

Performance Analysis of Equalizer Techniques for Modulated Signals

Gunjan Verma, Prof. Jaspal Bagga

(M.E in VLSI, SSGI University, Bhilai (C.G).

Associate Professor ETC, HOD IT Department, SSGI University, Bhilai (C.G).

ABSTRACT

In this work, the performance of two equalizers Least Mean Square (LMS) and Recursive Least Square (RLS) is observed by calculating the BER effect of Rician channels over low Doppler shift. AWGN is also added to the channel from -10 dB to 20 dB. The Bit Error Rate (BER) of 2, 4, 8-PSK (Phase Shift Keying) signals and 16, 64- QAM (Quadrature Amplitude Modulation) over Rayleigh and Rician channel is calculated.

Keywords - Additive White Gaussian Noise (AWGN), Least Mean Square (LMS), Recursive Least Square (RLS), Bit Error Rate (BER), Phase Shift Keying (PSK), Quadrature Amplitude Modulation (QAM).

I. INTRODUCTION

The wireless channel can be described as a function of time and space and the received signal is the combination of many replicas of the original signal impinging at receiver from many different paths. The presence of reflectors in the environment surrounding a transmitter and receiver create multiple paths that a transmitted signal can traverse. As a result, the receiver sees the superposition of multiple copies of the transmitted signal, each traversing a different path. Each signal copy will experience differences in attenuation, delay and phase shift while travelling from the source to the receiver. This can result in either constructive or destructive interference, amplifying or attenuating the signal power seen at the receiver. The general term fading is used to describe fluctuation in the envelope of transmitted radio signal. Equalizers are usually used to compensate the received signals which are corrupted by the inevitable noise, interference and signal power attenuation introduced by communication channel during transmission [1]. An iterative combination of blind equalization and soft clustering techniques [2], utilizes the CMA for initial reconstruction of received constellation and then uses an iterative approach in utilizing the decision adjusted modulus algorithm to refine choices. Few construct an error signal based on the cross relations between different channel in a novel systematic way [3]. The corresponding cost function is easy to

manipulate and facilitates the use of adaptive filtering method. Most of work deals with the use of individual equalizer like LMS, RLS, CMA and MMA etc or combined with some other technique like Artificial Neural Network (ANN), soft clustering etc for the reduction of fading effect. [4-10].

1.1 Rayleigh fading

When communications occur in a multi-path environment without LOS, the amplitude of the received signal has typically a Rayleigh distribution [6]. The Rayleigh distribution has a probability density function given by:

$$p(r) = \frac{r e^{-\frac{r^2}{2\sigma^2}}}{\sigma^2} \quad 0 \leq r < \infty \quad (1)$$

and

Two important statistics exist for determining error control codes and diversity schemes to be used in a communication system: the Level Crossing Rate (LCR) and the Average Fade Duration (AFD), respectively. The received signal in mobile radio communications often undergoes heavy statistical fluctuations; in digital communications, a heavy decline of the received signal directly leads to a drastic increase in the bit error rate. Suitable measures for characterizing this process are the LCR and the AFD. The number of level crossings per second is given by

$$N_R = \sqrt{2\pi} f_{max} \rho e^{-\rho^2} \quad (2)$$

Where, f_{max} is the maximum Doppler frequency, and $\rho = R/R_{rms}$ is the value of the specified signal level R normalized to the local Root Mean Square (RMS) amplitude of the fading envelope. AFD is defined as the mean time period during which the receiver signal is below a specified level R; it depends on the speed of the mobile and is given by

$$AFD = \bar{\tau} = \frac{e^{\rho^2}}{\rho f_{max} \sqrt{2\pi}} \quad (3)$$

Another mode to view the Rayleigh distribution is as the probability density function of the receiver signal amplitude to the noise ratio, which is proportional to the square of the signal envelope. Let A be the receiver signal-to-noise ratio; the probability density function of A is exponential and can be written

$$P_A(a) = \frac{1}{\rho} \exp\left(-\frac{a}{\rho}\right) \quad a \gg 0 \quad (4)$$

Where $\rho = E[A]$. The LCR can be written as

$$N_R(a) = \sqrt{\frac{2\pi a}{\rho}} f_{max} \exp\left(-\frac{a}{\rho}\right) \text{ or } N_R(a) = \sqrt{2\pi a \rho} f_{max} P_A(a) \quad (5)$$

1.2 Rician Fading

Rician fading is a stochastic model or radio propagation anomaly caused by partial cancellation of a radio signal by itself the signal arrives at the receiver by several different paths (hence exhibiting multipath interference), and at least one of the paths is changing (lengthening or shortening). Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others. In Rician fading, the amplitude gain is characterized by a Rician distribution.

A Rician fading channel can be described by two parameters: K and Ω . K is the ratio between the power in the direct path and the power in the other, scattered, path. Ω is the total power from both paths and acts as a scaling factor to the distribution.

The received signal amplitude (not the received signal power) R is then Rice distributed with parameters

$$\nu^2 = \frac{K}{1+K} \Omega \text{ and,} \quad (7)$$

$$\sigma^2 = \frac{\Omega}{2(1+K)}. \quad (8)$$

The resulting PDF then is:

$$f(x) = \frac{2(K+1)x}{\Omega} \exp\left(-K - \frac{(K+1)x^2}{\Omega}\right) I_0\left(2\sqrt{\frac{K(K+1)}{\Omega}}x\right), \quad (9)$$

Where, $I_0(\cdot)$ is the 0th order modified Bessel function of the first kind.

1.3 Equalizer

Equalizers are an important part of receivers, which minimizes the linear distortion produced by the channel. If channel characteristics are known a priori, than optimum setting for equalizers can be computed. But in practical systems the channel characteristics are not known a priori, so adaptive equalizers are used. Adaptive equalizers adapt, or change the value of its taps as time progresses [2].

1.3.1 Least Mean Squares Algorithm (LMS)

Least Mean Squares (LMS) algorithms are a class of adaptive filter used to mimic a desired filter by finding the filter coefficients that relate to producing the least mean squares of the error signal (difference between the desired and the actual signal). It is a stochastic gradient descent method in that the filter is only adapted based on the error at the current time [3]. LMS algorithm is built around a transversal filter, which is responsible for performing the filtering process. A weight control mechanism responsible for performing the adaptive control

process on the tape weight of the transversal filter as illustrated in Figure 1.

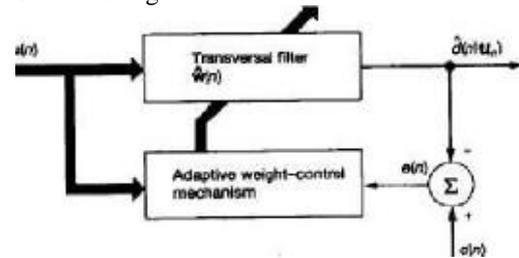


Fig.1 Block diagram of adaptive transversal filter employing LMS algorithm

The LMS algorithm in general, consists of two basic procedure:

Filtering process, which involve, computing the output ($d(n)$) of a linear filter in response to the input signal and generating an estimation error by comparing this output with a desired response as follows:

$$e(n) = d(n) - y(n) \quad (10)$$

$y(n)$ is filter output and is the desired response at time n .

Adaptive process, which involves the automatic adjustment of the parameter of the filter in accordance with the estimation error.

$$\hat{w}(n+1) = \hat{w}(n) + \mu(u)e^*(n) \quad (11)$$

Where μ is the step-size, $(n+1)$ = estimate of tape weight vector at time $(n+1)$ and If the prior knowledge of the tape weight vector (n) is not available set $(n)=0$.

The combination of these two processes working together constitutes a feedback loop. First, a transversal filter, around which the LMS algorithm is built; this component is responsible for performing the filtering process. Second, a mechanism for performing the adaptive control process on the tap weight of the transversal filter- hence the designated "adaptive weight -control mechanism".

1.3.2 Recursive Least Square Algorithm (RLS)

The RLS algorithm has the same to procedures as LMS algorithm, except that it provides a tracking rate sufficient for fast fading channel, moreover RLS algorithm is known to have the stability issues due to the covariance update formula $p(n)$, which is used for automatic adjustment in accordance with the estimation error as follows:

$$p(0) = \delta^{-1}I \quad (12)$$

Where p is inverse correlation matrix and δ is regularization parameter, positive constant for high SNR and negative constant for low SNR.

For each instance time $n=1, 2, 3, 4, \dots$

$$\pi(n) = p(n-1)u(n) \quad (13)$$

$$u(n) = \frac{\pi(n)}{\lambda + u^H(n)\pi(n)} \quad (14)$$

Time varying gain vector

$$\xi(n) = d(n) - \hat{w}^H(n-1)u(n) \quad (15)$$

$$\hat{w} = \hat{w}(n-1) + u(n)\xi^*(n) \quad (16)$$

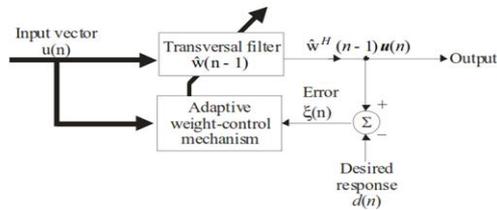


Fig. 2. Block diagram of adaptive transversal filter employing RLS algorithm

II. Methodology

Random signals are generated by using signal constellation. These signals are modulated using PSK and QAM scheme. Rician channel over 10 Hz Doppler shift noise has been introduced in channel with AWGN from -10 to 20 dB. The affected signal is then equalized through LMS and RLS equalizers. The equalized signal is the demodulated and the BER of these signals is calculated for the performance comparison of LMS and RLS equalizer. The block diagram of methodology is shown in Figure 3.

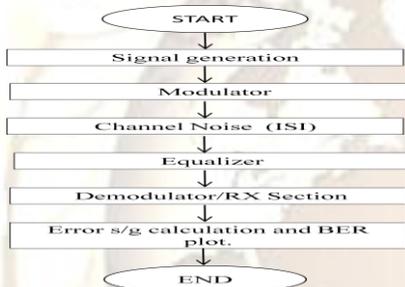


Fig. 3 Block diagram of Methodology

III. RESULT AND ANALYSIS

Equalization involves estimation of time dispersion characteristic i.e. the impulse response of the channel. Estimation is carried out by transmitting a known “training” signal and comparing the received signal with the training signal. The channel impulse response is time varying. So, the estimation of impulse response must be carried out regularly, via the regular transmission of the training signal. The equalizer is adaptive. This enables the channel estimation and equalizing filter to be updated on a continual basis during each training interval.

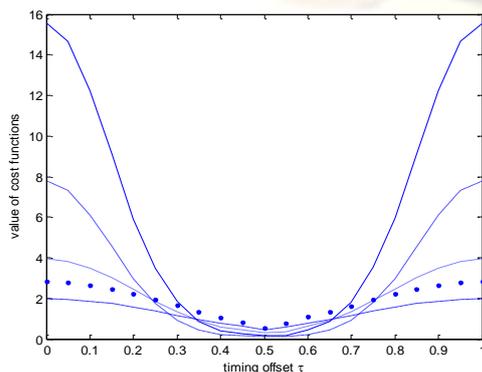


Fig 4. Value of cost function vs. timing offset

Cost function is calculated by “Monte Carlo” method, as shown in figure 4. It generates a relatively large number of realizations, or sample paths, so that it can aggregate across realizations. Here the five cost function are evaluated as :

$$cost1 = \frac{\sum(\sqrt{a^2+b^2})}{length(a)} \quad (17)$$

$$cost2 = \frac{\sum(a^2+b^2)}{length(a)} \quad (18)$$

$$cost3 = \frac{\sum(a^2+b^2)^2}{length(a)} \quad (19)$$

$$cost4 = \frac{\sum(|a|+|b|)}{length(a)} \quad (20)$$

$$cost5 = \frac{\sum(a^4+b^4)}{length(a)} \quad (21)$$

It can be concluded from the above graph that the cost function value are minimum at 0.4 to 0.6 timing offset. The cost function is the difference between the desired and the estimated signal, so during this timing offset the cost function or the error signal is minimum.

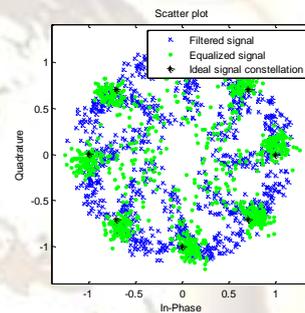


Fig. 5. Scatter plot of 8-PSK signal using LMS equalizer

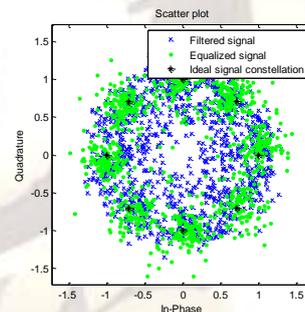


Fig. 6 Scatter plot of 8-PSK signal using RLS equalizer

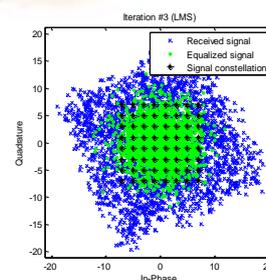


Fig. 7. Scatter plot of 64-QAM signal using LMS equalizer

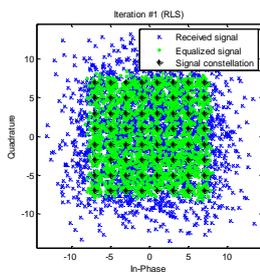


Fig. 8 Scatter plot of 64-QAM using RLS equalizer.

As it can be observed from the scatter plot of 8-PSK and 64-QAM signals shown in Figure 5, 6, 7 and 8, the performance of RLS equalizer is comparatively better than LMS equalizer. BER values of signal using Rician channel at 10 Hz Doppler shift, where the BER is calculated after demodulating the received signals. As SNR is increasing the performance of RLS equalizer is improving.

The BER of five different equalized signals are calculated and observed in Table 1:

Table 1. BER values of signal using Rician channel at 10 Hz Doppler shift.

Rician channel at 10 Hz Doppler shift using LMS equalizer										
SNR	2-PSK		4-PSK		8-PSK		16-QAM		64-QAM	
	LMS	RLS	LMS	RLS	LMS	RLS	LMS	RLS	LMS	RLS
-10	0.494	0.402	0.7293	0.412	0.8627	0.4762	0.4225	0.3845	0.3427	0.3538
-5	0.4153	0.274	0.6873	0.2907	0.8273	0.4371	0.4547	0.352	0.4758	0.3538
0	0.214	0.1367	0.482	0.1447	0.7093	0.3798	0.4727	0.3952	0.4982	0.2519
5	0.054	0.044	0.3027	0.0353	0.544	0.3102	0.4338	0.2438	0.4906	0.2217
10	0.0173	0.012	0.1313	0.004	0.4267	0.2282	0.4052	0.0574	0.4914	0.1987
15	0.004	0.002	0.0973	0.0033	0.3727	0.2056	0.4079	0.0299	0.4664	0.1576
20	0.0033	0.0013	0.0813	6.67E-04	0.3507	0.1884	0.382	0.0135	0.4991	0.1362

The BER of M-PSK and M-QAM signals is calculated for Rician channel at 10 Hz Doppler shift over -10 to 20 dB SNR. It can be observed from above values, for higher order QAM signals in LMS equalizer, the BER is increasing even at 20 dB SNR where as in RLS equalizer the value of BER is decreasing as SNR is increasing.

The BER plot of M-PSK and M-QAM signals is shown in Figure. 9 and 10 respectively. The performance of both the equalizers can be observed. It can be seen from above graph that for 16, 64-QAM signals, the performance of RLS equalizer is better at 10Hz Doppler shift.

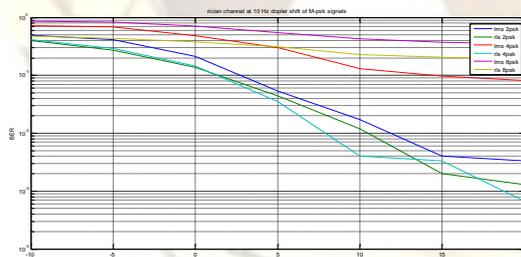


Fig.9 BER of M-psk signal using LMS and RLS equalizer

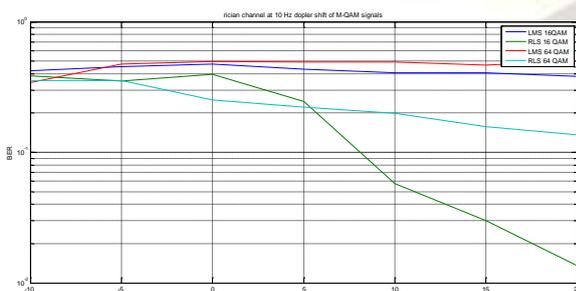


Fig.10 BER of M-QAM signal using LMS and RLS equalizer

IV. Conclusion and Future Scope

The performance of two equalizers i.e. LMS and RLS of 10 Hz Doppler shift has been studied by calculating its BER from -10 dB to 20dB SNR. As the order of modulated signals is increases the performance of equalizers fall, the performance of both the equalizer is almost same for lower order signals i.e. 2, 4, 8-PSK where as for higher order signals i.e. 16, 64-QAM RLS's performance is better.

For higher order, cascaded equalizers can be used and the performance of higher order signals can be improved. Blind equalization algorithm does not require training sequence so it can be used in communication system with effective use of bandwidth. Candidate signals can be increased for better realization of communication network.

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