sigma\) is defined as a polynomial:

\[
u(z) = \sum_{j=1}^{J} P_j z^j \quad (7)
\]

where the variable \(\square\) has \(J\) possible values and \(P_j\) is the probability that \(\square\) is equal to \(\square_j\).

The probabilistic characteristics of the random variable \(\square\) can be found using the function \(u(z)\). In particular, if the discrete random variable \(\square\) is the MSS stationary output performance, the availability \(E\) is given by the probability \(\text{Probab}(\square \geq D)\) which can be defined as follows:

\[
\text{Probab}(\Sigma \geq D) = \Psi((u(z) - D^-) \quad (8)
\]

where \(\Psi\) is a distributive operator defined by expressions (9) and (10):

\[
\Psi(Pz^{-\sigma-D}) = \begin{cases} P, & \text{if } \sigma \geq D \\ 0, & \text{if } \sigma < D \end{cases} \quad (9)
\]
It can be easily shown that equations (7)–(10) meet condition Probab(□ ≥ D) = \(\sum P_j\). By using the operator \(\Psi\), the coefficients of polynomial \(u(z)\) are summed for every term with \(\square_i \geq D\), and the probability that \(\square\) is not less than some arbitrary value \(D\) is systematically obtained. Consider single electro-pumpes with total failures and each electro-pumpes \(i\) has nominal performance \(\square_i\), and availability \(A_i\). Then, \(\text{Probab}(\square = \square_i) = A_i\), and \(\text{Probab}(\square \equiv \square) = 1 - A_i\). The UMGF of such an electro-pumpes has only two terms can be defined as:

\[
\psi(z) = (1 - A)z^0 + A_z^z = (1 - A) + A_z^z
\]

(11)

To evaluate the MSS availability of a series-parallel system, two basic composition operators are introduced. These operators determine the polynomial \(u(z)\) for a group of electro-pumpes.

**- parallel Electro-Pumpes**

Let consider a subsystem \(m\) containing \(J_m\) electro-pumpes connected in parallel. As the performance measure is related to the system productivity, the total performance of the parallel subsystem is the sum of performances of all its electro-pumpes. In power systems engineering, the term capacity is usually used to indicate the quantitative performance measure of an electro-pumpes in [6]. It may have different physical nature. Examples of electro-pumpes capacities are: generating capacity for a generator, pipeline capacity for a water carrier, carrying capacity for an electric transmission line, etc. The capacity of an electro-pumpes can be measured as a percentage of nominal total system capacity. In a electrical network, electro-pumpes are generators, transformers and electrical lines. Therefore, the total performance of the parallel electro-pumpes is the sum of performances in [19]. The \(\psi\)-function of MSS subsystem \(m\) containing \(J_m\) parallel electro-pumpes can be calculated by using the \(\Gamma\) operator:

\[
\Gamma(u_1(z), u_2(z), \ldots, u_n(z)) = \{P_i z^a_i, Q_j z^b_j\}
\]

\[
\sum_{i=1}^{n} P_i z^a_i + \sum_{j=1}^{m} Q_j z^b_j
\]

Therefore for a pair of electro-pumpes connected in parallel:
IV. THE ANT COLONY APPROACH

The problem formulated in this paper is a complicated combinatorial optimization problem. The total number of different solutions to be examined is very large, even for rather small problems. An exhaustive examination of the enormous number of possible solutions is not feasible given reasonable time limitations. Thus, because of the search space size of the ROP for MSS, a new meta-heuristic is developed in this section. This meta-heuristic consists in an adaptation of the ant colony optimization method.

The ACO principal

Recently, in [8] introduced a new approach to optimization problems derived from the study of any colonies, called “Ant System”. Their system inspired by the work of real ant colonies that exhibit the highly structured behavior. Ants lay down in some quantity an aromatic substance, known as pheromone, in their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone in laid down by others ants, therefore the best paths has more intensive pheromone and higher probability to be chosen. This simple behavior explains why ants are able to adjust to changes in the environment, such as new obstacles interrupting the currently shortest path.

Artificial ants used in ant system are agents with very simple basic capabilities mimic the behavior of real ants to some extent. This approach provides algorithms called ant algorithms. The Ant System approach associates pheromone trails to features of the solutions of a combinatorial problem, which can be seen as a kind of adaptive memory of the previous solutions. Solutions are iteratively constructed in a randomized heuristic fashion biased by the pheromone trails, left by the previous ants. The pheromone trails, \( \tau_n \), are updated after the construction of a solution, enforcing that the best features will have a more intensive pheromone. An Ant algorithm presents the following characteristics. It is a natural algorithm since it is based on the behavior of ants in establishing paths from their colony to feeding sources and back. It is parallel and distributed since it concerns a population of agents moving simultaneously, independently and without supervisor. It is cooperative since each agent chooses a path on the basis of the information, pheromone trails, laid by the other agents with have previously selected the same path. It is versatile that can be applied to similar versions the same problem. It is robust that it can be applied with minimal changes to other combinatorial optimization problems. The solution of the travelling salesmen problem (TSP) was one of the first applications of ACO.

Various extensions to the basic TSP algorithm were proposed, notably by Dorigo and Gambardella in [9]. The improvements include three main aspects: the state transition rule provides a direct way to balance between exploration of new edges and exploitation of a priori and accumulated knowledge about the problem, the global updating rule is applied only to edges which belong to the best ant tour and while ants construct solution, a local pheromone updating rule is applied. These extensions have been included in the algorithm proposed in this paper.
ACO-based Solution Approach

In our reliability optimization problem, we have to select the best combination of parts to minimize the total cost given a reliability constraint. The parts can be chosen in any combination from the available electro-pumpess. Electro-pumpess are characterized by their reliability, capacity and cost. This problem can be represented by a graph (figure 2) in which the set of nodes comprises the set of subsystems and the set of available electro-pumpess (i.e. max (Mi), j = 1..n) with a set of connections partially connect the graph (i.e. each subsystem is connected only to its available electro-pumpess). An additional node (blank node) is connected to each subsystem

![Figure 2. Series-Parallel Petroleum Pape Lines Transportation Represented In Graph](image)

In figure 2, a series-parallel petroleum transport system is illustrated. At each step of the construction process, an ant uses problem-specific heuristic information, denoted by ηij to choose the optimal number of electro-pumpess in each subsystem. Imaginary heuristic information is associated to each blank node. These new factors allow us to limit the search surfaces (i.e. tuning factors). An ant positioned on subsystem i chooses a electro-pumpess j by applying the rule given by:

\[
j = \left\{ \begin{array}{ll}
\operatorname{arg} \max_{m \in AC_i} \left( (\tau_{im})^\alpha (\eta_{lim})^\beta \right) & \text{if } q \leq q_o \\
J & \text{if } q > q_o
\end{array} \right.
\]

(14)

and J is chosen according to the probability:

\[
p_{ij} = \left\{ \begin{array}{ll}
\frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{m \in AC_i} [\tau_{im}]^\alpha [\eta_{lim}]^\beta} & \text{if } j \in AC_i \\
0 & \text{otherwise}
\end{array} \right.
\]

(15)

α : The relative importance of the trail.
β : The relative importance of the heuristic information ηij.
ACi : The set of available electro-pumpess choices for subsystem i.
q : Random number uniformly generated between 0 and 1.

The heuristic information used is: \( \eta_{ij} = 1/(1+c_{ij}) \) where \( c_{ij} \) represents the associated cost of electro-pumpess j for subsystem i. A “tuning” factor \( t_i = \eta_{ij} = 1/(1+c_{i,j,M+1}) \) is associated to blank electro-pumpess (Mi+1) of subsystem i. The parameter qo determines the relative importance of exploitation versus exploration: every time an ant in subsystem i have to choose a electro-pumpess j, it samples a random number 0 < q < 1. If \( q \leq q_o \) then the best edge, according to (14), is chosen (exploitation), otherwise an edge is chosen according to (15) (biased exploration).

The pheromone update consists of two phases: local and global updating. While building a solution of the problem, ants choose electro-pumpess and change the pheromone level on subsystem-electro-pumpess edges. This local trail update is introduced to avoid premature convergence and effects a temporary reduction in the quantity of pheromone for a given subsystem-electro-pumpess edge so as to discourage the next ant from choosing the same electro-pumpess during the same cycle. The local updating is given by:

\[
\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho \tau_o
\]

(16)

where \( \rho \) is a coefficient such that (1-\( \rho \)) represents the evaporation of trail and \( \tau_o \) is an initial value of trail intensity. It is initialized to the value \( n.TC_{mn} \) with n is the size of the problem (i.e. number of subsystem and total number of available electro-pumpess) and \( TC_{mn} \) is the result of a solution obtained through some simple heuristic.

After all ants have constructed a complete system, the pheromone trail is then updated at the end of a cycle (i.e. global updating), but only for the best solution found. This choice, together with the use of the pseudo-random-proportional rule given by (14) and (15), is intended to make the search more directed: ants search in a neighborhood of the best solution found up to the current iteration of the algorithm. The pheromone level is updated by applying the following global updating rule:

\[
\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho \Delta \tau_{ij}
\]

(17)

\[
\Delta \tau_{ij} = \begin{cases} 
\frac{1}{TC_{best}} & \text{if } (i, j) \in \text{ best tour} \\
0 & \text{otherwise}
\end{cases}
\]

(18)

The Algorithm

An ant-cycle algorithm is stated as follows. At time zero an initialization phase takes place during wish NbAnt ants select electropumpess in each
subsystem according to the Pseudo-random-proportional transition rule given by (14) and (15). When an ant selects an electro-pumpes, a local update is made to the trail for that subsystem-electro-pump edge according to equation (16). In this equation, $\rho$ is a parameter that determines the rate of reduction of the pheromone level. The pheromone reduction is small but sufficient to lower the attractiveness of precedent subsystem-electro-pump edge. At the end of a cycle, for each ant $k$, the value of the system’s reliability $A_k$ and the total cost $TC_k$ are computed. The best feasible solution found by ants (i.e. total cost and assignments) is saved. The pheromone trail is then updated for the best solution obtained according to (17) and (18). This process is iterated until the tour counter reaches the maximum number of cycles $NC_{\text{max}}$ or all ants make the same tour (stagnation behavior).

V. ILLUSTRATIVE EXAMPLE

1) Description of The System to be Optimized

In order to illustrate the proposed ant colony algorithm, a petroleum transportation pape-lines system is considered. The petroleum feeding system station (Gaz or quandonsa liquid) supplies the boat transport. It consists of five basic subsystems (type of parallel electro-pump coupled to diferrent section pape-lines).

1- Reservoir 1 is composed by a set of parallel electro-pumpes, loads the liquid from reservoir to the first station distributor where it is evacuated.

2- The first station is composed by electro-pumpes to carry the liquid and supply the second stations.

3- The second station accumulate the gaz or liquid which is distribube to other pape-lines with diferrent sections with high debit by a specific electro-pumpes.

4- The third station received and select between different density of Gaz or liquid (rafiner) and supply the four station.

5- In the end the four station condense the Gaz and transform it to liquid with a set of refrigerator electro-pump and loded it to the boat transport.

Each electro-pumpe of the system is considered as unit with one mode failures. Table 1 shows the numerical data for each electro-pump. Each electro-pump of the subsystem is considered as a unit with total failures. Table 2 contains the data of cumulative demand.

The numbers of machines $C_{\text{max}}$ in parallel are set to (4,5,4,6,4). The number of ants used to find the best solution is 30. The simulation results depend greatly on the values of the coefficients $\alpha$ and $\beta$. Different $t_i$ values (tuning factors associated to blank electro-pumpes) were tested and shown to influence greatly the algorithm. The best found values of $t_i$ are ($t_1= -0.13$, $t_2= -0.04$, $t_3= 2.3$, $t_4= -0.35$, $t_5= 0.35$). Several simulations are made for $\alpha=5$ and $\beta=1$ and the best solution is obtained in 500 cycle. Table 3 presents the obtained configuration.

![Figure 3. Detailed Series-parallel petroleum Pape lines Transportation](image)

The characteristics of the products available on the market for each type of electro-pumpes are presented in table 1. This table shows for each subsystem availability $A$, nominal capacity or debit $\Sigma$ and cost per unit $C$. With out loss of generality both the electro-pumpes capacity and the demand levels can be measured as a percentage of the maximum capacity.

<table>
<thead>
<tr>
<th>Comp#</th>
<th>Vers#</th>
<th>Availability $A$</th>
<th>Cost $C$</th>
<th>Debit $\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.980</td>
<td>0.590</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.977</td>
<td>0.535</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.982</td>
<td>0.470</td>
<td>85</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.978</td>
<td>0.420</td>
<td>85</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.995</td>
<td>0.205</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.996</td>
<td>0.189</td>
<td>92</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.997</td>
<td>0.091</td>
<td>53</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.997</td>
<td>0.056</td>
<td>28</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.998</td>
<td>0.042</td>
<td>21</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.971</td>
<td>7.525</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.973</td>
<td>4.720</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.971</td>
<td>3.590</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.976</td>
<td>2.420</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.977</td>
<td>0.180</td>
<td>115</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.978</td>
<td>0.160</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.978</td>
<td>0.150</td>
<td>91</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.983</td>
<td>0.121</td>
<td>72</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.981</td>
<td>0.102</td>
<td>72</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.984</td>
<td>7.525</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.983</td>
<td>4.720</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.987</td>
<td>3.590</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.981</td>
<td>2.420</td>
<td>20</td>
</tr>
</tbody>
</table>

![Table 1. Data Example of Machines](image)
2) Optimal Design Solution and Results Discussion

Our natural objective function is to define the minimal cost of design of petroleum transport system configuration which provides the requested level of availability. The whole of the results obtained by the proposed ant algorithm for different given values of \( A_0 \) are illustrated in Table 3. This latter also shows the computed availability index \( A \), the cost \( C \) of the system and their corresponding structures. Three different solutions for \( A_0 = 0.975 \) is represented. In these experiments the values parameters of the ACO algorithm are the set of the following values: \( \alpha = 5, \beta = 1, \tau_0 = 0.05 \) and \( \rho = 0.080 \). The choice of these values affects strongly the solution. These values were obtained by a preliminary optimization phase. The ACO algorithm is tested well for quite a range of these values. In the ACO algorithm 30 ants are used in each iteration. The stopping criterion is when the number of iterations attempt 500 cycles. The space search visited by the 30 ants is composed of 15000 solutions (30*500 cycles) and the huge space size of an exhaustive search (combinatorial algorithm) is not realistic. Indeed, a large comparison between the ACO and an exhaustive one, clearly the goodness of the proposed ACO meta-heuristic which respect to the calculating time.

### TABLE 3
OPTIMAL SOLUTION OBTAINED BY ANT COLONY ALGORITHM

<table>
<thead>
<tr>
<th>( A_0 )</th>
<th>Structure</th>
<th>Optional Structure Electro-pumpes</th>
<th>Availability</th>
<th>Cost C A</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.975</td>
<td>Electro-pumpes 1</td>
<td>1-2-3-4</td>
<td>0.986</td>
<td>18.822</td>
<td></td>
</tr>
<tr>
<td>0.975</td>
<td>Electro-pumpes 2</td>
<td>1-2-3-4-5</td>
<td>0.986</td>
<td>18.822</td>
<td></td>
</tr>
<tr>
<td>0.975</td>
<td>Electro-pumpes 3</td>
<td>2-3-4-4</td>
<td>0.986</td>
<td>18.822</td>
<td></td>
</tr>
<tr>
<td>0.975</td>
<td>Electro-pumpes 4</td>
<td>1-2-3-4-5-6</td>
<td>0.986</td>
<td>18.822</td>
<td></td>
</tr>
<tr>
<td>0.975</td>
<td>Electro-pumpes 5</td>
<td>2-3-4-4</td>
<td>0.986</td>
<td>18.822</td>
<td></td>
</tr>
</tbody>
</table>

### VI. CONCLUSION

A new algorithm for choosing an optimal series-parallel electro-pumpes pape-lines structure configuration is proposed which minimizes total investment cost subject to availability constraints. This algorithm seeks and selects electro-pumpes or pape-lines among a list of available products according to their availability, nominal capacity (debit) and cost. Also defines the number and the kind of parallel electro-pumpes in each subsystem. The proposed method allows a practical way to solve wide instances of reliability optimization problem of multi-state petroleum transport systems without limitation on the diversity of versions of electro-pumpes put in parallel. A combination is used in this algorithm is based on the universal moment generating function and an ant colony optimization algorithm.

### References


