On The Onset of Rayleigh-Bénard Convection in a Layer of Ferrofluid

Ramesh Chand[1] and Ankuj Bala[2]
[1]Department of Mathematics, Government College, Dhaliara (Kangra), Himachal Pradesh, INDIA
[2]Department of Mathematics, Himachal Institute of Engineering and Technology, Shahpur, Himachal Pradesh, INDIA

ABSTRACT

The onset of Rayleigh-Bénard convection in a horizontal layer of ferrofluid is investigated by using Galerkin weighted residuals method. Linear stability theory based upon normal mode analysis is employed to find expressions for Rayleigh number and critical Rayleigh number. The boundaries are considered to be free-free, rigid-free and rigid-rigid. It is also observed that the system is more stable in the case of rigid-rigid boundaries and least stable in case of free-free boundaries. 'Principle of exchange of stabilities' is valid and the oscillatory modes are not allowed. The effect of magnetic parameter on the stationary convection is investigated graphically for all types of boundary conditions.

Key words: Ferrofluid, Convection, Magnetic thermal Rayleigh number, Galerkin method, Prandtl number.

Nomenclature

a	wave number
B	magnetic induction
g	acceleration due to gravity
H	magnetic field intensity
k	thermal conductivity
K_1	pyomagnetic coefficient
M	magnetization
M_1	buoyancy magnetization
M_3	magnetic parameter
N	magnetic thermal Rayleigh number
n	growth rate of disturbances
p	pressure (Pa)
P_r	Prandtl number
q	fluid velocity
R	Rayleigh number
R_c	critical Rayleigh number
t	time
T	temperature
Ta	average temperature
u, v, w	fluid velocity components
(x, y, z)	space co-ordinates

Greek symbols

α	thermal expansion coefficient
β	uniform temperature gradient
μ_o	magnetic permeability
μ	viscosity
ρ	density of the fluid
(ρc)	heat capacity of fluid
κ	thermal diffusivity
φ'	perturbed magnetic potential
ω	dimensional frequency
χ	magnetic susceptibility

Superscripts

'	non dimensional variables
' '	perturbed quantity

Subscripts

0	lower boundary
1	upper boundary
H	horizontal plane

I. Introduction

Ferrofluid formed by suspending submicron sized particles of magnetite in a carrier medium such as kerosene, heptanes or water. The attractiveness of ferrofluids stems from the combination of a normal liquid behavior with sensitivity to magnetic fields. Ferrofluid has three main constituents: ferromagnetic particles such as magnetite and composite ferrite, a surfactant, and a base liquid such as water or oil. The surfactant coats the ferromagnetic particles, each of which has a diameter of about 10 nm. This prevents coagulation and keeps the particles evenly dispersed throughout the base liquid. Its dispersibility remains stable in strong magnetic fields.

Ferromagnetic fluid has wide ranges of applications in instrumentation, lubrication, printing, vacuum technology, vibration damping, metals recovery, acoustics and medicine, its commercial usage includes vacuum feed through for semiconductor manufacturing in liquid-cooled loudspeakers and computer disk drives etc. Owing the applications of the ferrofluid its study is important to the researchers. A detailed account on the subject is given in monograph has been given by Rosensweig (1985). This monograph reviews several applications of heat transfer through ferrofluid. One such phenomenon is enhanced convective cooling having a temperature-dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of the magnetic field, temperature, salinity and density of the fluid. In our analysis, we assume that the magnetization is aligned with the magnetic field.

Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson (1970). He explained the concept of thermo-mechanical interaction in ferromagnetic fluids. Thermoconvective stability of ferromagnetic fluids without considering buoyancy effects has been investigated by Lalas and Carmi (1971). Linear and nonlinear convective instability of a ferromagnetic fluid for a fluid layer heated from below under various assumptions is studied by many authors Shliomis (2002), Blennerhassett et.al.(1991), Gupta and Gupta (1979), Stiles and Kagan (1990), Sunil et.al. (2005, 2006), Sunil, Mahajan (2008), Venkatasubramanian and Kaloni (1994), Zebib (1996), Mahajan (2010). However, a limited effort has been put to investigate the instability in rigid-rigid and rigid-free boundaries. In this paper an attempt has been made to study the linear convective instability of a ferromagnetic fluid for a fluid layer heated from below by Galerkin weighted residuals method for all type of boundary conditions.

II. Mathematical Formulation of the Problem

Consider an infinite, horizontal layer of an electrically non-conducting incompressible ferromagnetic fluid of thickness 'd', bounded by plane z = 0 and z = d. Fluid layer is acted upon by gravity force **g** (0, 0, -g) and a uniform magnetic field $\mathbf{H} = H_0^{ext}\hat{k}$ acts outside the fluid layer. The layer is heated from below such that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is to be maintained. The temperature T at z = 0 taken to be T_0 and T_1 at z = d, ($T_0 > T_1$) as shown in Fig.1.

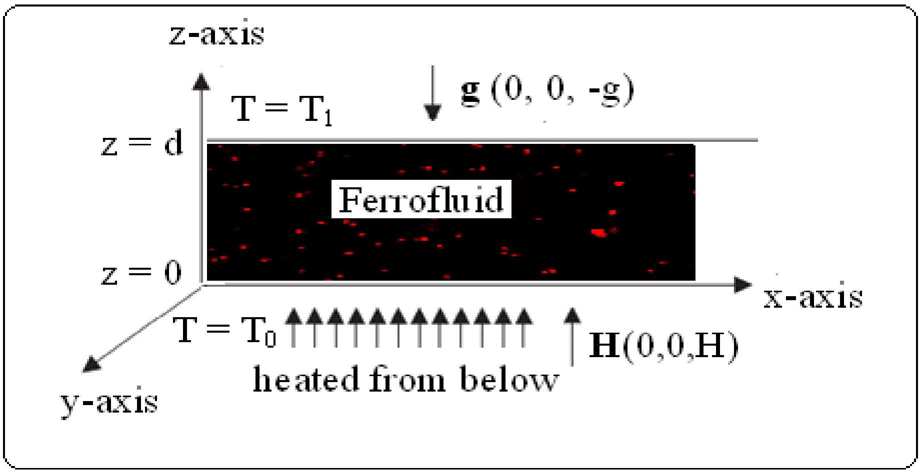

Fig.1 Geometrical configuration of the problem

The mathematical governing equations under Boussinesq approximation for the above model (Finlayson (1970), Resenweig (1997), and Mahajan (2010) are:

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$\rho_0\frac{d\mathbf{q}}{dt} = -\nabla p + \rho_0\mathbf{g} + \mu\nabla^2\mathbf{q} + \mu_0(\mathbf{M}.\nabla)\mathbf{H}, \tag{2}$$

$$(\rho_\circ C_0)_f\frac{dT}{dt} + (\rho_\circ C_0)_f\, q.\nabla T = k\nabla^2 T, \tag{3}$$

Maxwell's equations, in magnetostatic limit:

$$\nabla.\mathbf{B} = 0, \nabla\times\mathbf{H} = 0, \quad B = \mu_0(H + M). \tag{4}$$

The magnetization has the relationship

$$\mathbf{M} = \frac{\mathbf{H}}{H}\left[M_0 + \chi(H - H_0) - K_1(T - T_1)\right]. \tag{5}$$

The density equation of state is taken as

$$\rho = \rho_\circ\left[1 - \alpha(T - T_a)\right]. \tag{6}$$

Here ρ, ρ_0, **q**, t, p, μ, μ_0, **H**, **B**, C_0, T, M, K_1, and α are the fluid density, reference density, velocity, time, pressure, dynamic viscosity (constant), magnetic permeability, magnetic field, magnetic induction, specific heat at constant pressure, temperature, magnetization, thermal conductivity and thermal expansion coefficient, T_a is the average temperature given by $T_a = \left(\frac{T_0 + T_1}{2}\right)$, $H = |\mathbf{H}|$, $M = |\mathbf{M}|$ and $M_0 = M(H_0, T_a)$. The magnetic susceptibility and pyomagnetic coefficient are defined by $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_\circ, T_a}$ and $K_1 = \left(\frac{\partial M}{\partial T}\right)_{H_\circ, T_a}$ respectively.

Since the fluid under consideration is confined between two horizontal planes z = 0 and z =

d, on these two planes certain boundary conditions must be satisfied. We assume the temperature is constant at z = 0, z = d, thus boundary conditions [Chandrasekhar (1961), Kuznetsov and Nield (2010)] are

$$w = 0, \frac{\partial w}{\partial z} + \lambda_1 d \frac{\partial^2 w}{\partial z^2} = 0,\ T = T_0, D\varphi = 0 \text{ at } z = 0,$$

$$w = 0, \frac{\partial w}{\partial z} - \lambda_2 d \frac{\partial^2 w}{\partial z^2} = 0,\ T = T_1,\ D\varphi = 0 \text{ at } z = d. \quad (7)$$

The parameters λ_1 and λ_2 each take the value 0 for the case of a rigid boundary and ∞ for a free boundary.

2.1 Basic Solutions

The basic state is assumed to be a quiescent state and is given by

$$q(u,v,w) = q_b(u,v,w) = 0 \quad , p = p_b(z) \, ,$$

$$T = T_b(z) = -\beta z + T_a \, ,$$

$$H_b = \left[H_\circ + \frac{K_1(T_b - T_a)}{1+\chi}\right]\hat{k},$$

$$M_b = \left[M_\circ - \frac{K_2(T_b - T_a)}{1+\chi}\right]\hat{k} \quad],$$

$$H_\circ + M_\circ = H_\circ ext\,. \quad (8)$$

2.2 The Perturbation Equations

We shall analyze the stability of the basic state by introducing the following perturbations:

$$q = q_b + q',\ p = p_b(z) + \delta p,\ T = T_b(z) + \theta\,,$$

$$H = H_b(z) + H'\ \ M = M_b(z) + M' \quad (9)$$

where $q'(u,v,w)$, δp, θ, $H'(H'_1,H'_2,H'_3)$ and $M'(M'_1,M'_2,M'_3)$ are perturbations in velocity, pressure, temperature, magnetic field and magnetization. These perturbations are assumed to be small and then the linearized perturbation equations are

$$\nabla.\mathbf{q}' = 0, \quad (10)$$

$$\rho_\circ \frac{\partial \mathbf{q}'}{\partial t} = -\nabla\delta p + \mu\nabla^2\mathbf{q}' + \rho_0\alpha g\theta\hat{k} - \frac{\mu_\circ K_1\beta}{1+\chi}\left((1+\chi)\frac{\partial\varphi'_1}{\partial z}\hat{k} - K_1\theta\hat{k}\right) \quad (11)$$

$$\frac{\partial\theta}{\partial t} = \kappa\nabla^2\theta + \beta w\,, \quad (12)$$

$$\left(1 + \frac{M_0}{H_0}\right)\nabla^2\phi'_1 - \left(\frac{M_0}{H_0} - \chi\right)\frac{\partial^2\phi'_1}{\partial z^2} = K_1\frac{\partial\theta}{\partial z} \quad (13)$$

where $\mathbf{H}' = \nabla\varphi'_1$ and φ' is the perturbed magnetic potential and $\kappa = \frac{k}{(\rho_0 c_0)_f}$ is thermal diffusivity of the fluid.

And boundary conditions are

$$w' = 0, \frac{\partial w'}{\partial z} + \lambda_1 d\frac{\partial^2 w'}{\partial z^2} = 0,\ T = T_0, D\varphi = 0 \text{ at } z = 0,$$

$$w' = 0, \frac{\partial w'}{\partial z} - \lambda_2 d\frac{\partial^2 w'}{\partial z^2} = 0,\ T = T_1, D\varphi = 0 \text{ at } z = d \quad (14)$$

We introduce non-dimensional variables as

$$(x'', y'', z'') = \left(\frac{x', y', z'}{d}\right),\ \mathbf{q}'' = \mathbf{q}'\frac{d}{\kappa},$$

$$t' = \frac{\kappa}{d^2}t,\ \delta p' = \frac{d^2}{\mu\kappa}\delta p,\ \theta' = \frac{\theta}{\beta d},$$

$$\varphi''_1 = \frac{(1+\chi)}{K_1\beta d^2}\varphi'_1\,.$$

There after dropping the dashes (") for simplicity. Equations (10)-(14), in non dimensional form can be written as

$$\nabla.\mathbf{q} = 0, \quad (15)$$

$$\frac{1}{Pr}\frac{\partial\mathbf{q}}{\partial t} = -\nabla\delta p + \nabla^2\mathbf{q} + R(1+M_1)\theta\hat{k} - RM_1\frac{\partial\varphi_1}{\partial z}\hat{k}, \quad (16)$$

$$\frac{\partial\theta}{\partial t} = \nabla^2\theta + w\,, \quad (17)$$

$$M_3\nabla^2\phi_1 - (M_3 - 1)\frac{\partial^2\phi_1}{\partial z^2} = \frac{\partial\theta}{\partial z}\,. \quad (18)$$

where non-dimensional parameters are:

$P_r = \frac{\mu}{\rho\kappa}$ is Prandtl number; $R = \frac{\rho_0 g\alpha\beta d^4}{\mu\kappa}$ is Rayleigh number; $M_1 = \frac{\mu_0 K_1^2\beta}{\alpha\rho_0 g(1+\chi)}$ measure the ratio of magnetic to gravitational forces, $N = RM_1 = \frac{\mu_0 K_1^2\beta^2 d^4}{\mu\kappa(1+\chi)}$ is magnetic thermal Rayleigh number; $M_3 = \frac{\left(1 + \frac{M_0}{H_0}\right)}{(1+\chi)}$ measure the departure of linearity in the magnetic equation of state and values from one $(M_0 = \chi H_0)$ higher values are possible for the usual equation of state.

The dimensionless boundary conditions are

$$w = 0, \frac{\partial w}{\partial z} + \lambda_1\frac{\partial^2 w}{\partial z^2} = 0,\ T = 1,\ D\varphi = 0 \text{ at } z = 0$$

and $w = 0, \frac{\partial w}{\partial z} - \lambda_2\frac{\partial^2 w}{\partial z^2} = 0,\ T = 0\ \ D\varphi = 0 \text{ at } z = 1$

. (19)

Operating equation (10) with $\hat{k}$.curl curl, we get

$$\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 w = \nabla^4 w + R(1+M_1)\nabla_H^2\theta - RM_1\nabla_H^2 D\varphi_1 . \tag{20}$$

where ∇_H^2, is two-dimensional Laplacian operator on horizontal plane.

III. Normal Mode Analysis

Analyzing the disturbances of normal modes and assume that the perturbation quantities are of the form

$$[w, \theta, \varphi_1] = [W(z), \Theta(z), \Phi(z)]\exp(ik_x x + ik_y y + nt), \tag{21}$$

where, k_x, k_y are wave numbers in x- and y- direction and n is growth rate of disturbances.
Using equation (21), equations (20) and (17) - (18) becomes

$$\left(D^2 - a^2 - \frac{n}{Pr}\right)(D^2 - a^2)W - a^2R(1+M_1)\Theta + a^2RM_1D\Phi = 0, \tag{22}$$

$$W + (D^2 - a^2 - n)\Theta = 0, \tag{23}$$

$$D\Theta - (D^2 - a^2M_3)\Phi = 0 . \tag{24}$$

where $D = \frac{d}{dz}$, and $a^2 = k_x^2 + k_y^2$ is dimensionless the resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

(i) when both boundaries free

$$W = 0, D^2W = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 0,1 . \tag{25a}$$

(ii) when both boundaries rigid

$$W = 0, DW = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 0,1 . \tag{25b}$$

(ii) when lower rigid and upper free boundaries

$$W = 0, DW = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 0,$$

$$W = 0, D^2W = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 1. \tag{25c}$$

IV. Method of solution

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of equations (22) – (24) with the corresponding boundary conditions (25). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W = \sum_{p=1}^{n} A_p W_p, \Theta = \sum_{p=1}^{n} B_p\Theta_p, D\Phi = \sum_{p=1}^{n} C_p D\Phi_p . \tag{26}$$

Where A_p, B_p and C_p are unknown coefficients, p =1, 2, 3,...N and the base functions W_p, Θ_p and $D\Phi_p$ are assumed in the following form for free-free, rigid-rigid and rigid-free boundaries respectively:

$$W_p = \cos p\pi z, \Theta_p = \cos p\pi z, D\Phi_p = \cos p\pi z, \tag{27}$$

$$W_p = z^{p+1} - 2z^{p+2} + z^{p+3}, \Theta_p = z^p - z^{p+1}, D\Phi_p = z^p - z^{p+1}, \tag{28}$$

$$W_p = z^2(1-z)[(p+2) - 2z^p], \Theta_p = z^p - z^{p+1}, D\Phi_p = z^p - z^{p+1}. \tag{29}$$

such that W_p, Θ_p and Φ_p satisfy the corresponding boundary conditions. Using expression for W, Θ and DΦ in equations (22) – (24) and multiplying first equation by W_p second equation by Θ_p and third by $D\Phi_p$ and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations in 3N unknown A_p, B_p and C_p; p =1,2,3,...N. For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number R.

V. Linear Stability Analysis

5.1 Solution for free boundaries:

We confined our analysis to the one term Galerkin approximation; for one term Galerkin approximation, we take N=1, the appropriate trial function are given as

$$W_p = \cos\pi z, \Theta_p = \cos\pi z, D\Phi_p = \cos\pi z, \tag{30}$$

which satisfies boundary conditions

$$W = 0, D^2W = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 0 \ \text{ and}$$

$$W = 0, D^2W = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 1. \tag{31}$$

Using trial function (30) boundary conditions (31) we get the expression for Rayleigh number R as:

$$R = \frac{(\pi^2 + a^2M_3)\left(\pi^2 + a^2 + \frac{n}{Pr}\right)(\pi^2 + a^2 + n)(\pi^2 + a^2)}{a^2(\pi^2 + a^2M_3 + a^2M_1M_3)} . \tag{32}$$

For neutral stability, the real part of n is zero. Hence we put n = iω, in equation (32), where ω is real and is dimensionless frequency, we get

$$R = \frac{(\pi^2 + a^2M_3)\left(\pi^2 + a^2 + \frac{i\omega}{Pr}\right)(\pi^2 + a^2 + i\omega)(\pi^2 + a^2)}{a^2(\pi^2 + a^2M_3 + a^2M_1M_3)} . \tag{33}$$

Equating real and imaginary parts, we get

$$R = \Delta_1 + i\omega\Delta_2 , \tag{34}$$

where

$$\Delta_1 = \frac{(\pi^2 + a^2M_3)(\pi^2 + a^2)\left((\pi^2 + a^2)^2 - \frac{\omega^2}{\Pr}\right)}{a^2(\pi^2 + a^2M_3 + a^2M_1M_3)}, \qquad (35)$$

and

$$\Delta_2 = \frac{(\pi^2 + a^2M_3)(\pi^2 + a^2)\left(1 + \frac{1}{\Pr}\right)}{a^2(\pi^2 + a^2M_3 + a^2M_1M_3)}. \qquad (36)$$

Since R is a physical quantity, so it must be real. Hence, it follow from the equation (34) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \# 0$ overstability or oscillatory onset).
But $\Delta_2 \# 0$, we must have $\omega = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied. This is the good agreement of the result as obtained by Finlayson (1970).

(a) Stationary Convection

Consider the case of stationary convection i.e., $\omega = 0$, from equation (33), we have

$$R = \frac{(\pi^2 + a^2)^3 + (\pi^2 + a^2M_3)}{a^2(\pi^2 + a^2M_3 + a^2M_1M_3)}. \qquad (37)$$

This is the good agreement of the result as obtained by Finlayson (1970).
In the absence of magnetic parameters $M_1=M_3=0$, the Rayleigh number R for steady onset is given by

$$R = \frac{(\pi^2 + a^2)^3}{a^2}. \qquad (38)$$

Consequently critical Rayleigh number is given by

$$Rc = \frac{27\pi^2}{4}.$$

This is exactly the same the result as obtained by Chandrasekhar (1961) in the classical Bénard problem.

5.2 Solution Rigid-Rigid Boundaries

We confined our analysis to the one term Galerkin approximation; the appropriate trial function for rigid-rigid boundary conditions is given by

$$W_p = z^2(1-z)^2, \Theta_p = z(1-z), D\Phi_p = z(1-z), \qquad (39)$$

which satisfied boundary conditions

$$W = 0, DW = 0, \Theta = 0, D\Phi = 0 \text{ at } z = 0 \text{ and}$$
$$W = 0, DW = 0, \Theta = 0, D\Phi = 0 \text{ at } z = 1. \qquad (40)$$

Using trial function (39) boundary conditions (40) we get the expression for Rayleigh number R as:

$$R = \frac{28}{27a^2}\left[\frac{(13a^2M_3 + 42)\left[(a^4 + 24a^2 + 504) + n\left(12 + \frac{1}{\Pr}\right)(a^2 + 10 + n)\right]}{13a^2M_3 + 13a^2M_1M_3 + 42}\right]. \qquad (41)$$

For neutral stability, the real part of n is zero. Hence we put $n = i\omega$, in equation (41), we get

$$R = \frac{28}{27a^2}\left[\frac{(13a^2M_3 + 42)\left[(a^4 + 24a^2 + 504) + i\omega\left(12 + \frac{1}{\Pr}\right)(a^2 + 10 + i\omega)\right]}{13a^2M_3 + 13a^2M_1M_3 + 42}\right]. \qquad (42)$$

Equating real and imaginary parts, we get

$$R = \Delta_3 + i\omega\Delta_4, \qquad (43)$$

where

$$\Delta_3 = \frac{28}{27a^2}\left[\frac{(13a^2M_3 + 42)\left[(a^4 + 24a^2 + 504)(a^2 + 10) - \omega^2\left(12 + \frac{1}{\Pr}\right)\right]}{13a^2M_3 + 13a^2M_1M_3 + 42}\right], \qquad (44)$$

and

$$\Delta_4 = \frac{28}{27a^2}\left[\frac{(13a^2M_3 + 42)\left[(a^4 + 24a^2 + 504) + \left(12 + \frac{1}{\Pr}\right)(a^2 + 10)\right]}{13a^2M_3 + 13a^2M_1M_3 + 42}\right]. \qquad (45)$$

Since R is a physical quantity, so it must be real. Hence, it follow from the equation (43) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \# 0$ overstability or oscillatory onset).
But $\Delta_2 \# 0$, we must have $\omega = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for rigid –rigid boundaries.

(b) Stationary Convection

Consider the case of stationary convection i.e., $\omega = 0$, from equation (42), we have

$$R = \frac{28}{27a^2}\left[\frac{(a^4 + 24a^2 + 504)(a^2 + 10)(13a^2M_3 + 42)}{13a^2M_3 + 13a^2M_1M_3 + 42}\right]. \qquad (46)$$

In the absence of magnetic parameters $M_1=M_3=0$, the Rayleigh number R for steady onset is given by

$$R = \frac{28}{27a^2}(a^4 + 24a^2 + 504)(a^2 + 10). \qquad (47)$$

This is exactly the same the result as obtained by Chandrasekhar (1961) in the classical Bénard problem for rigid -rigid boundaries.

5.3 Solution Rigid-Free Boundaries

The appropriate trial function to the one term Galerkin approximation for rigid-free boundary conditions is given by

$$W_p = z^2(1-z)(3-2z), \Theta_p = z(1-z), D\Phi_p = z(1-z) \quad , \tag{48}$$

which satisfied boundary condition

$$W = 0, DW = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 0 \text{ and}$$

$$W = 0, D^2W = 0, \ \Theta = 0, \ D\Phi = 0 \ \text{ at } \ z = 1. \tag{49}$$

It is observed that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for rigid -free boundaries.

The eigenvalue equation for stationary case takes the form

$$R = \frac{28}{507a^2}\frac{(13a^2M_3 + 42)(19a^4 + 432a^2 + 4536)(a^2 + 10)}{13a^2M_3 + 13a^2M_1M_3 + 42} . \tag{50}$$

In the absence of magnetic parameter M_1=M_3=0, the Rayleigh number R for stationary convection is given by

$$Ra = \frac{28}{507a^2}(19a^4 + 432a^2 + 4536)(a^2 + 10) . \tag{51}$$

This is the good agreement of the result as obtained by Chandrasekhar (1961) in the classical Bénard problem.

VI. Results and Discussion

Expressions for Stationary convection Rayleigh number are given by equations (37), (46) and (50) for the case of free-free, rigid-rigid and rigid-free boundaries. It is observed that oscillatory modes not allowed for layer of ferrofluid heated from below. We have discussed the results numerically and graphically. The stationary convection curves in (R, a) plane for various values of magnetization M_3 and fixed values of M_1=1000, other parameters as shown in Fig. 2. It is clear that the linear stability criteria to be expressed in thermal Rayleigh number, below which the system is stable and unstable above. It has been found that the Rayleigh number decrease with increase in the value of magnetization M_3 thus magnetization M_3 destabilizing effect on the system. It is also found that stability of fluid layer in most stable in rigid-rigid boundaries and least stable free-free boundaries. It is also observed that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for all type of boundary conditions i.e free-free, rigid-rigid and rigid -free boundaries.

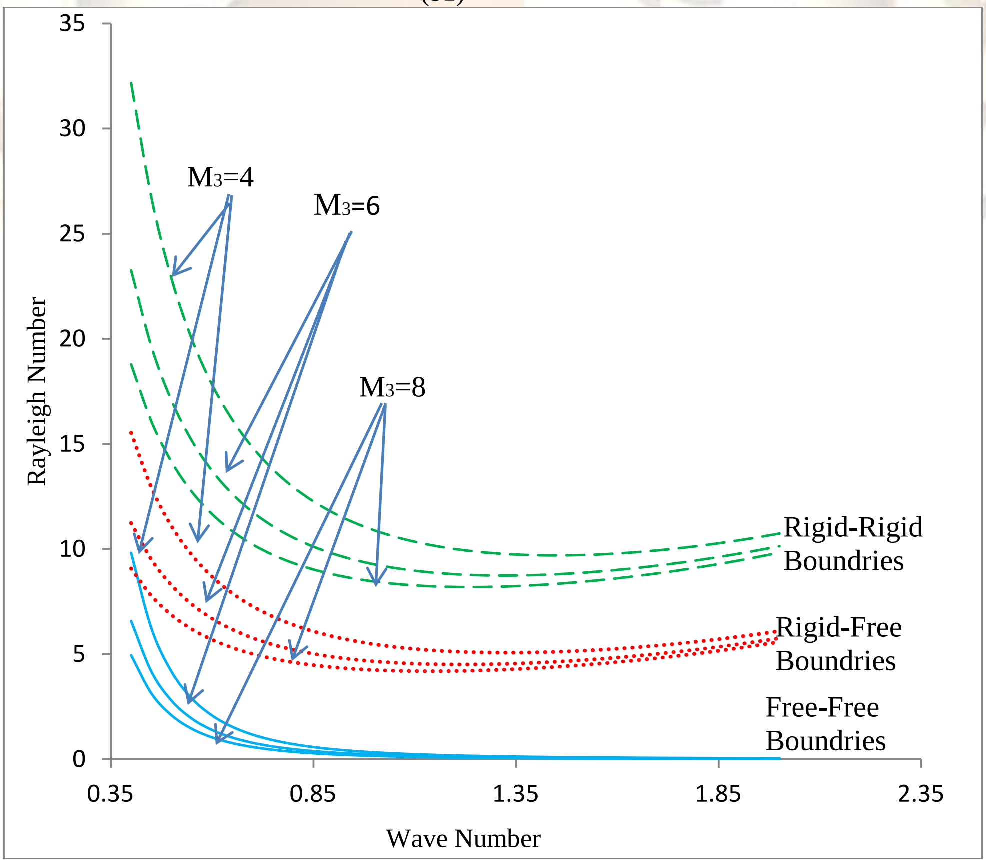

Fig.2 Variation of critical Rayleigh number R with wave number a for different value of magnetization M

VII. Conclusions

A linear analysis of thermal instability for ferrofluid for free-free, rigid-rigid and rigid -free boundaries is investigated. Galerkin-type weighted residuals method is used for the stability analysis. The behavior of magnetization on the onset of convection analyzed for all type of boundaries. Results has been depicted graphically.

The main conclusions are as follows:

1. For the case of stationary convection, the magnetization parameter destabilized the fluid layer for all type of boundary conditions.
2. The 'principle of exchange of stabilities' is valid for all type of boundary conditions.
3. The oscillatory modes are not allowed for the ferromagnetic fluid heated from below.
4. It is also found that stability of fluid layer in most stable in rigid-rigid boundaries and least stable in free-free boundary conditions.

References

[1] Blennerhassett, P. J., Lin, F. & Stiles, P. J.: Heat transfer through strongly magnetized ferrofluids, Proc. R. Soc. A 433, 165-177, (1991).

[2] Chandrasekhar, S.: *Hydrodynamic and hydromagnetic stability*, Dover, New York (1981)

[3] Finlayson, B.A.: Convective instability of ferromagnetic fluids. J. Fluid Mech. **40**, 753-767 (1970).

[4] Gupta, M.D., Gupta, A.S.: Convective instability of a layer of ferromagnetic fluid rotating about a vertical axis, Int. J. Eng. Sci. **17**, 271-277, (1979).

[5] Kuznetsov, A. V., Nield, D. A.: Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman Model, Transp. Porous Medium, 81, 409-422, (2010).

[6] Lalas, D. P. & Carmi, S.: Thermoconvective stability of ferrofluids. Phys. Fluids 14, 436-437, (1971).

[7] Mahajan A. : *Stability of ferrofluids: Linear and Nonlinear*, Lambert Academic Publishing, Germany, (2010).

[8] Rosensweig, R.E.: *Ferrohydrodynamics*, Cambridge University Press, Cambridge (1985).

[9] Shliomisa, M.I and Smorodinb, B.L.: Convective instability of magnetized ferrofluids, Journal of Magnetism and Magnetic Materials, 252,197-202, (2002).

[10] Stiles, P. J. & Kagan, M. J.:Thermo-convective instability of a horizontal layer of ferrofluid in a strong vertical magnetic field, J. Magn. Magn. Mater, 85, 196-198, (1990).

[11] Sunil, Sharma, D. and Sharma, R. C.: Effect of dust particles on thermal convection in ferromagnetic fluid saturating a porous medium, J. Magn. Magn. Mater. 288, 183-195, (2005).

[12] Sunil, Sharma, A. and Sharma, R. C.: Effect of dust particles on ferrofluid heated and soluted

[13] from below, Int. J. Therm. Sci. 45, 347-358, (2006).

[14] Sunil, Mahajan, A.: A nonlinear stability analysis for magnetized ferrofluid heated from below. Proc. R. Soc. A 464, 83-98 (2008).

[15] Venkatasubramanian, S., Kaloni, P.N.: Effects of rotation on the thermoconvective instability of a horizontal layer of ferrofluids. Int. J. Eng. Sci. **32**, 237–256 (1994).

[16] Zebib, A : Thermal convection in a magnetic fluid, J. Fluid Mech., 32 , 121-136, (1996).