Analysis of pre-stressed floor grid system.

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***Abstract*—**
the sequential pre-stressing of grid structure is very important parameter in the waffle slab. Such slabs have several advantages over the more conventional slabs when used as decks for concrete bridges and as process floors in semiconductor manufacturing plants.Generally waffle slab is a monolithically construction of slab and beams, it may be one directional rib or in the both direction. In the grid structure (i.e Beam) is the key sectional properties which takes maximum contribution of load carrying in the waffle slab. This paper presents analysis in to two formats. First format is sequential and immediate pre-stressing of grid (waffle) slab for two selected patterns. Second format is to take a single cross beam from same grid (waffle) slab and analyze the same beam for differential pre-stressed forces. Here cross beam is analyzed by using direct stiffness method which is based on finite element method. Results was shows that while pre-stressing of main beam with stretching of one cable will reduce the bending moment and further alternate stretching will case cracks propagations in the secondary beam . In the waffle slab immediate pre-stressing and sequential pre-stressing were Comair in the form of bending moment and deflections and such effect after all losses occurs in the tendons were same in all such cases. But on the same time it was observed that while pre-stress a grid (waffle) slab it is very important to select the pattern in such way that the pre-stressing of any beam will not propagate the high amount of concentrated bending stress in other beams.

Keywords: *Cross beam analysis, waffle slab, grid structure, sequential pre-stressed.*

I. INTRODUCTION

Pre-stressed waffle slab is generally constructed with monolithic connection between beams and slab. Waffle slabs have a thin topping slab and narrow ribs spanning in both directions between column heads, slab generate a torsion moments in the corner zone. In waffle slab maximum downward load (i.e. due to dead load, live load etc.) is carried by beams. Ribbed and Waffle slabs provide a lighter and stiffer slab than an equivalent flat slab, reducing the extent of foundations. They provide a very good form where slab vibration is an issue, such as laboratories and hospitals. Ribbed slabs are made up of wide band beams running between columns with equal depth narrow ribs spanning the orthogonal direction. A thin topping slab completes the system.

Post-tensioned waffle slabs provide added economy where waffle forms are readily available and concrete is relatively expensive. Waffle slabs are analyses by using frame analysis or finite element analysis methods and designed using the concept of load path designation. Tendon layout in waffle Slab construction follows the same general procedure as flat slabs. The preferred procedure is to place a minimum of one tendon in each waffle stem in one direction. In the perpendicular direction, the tendons are banded along the support lines. Contrary to the typical tendon layout for flat slabs, uniform distribution of the tendons among the waffle stems in both directions is also widely used.

II. OBJECTIVE OF STUDY

The main objective is studied in this dissertation for the grid floor by sequential pre-stressing of orthogonal beams and its behavior. In grid (waffle) slab beams are stretched one by one on both directions and also stressed by immediate pre-stressing of all beams and compare the behavior of bending moment, shear force and deflection for each alternate stretching of tendons.

III. METHODS OF ANALYSIS

The three methods commonly used for analysis of concrete structures are: Simple Frame, Equivalent Frame, and Finite Element analysis.

A. Simple frame method

In the Simple Frame method, the slab is divided up into design strips. The geometry of the structure is modeled exactly, i.e., the frames are analyzed using the stiffness's of the columns and associated slabs as calculated from their geometries. As a result, the analysis does not account for the influence of biaxial plate bending.

B. Equivalent Frame method

Equivalent frame method is a refinement of the Simple Frame method. It is somewhat more exact than the Simple Frame method since the relative column and slab stiffness's are adjusted to account for the biaxial plate bending.

In typical design, most column-supported floors are analyzed with the Equivalent Frame method. The information required as far as geometry, loading and boundary conditions is the same for both the Simple Frame and the Equivalent Frame methods. Although both methods are approximate they both yield lower bound (safe) solutions. The degree of approximation depends on the extent to which a floor system deviates from a uniform, orthogonal support layout and constant slab thickness.

C. Finite Element method

The third method of analysis, the Finite Element method (FEM), is based on the division of the structure into small pieces (elements) whose behavior is formulated to capture the local behavior of the structure. Each element's definition is based on its material properties, geometry, and location in the structure, and relationship with surrounding elements. The mathematical assemblage of these elements into the complete structure allows for automated computation of the response of the entire structure. FEM inherently incorporates the biaxial behavior of the floor system when determining the actions in the floor.

D. Direct stiffness method. D.1 axially loaded member

Figure-1 shown an axially loaded member of constant cross sectional area with element forces q_1 and q_2 and displacement δ_1 and δ_2 they are shown in their respective positive direction.

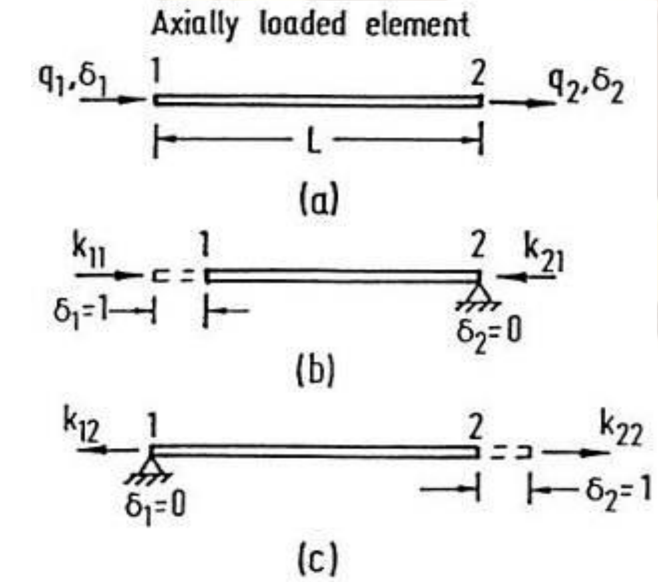


Fig.1 axially loaded member

With unit displacement δ_1 =1 at node 1, as shown in Figure -1, axial forces at nodes 1 and 2 are obtained as

$$k_{11} = \frac{AE}{L}, k_{21} = -\frac{AE}{L}$$

In the same way by setting δ_2 =1 as shown in Figure 3.1 the corresponding forces are obtained as

$$k_{12} = -\frac{AE}{L}, k_{22} = \frac{AE}{L}$$

The stiffness matrix is written as

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{21} \\ k_{12} & k_{22} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \text{---------- (Equation 1)}$$

A-2) Flexural Member

The stiffness matrix for the flexural element shown in Figure-2 can be constructed as follows. The forces and the corresponding displacements, namely the moments, the shears, and the

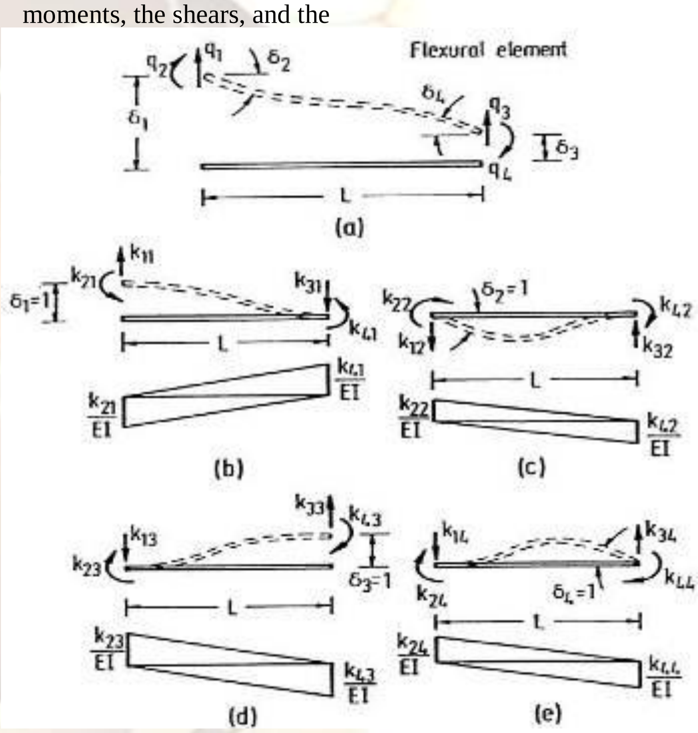


Fig.2 Beam element stiffness matrix

Corresponding rotations and translations at the ends of the member are defined in the figure. The matrix equation that relates these forces and displacements can be written in the form

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{21} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix}$$

The terms in the first column consist of the element forces q_1 through q_4 that result from displacement. When $\delta_1 = 1$ $\delta_2 = \delta_3 = \delta_4 = 0$. This means that a unit vertical displacement is imposed at the left end of the member while translation at the right end and rotation at both ends are prevented as shown in Figure-2. The four member forces corresponding to this deformation can be obtained using the moment-area method.

The change in slope between the two ends of the member is zero and the area of the *M/EI* diagram between these points must, therefore, vanish. Hence,

$$\frac{k_{41}L}{2EI} - \frac{k_{21}L}{2EI} = 0$$

And $k_{21} = k_{41}$-------- (Equation 2)

The moment of the *M=EI* diagram about the left end of the member is equal to unity. Hence,

$$\frac{k_{41}L}{2EI}\left(\frac{2L}{3}\right) - \frac{k_{21}L}{2EI}\left(\frac{L}{3}\right) = 1$$

And in view of Equation 2,

$$k_{21} = k_{41} = \frac{6EI}{L^2}$$

Finally, moment equilibrium of the member about the right end leads to

$$k_{11} = \frac{k_{21} + k_{41}}{L} = \frac{12EI}{L^3}$$

And from equilibrium in the vertical direction we obtain

$$k_{11} = k_{31} = \frac{12EI}{L^3}$$

Forces act in the directions indicated in Figure -2. To obtain the correct signs, one must compare the forces with the positive directions defined in Figure -2a. Thus,

$$k_{11} = \frac{12EI}{L^3},\ k_{21} = -\frac{6EI}{L^2},\ k_{31} = -\frac{12EI}{L^3},\ k_{41} = -\frac{6EI}{L^2}$$

The second column of the stiffness matrix is obtained by letting $\delta_2 = 1$ and setting the remaining three displacements equal to zero as indicated in Figure-2c. The area of the *M/EI* diagram between the ends of the member for this case is equal to unity, and hence,

$$\frac{k_{22}L}{2EI} - \frac{k_{42}L}{2EI} = 1$$

The moment of the *M/EI* diagram about the left end is zero, so that

$$\frac{k_{22}L}{2EI}\left(\frac{L}{3}\right) - \frac{k_{42}L}{2EI}\left(\frac{2L}{3}\right) = 0$$

Therefore, one obtains

$$k_{22} = \frac{4EI}{L},\ k_{42} = \frac{2EI}{L}$$

From vertical equilibrium of the member,

$$k_{12} = k_{32}$$

And moment equilibrium about the right end of the member leads to

$$k_{12} = \frac{k_{22} + k_{42}}{L} = \frac{6EI}{L^2}$$

Comparison of the forces in Figure-2c with the positive directions defined in Figure-2a indicates that all the influence coefficients except k_{12} are positive. Thus,

$$k_{12} = -\frac{6EI}{L^2},\ k_{22} = \frac{4EI}{L},\ k_{32} = \frac{6EI}{L^2},\ k_{42} = \frac{2EI}{L}$$

Using Figures-2d and e, the influence coefficients for the third and fourth columns can be obtained. The results of these calculations lead to the following element-stiffness matrix:

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix}$$

---- (Equation 3.3)

IV. PROBLEM FORMULATION AND MODELING.

Format-1. Analysis of cross beam.

Span 15m , Live load = 10KN/m on beam B1 , Beam size 450 x 800 mm for both beam , concrete grade M40, Cables 7wires of 5mm dia. with pre-stressed with 1488Mpa . Pre-stress loss 10.265%.

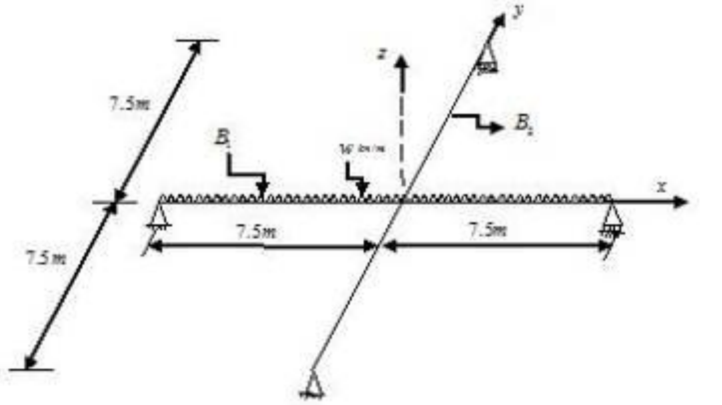


Fig. 3 Simply supported cross beam.

Analysis Results.

Bending moment and Shear force results of pre-stressed member before all losses.

TABLE:1 Element Forces – Bending moment for transfer case.

Line	Station	OutputCase	P	M3	StressTop	StressBot
Text	m	Text	kN	kN-m	N/mm2	N/mm2
		1.0(DL)		240.00	-5.00	-5.00
1	7.50	1.0(DL+PTT T1)	-204.52	227.48	-5.31	4.17
1	7.50	1.0(DL+PTT T4)	-818.09	180.93	-6.04	1.50
1	7.50	1.0(DL+PTT T8)	-1636.19	118.86	-7.02	-2.07
1	7.50	1.0(DL+PTT T12)	-2454.28	56.79	-8.00	-5.63
2	7.50	1.0(DL+PTT T1)	0.00	217.61	-4.53	4.53
2	7.50	1.0(DL+PTT T4)	0.00	141.45	-2.95	2.95
2	7.50	1.0(DL+PTT T8)	0.00	39.91	-0.83	0.83
2	7.50	1.0(DL+PTT T12)	0.00	-61.64	1.28	-1.28

DL = Dead load; LL = Live load; PTT = Pre-stressed in transfer; PTT T1= Pre-stressed in transfer with 1 tendons. (i.e 1,4,8,12 indicate no. of tendons in the beam); PTF = Pre-stressed after losses (final).

TABLE:2 Element Forces – Shear force for transfer case.

Line	Station	OutputCase	P	V2	Stress Top	Stress Bottom
Text	m	Text	kN	kN	N/mm2	N/mm2
		1.0(DL)		-64.80		
1	0.00	1.0(DL+PTT T1)	-204.28	-58.10	-0.57	-0.57
1	0.00	1.0(DL+PTT T4)	-817.11	-37.98	-2.27	-2.27
1	0.00	1.0(DL+PTT T8)	-1634.21	-11.17	-4.54	-4.54
1	0.00	1.0(DL+PTT T12)	-2451.31	15.65	-6.81	-6.81
2	0.00	1.0(DL+PTT T1)	0.00	-61.42	0.00	0.00
2	0.00	1.0(DL+PTT T4)	0.00	-51.26	0.00	0.00
2	0.00	1.0(DL+PTT T8)	0.00	-37.72	0.00	0.00
2	0.00	1.0(DL+PTT T12)	0.00	-24.18	0.00	0.00

Bending moment and Shear force results of pre-stressed member after all losses.

TABLE: 3 Element Forces – Bending moment for transfer case after all losses.

Line	Station	OutputCase	P	M3	Stress Top	Stress Bottom
Text	m	Text	kN	kN-m	N/mm2	N/mm2
1	7.50	1.0(DL+PTF T1)	-186.51	221.80	-5.28	4.25
1	7.50	1.0(DL+PTF T4)	-746.05	186.19	-5.95	1.81
1	7.50	1.0(DL+PTF T8)	-1492.10	129.37	-6.84	-1.45
1	7.50	1.0(DL+PTF T12)	-2238.15	72.56	-7.73	-4.71
2	7.50	1.0(DL+PTF T1)	0.00	219.90	-4.58	4.58
2	7.50	1.0(DL+PTF T4)	0.00	150.60	-3.14	3.14
2	7.50	1.0(DL+PTF T8)	0.00	58.21	-1.21	1.21
2	7.50	1.0(DL+PTF T12)	0.00	-34.19	0.71	-0.71

TABLE:4 Element Forces – Shear force for transfer case after all losses.

Line	Station	Output Case	P	V2
Text	M	Text	kN	kN
1	0.00	1.0(DL+PTF T1)	-185.55	-58.72
1	0.00	1.0(DL+PTF T4)	-742.20	-40.46
1	0.00	1.0(DL+PTF T8)	-1484.40	-16.13
1	0.00	1.0(DL+PTF T12)	-2226.60	8.21
2	0.00	1.0(DL+PTF T1)	0.00	-61.72
2	0.00	1.0(DL+PTF T4)	0.00	-52.48
2	0.00	1.0(DL+PTF T8)	0.00	-40.16
2	0.00	1.0(DL+PTF T12)	0.00	-27.84

Format-2. Analysis of grid (waffle) slab for sequential pre-stressing.

Section properties

1) Span of 20m. 2)
Slab thickness considered 100mm. 3)
Live load =10KN/m. 4)
Beam size = Width 400mm, Overall depth800mm. 5)
Eccentricity at support is zero in both directions. 6)
Eccentricity at mid-span in x direction is 350mm. 7)
Eccentricity at mid-span in y direction is 300mm. 8)
Pre-stress losses = 16.255%

Material properties

1) Freyssinet cables (7-5)mm diameter with pre-stressing stress of 1488MPa.
2) Concrete grade M40.

Patterns for analysis of sequential pre-stressing.

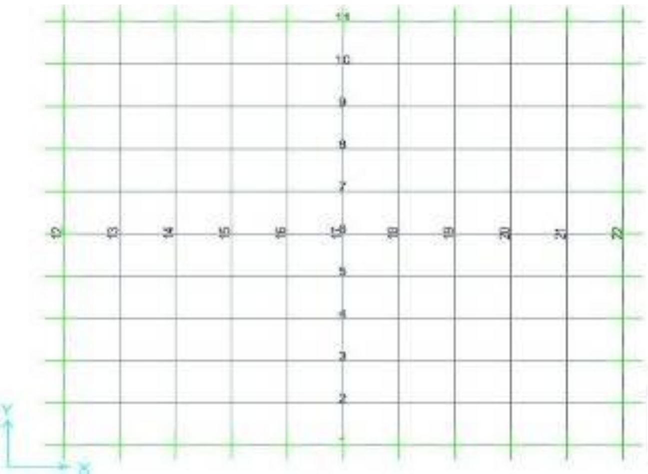


Fig.4 Plan area 20m x 20m

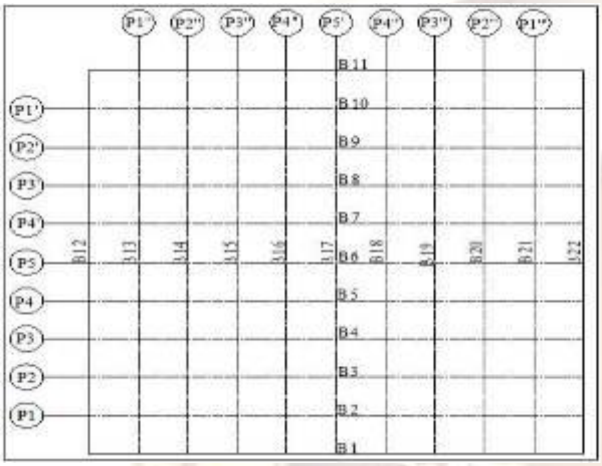


Fig.5 Sequential pre-stress pattern-1

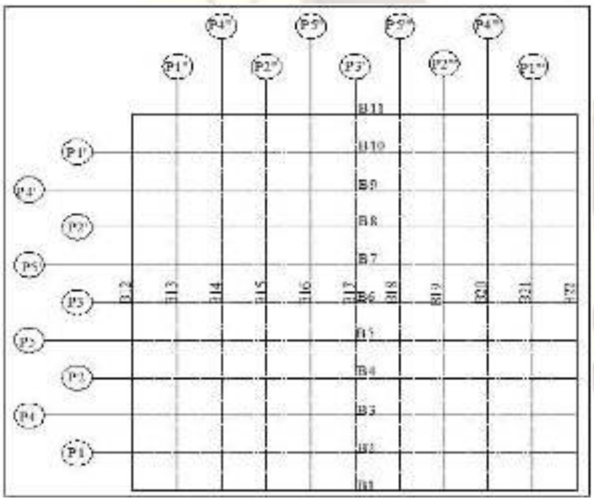
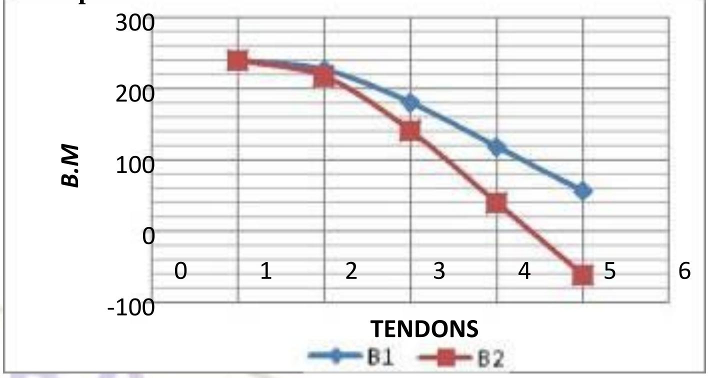


Fig.6 Sequential pre-stress pattern-2

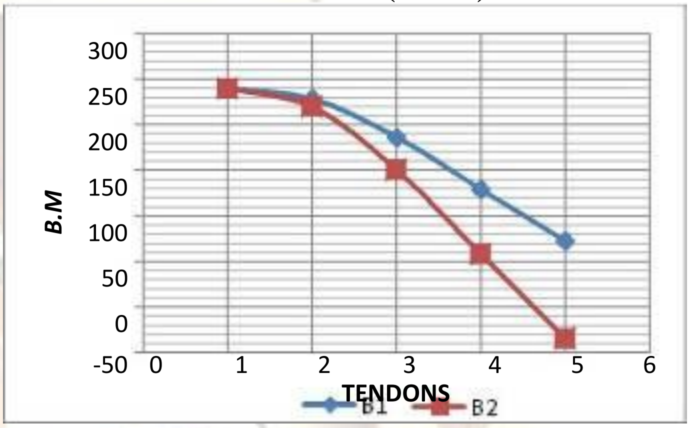
P1''' = P indicates pre-stressing; 1, 2, 3.... Indicates sequence of pre-stressing beam and comma ''' Indicates increasing order of sequential pre-stressed of individual beam number.

Analysis Results

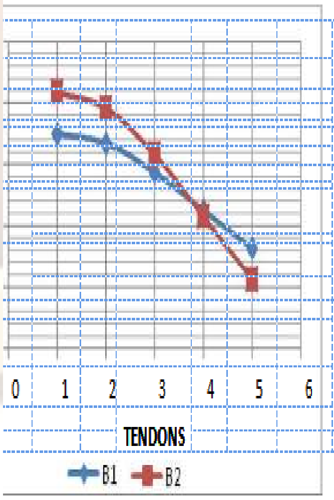
Graphical results for format-1

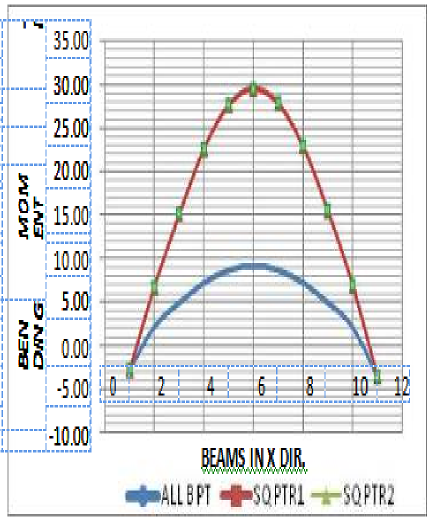


Graph-1. Comparison between bending moment of B1 and B2 beam in transfer case 1.0(DL+PT).

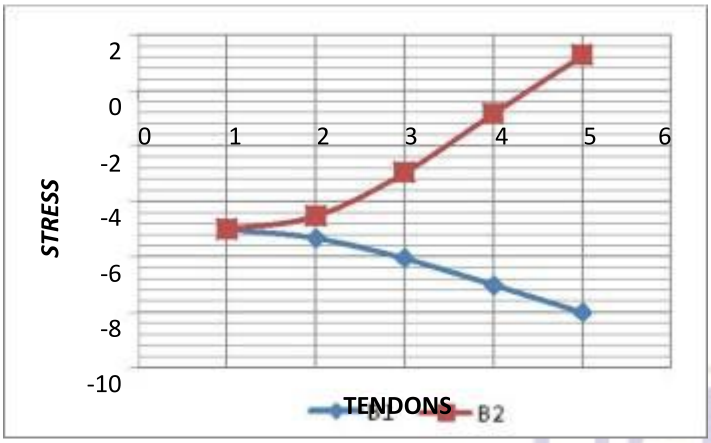


Graph-2 Comparison between bending moment of B1 and B2 beam in transfer case after all losses 1.0(DL+PT) (-ve) sig. Indicate compressive stress.

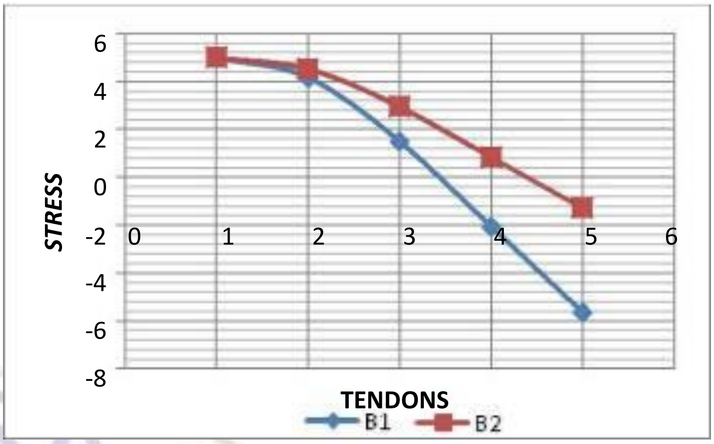




Graph-3 Comparison between bending moment of B1 and B2 beam in service case 1.0(DL+LL+PT).
Graph-6 Comparison of bending moment in between immediate pre-stressing and sequential patters for transfer case 1.0(DL+PT) in X-dir.

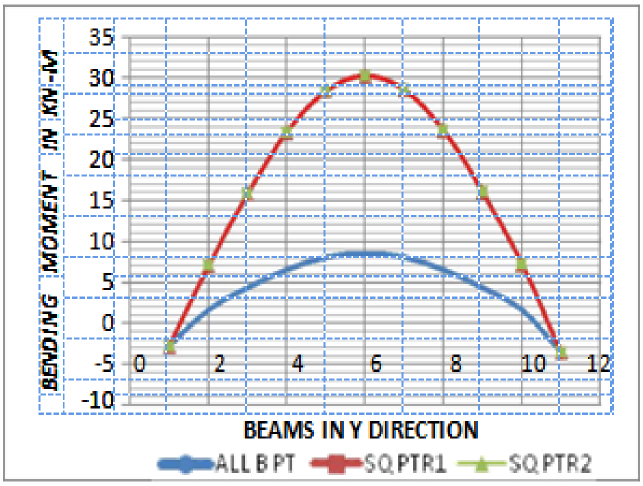


Graph-4 Comparison of flexural stress on top fibre in transfer case 1.0(DL+PT)

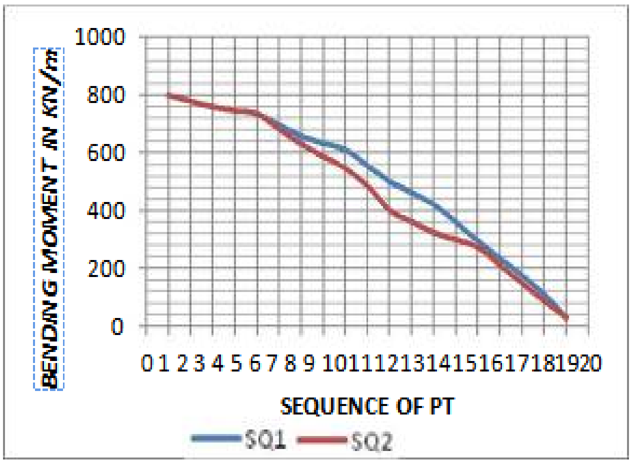


Graph-5 Comparison of flexural stress on bottom fibre in transfer case 1.0(DL+PT)

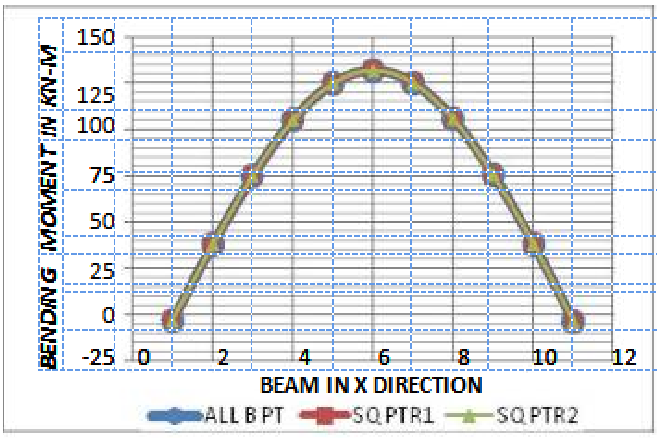
Graphical results for format-2



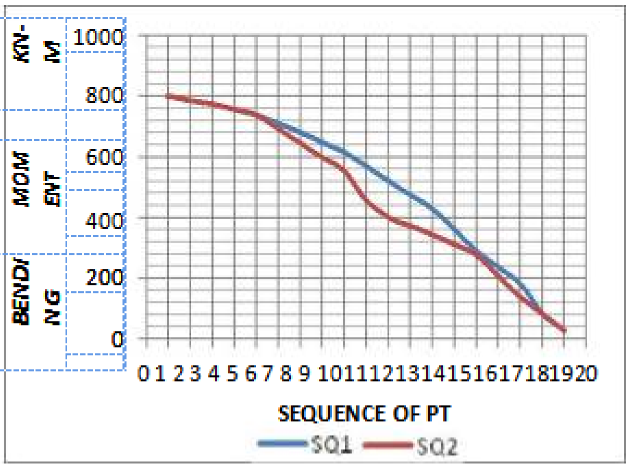
Graph-7 Comparison of bending moment in between immediate pre-stressing and sequential patters for transfer case 1.0(DL+PT) in Y-dir.



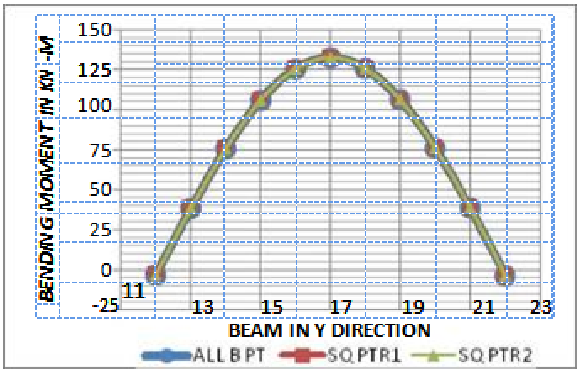
Graph-10 Comparison of bending moment in between sequential pre-stress pattern 1 and pattern 2 in X –direction for beam number 6.



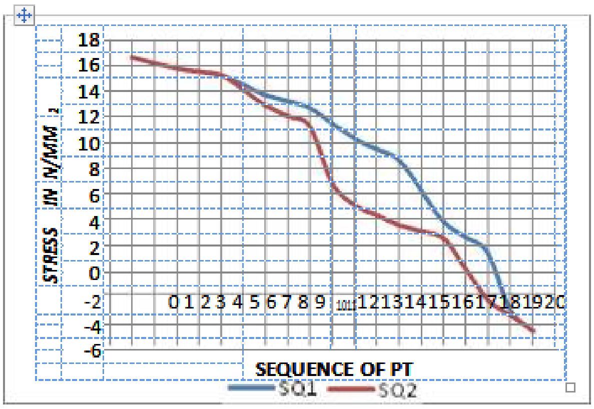
Graph-8 Comparison of bending moment in between immediate pre-stressing and sequential patters for transfer case 1.0(DL+PF) in X-dir.



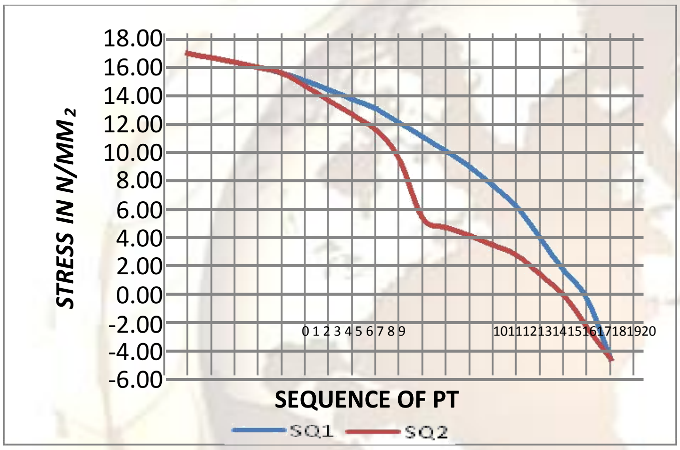
Graph-11 Comparison of bending moment in between sequential pre-stress pattern 1 and pattern 2 in Y – direction for beam number 17.



Graph-9 Comparison of bending moment in between immediate pre-stressing and sequential patters for transfer case 1.0(DL+PF) in Y-dir.



Graph-12 Comparison of bending stress in between sequential pre-stress pattern 1 and pattern 2 in X –direction for beam number 6.



Graph-13 Comparison of bending stress in between sequential pre-stress pattern 1 and pattern 2 in Y – direction for beam number 17.

V. DISCUSSION

1) It is observed that stretching of one cable in the loaded beam will reduces the bending moment in the same beam as well as in the unloaded beam (i.e. B2 beam).

2) When the pre-stressing force is increases (i.e. no of cables are more than 1 will cause sudden reduction in bending moment and shear force in the unloaded beam. Further increasing the forces moments changes it's sign by sagging moment to hogging moment in the unloaded beam in transfer case 1.0(DL +PT) . But in the loaded beam moment is in the sagging ($+^{ve}$) condition.

3) While sudden increase in stresses in the transfer case 1.0(DL+PT) for unloaded beam will case increase in strain in the same beam as stress is proportional to the strain. Therefore as long as increasing the pre-stress force will cause cracking in the unloaded beam.

4) While in the transfer case development of final bending moment in the immediate pre-stressing of grid (waffle) slab is lower than the pre-stressing of sequential patterns as incorporated in this dissertation.

5) Immediate pre-stressing and sequential patterns for pre-stress as incorporated in this dissertation are same bending moment and deflection after pre-stress losses.

6) After pre-stress losses deflection is same in the immediate pre-stressing and sequential pre-stressing.

7) Individual behavior of pattern 1 and 2 creates a differential stress concentration while pre-stressing of beam. In pattern 1 stress concentration is approximately parabolic in nature while on the same time pattern-2 creates differential stress behavior in the beams.

VI. CONCLUSION

- As pre-stress forces is increases moments changes it's sign by sagging moment to hogging moment in the unloaded beam in transfer case 1.0(DL +PT) , that beam will be unsafe for hogging moment .

- Sudden increasing the stresses will accelerate the strain and such strain will be propagating the cracks.
- In the grid structure while pre-stressing the main beam, behavior on secondary beam should be checked.
- While selecting a sequence pattern for grid (waffle) slab such development of differentia stress concentration should be avoid. i.e. in the pattern-2 such concentration will be avoided by pre-stressing the alternate beam with smaller pre-stress force.

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