Critical Review On Structural Dynamic Modification On Beam Structures

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Abstract:

The present paper illustrates the various developments in the field of structural dynamic modification (SDM). SDM techniques can be defined as the methods by which the dynamic behavior of a structure is improved by predicting the modified behavior brought about by adding modifications like those of lumped masses, rigid links, dampers, beams etc or by variations in the configuration parameters of the structures itself.

The theory of SDM started in the late 70s. But intensive research has taken place only after a decade and subsequently light has been put on this subject in recent years. The contribution of many researchers to this field has taken the subject to a new era of investigation. The modification in any structure to improve its natural frequencies has received a lot of attention in many areas as structure response is heavily influenced by its natural frequencies. FEM is a basic tool that is used for analysis of such structures, which simplifies the laborious calculations

Keywords: Beam Analysis, Eigen Values, Natural frequencies, SDM

Abbreviations:

SDM : Structural Dynamic Modification
3D : 3 Dimensional
DOF : Degrees of Freedom
IMAC : International Modal Analysis Conference
EM : Eigen value Modification
LEMP : Local Eigen Value Modification procedure
FRF : Frequency Response Function
LMM : Linear Modification Method
DMM ; Direct Modification Method
MUCO: Modal updating using constrained optimization
FEA : Finite Element Analysis
MAC : Modal Assurance Criterion
EMA : Experimental Modal Analysis
IESM : Iterative method or inverse Eigen sensitivity Method

I. Introduction:

The first useful structural element in the SDM process to be considered was a general 3D beam element (useful for beam and rib types of modification studies). However, there was an inherent problem when these realistic structural elements were used in conjunction with modal data obtained from a modal test. The Lack of rotational DOF was a major obstacle at the introduction of the SDM technique and still presents unique obstacles to efficient implementation. A great deal of research effort was expended in the 1980s to develop techniques for the estimation of rotational DOF as well as the development of structural elements

Since rotational measurements may not exist or be available, efforts were focused on approximations of general 3D structural elements using only available translational information. Beam approximations using 3-point bending equations were the first estimates to be used. These provided reasonably good results for systems that behaved with beam-like responses in the modified system characteristics. In general, the first ten years of the International Modal Analysis Conference (IMAC) were seen to be the birth and development of the Structural Dynamic Modification Technique. The development of the proportional and complex mode Eigen value modification technique with the computationally efficient Local Eigen value Modification Technique (LEMP) was the subject of many papers in the early years of IMAC. This was followed by the development of more realistic structural elements for component modification studies as well as system models from component modes. The development of tools to estimate rotational DOF was evident during the same period.

II. Origin for research -1980

Around in 1980s research was done by B.P. Wang *etal* on S.D.M. on some existing structures and presented the paper, 'Structural Dynamic Modification using Modal Analysis Data [1]. The Frequency Response Functions (FRF) of the modified structure was studied using the frequency response data of the existing structure. An experiment was performed on a 31 cm square Aluminum plate. Graphs were plotted with frequency on X axis and magnitude of transfer function on Y axis. For every step of the increase in mass it was seen that the accuracy of the synthesized function decreased.

Later B.P.Wang [2] studied the limitation of the local modification on the distribution of natural frequencies. For finite element models, it was shown that the distribution of natural frequencies cannot be altered arbitrarily by local modifications. Three components were taken for analysis viz, 1) 5 DOF system, 2) Truss Members and 3) Spring Mass System. In all the systems, it was seen that the fundamental frequency could not be raised after reaching a saturation point even by modifying the stiffness of the system.

In 1985, Fabrizo Pimazio and Lucion Ricciardiello [3] studied the effects of structural variations on the dynamic behavior of a system. Two structures were studied. The first structure was simple; as was used to correlate the energy of an area with the efficiency of a modification in the area itself. The second structure was more complex to permit analysis of perturbation formulae. The analysis revealed that even a small change to area of any structure having high energy content; brought a notable variation in the frequency and if the energy of the modified area is higher, the forecast and reference frequencies have less error. This error increased when the degree of modification was increased.

. The dynamic behavior of structure is usually modeled using the below equation,

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (1)$$

Where 'f' is the excitation applied to the structure,
'x' is the instantaneous deformation and
m, k and c represent the capacity of the structure to store kinetic and strain energy and dissipate energy viscously respectively.

The homogeneous form of equation (1) is solved in terms of an Eigen value problem

$$(\lambda^2 m + \lambda C + k)q = 0$$

The research work carried out by J.A Brandon *etal* [4] presents the analysis of the above equations in terms of matrix algebra where matrices are of the order 'n'. It is the utilization of the known solution of an existing problem to deduce the solution to a modified model.

A new modification approach was presented by A.Sestieri and W.D'Ambrogio [5]. The method may be used to handle useful applications and allows the achievement of optimal modifications accomplished with different dynamic requirements like FRF Modulas, response power spectral density and response mean square value. The structural modifications that can be accounted for are lumped masses, springs, viscous dampers, dynamic absorbers and stiffening rods.

The method devised by them involves 3 steps viz,.

(a) Synthesis of the modified structure: Here the FRF was used to express the dynamic behavior of the modified structure which is called as FRF matrix with 'N' degrees of freedom of order NxN.
(b) Definition of structural modification goals: Goal of structural modification is to limit the key response parameters with in established boundaries.
(c) Formation of the mathematical of structural modification: The problem stated above was put under non linear programming technique and is defined as a constrained optimization problem.

A further step in the area of optimization was presented by the same author in the paper- SDM Variables Optimization [6]. Here the inverse of optimization problem was presented and FRFs were used and not the modal parameters to define the dynamic behavior.

Two cases were examined viz considering a) Deterministic forces and b) Random Forces
It is to be noted that, when there is no sufficient information on the existing forces, the estimate of the dynamic behavior of the structure may be performed by means of the FRF. By it, it is possible to determine the response in several points of the structure starting from a general estimate of the existing force.

If the existing forces are known, it is possible to estimate the dynamic behavior of the structure by considering the response to such forces. The author also defines sensitivity analysis and applies it to the non linear optimization problems to reduce the number of design variables. By sensitivity analysis one can determine the effectiveness in performing the required goals.

` The applications of SDM were studied by A.Sesteiri [7]. The method as applied to two real life structures viz a) Reduction of kinematic vibration transmission path in case of engines and b) lowering of peaks of spindle drive point on a flexible manufacturing cell.

As the engine element is designed for static criteria, it is chosen for modification. Hence addition of small amount of masses at a few points on the engine showed significant sensitivity i.e. it reduced the vibration transmission. It was seen that the high frequency range is mass controlled and stiffness modification gave more negligible results.

In the latter case structural modification technique was applied on the Flexible Manufacturing Cell. The aim of this case was to improve the dynamic behavior of the machine by some modifications to it without hindering the normal machine operation. The sensitivity analysis was performed by considering effects of adding a single mass of increasing magnitude.

At first masses were added to the back of the machine. Only a very slight effect on the spindle related FRFs was seen. Then masses on the spindle housing were examined. The effect was not encouraging. Finally the effect of a dynamic absorber on the spindle housing was examined which showed a significant improvement in the spindle FRFs. But the cost of modification on the back was assessed to be ten times lower than the cost of modifications on the spindle housing. This demonstrated that the optimization technique based on FR is effective also on complex structures like machine tools.

H.N. Ozguven [8] showed that when the modification does not alter the DOF of the structure, then the computational speed is also increased. The method was applicable to complex systems and several inaccuracies were also eliminated.

III. After a decade- 1990

The dynamic behavior of a vibrating system after modification was studied by Y.M.Ram and J.J. Bleech [9]. A mass was freely attached to a vibrating system through a spring. This decreased all the natural frequencies of the modified system. The lowest Eigen value of this system determines the trend on which the Eigen values of the first system change and conversely.

S.Lammens *etal* [10] studied the modal updating and structural optimization of a tennis racket. Changes were applied to the Aluminum Tennis Racket and the dynamic characteristics were calculated. An experimental modal analysis test was performed. With the updated model, the structural optimization calculation was performed. Finally the results of the optimization were verified on the real structure. Ten iterations were done. The frequency differences between calculated and measured frequencies were less than 5%.

The effect of SDM was studied on the damped beam elements by R.K. Srivatsava and T.K. Kundra [11]. They formulated an algorithm using hysterically damped beam elements. Considering a numerical example it was observed that the modal damping is increased with increased damping of the added beam. Also there is a decrease in trend of vibration levels with increased damping.

In the event that a vibration problem exists, the question arises as to how the characteristics of the structure may be modified in such a manner as to minimize the problem. Staneley G.Hutton [12] presented a paper restricting to the case in which it is required to modify a natural frequency of the structure. For the modification of the structure two basic approaches were available. The author throws the light on these two approaches firstly Forward modification using modal truncation which is a trial and error method and an iterative procedure. Initially the change is made to the structure and the resulting equations are solved to check if the prescribed changes had the desired effect if not, further changes are made and the progress is repeated. In the second approach, algorithms are developed in which the prescribed changes to the natural frequency are specified as input data and the output defines the mass and stiffness changes required to the effect of the required modification. Such procedures are called as Inverse Modification Procedures. Here it is required to determine what structural changes will produce a prescribed changed in a given frequency. This problem is complicated by the fact that, in general there is no simple relationship between frequency change and structural change.

Structural Dynamic Modifications are the methods to obtain the modified system dynamic characteristics due to structural modifications without going for a repeated analytical solution or experimentation. Among the various methods of SDM, the perturbation method is very useful and gives reasonably accurate estimates of the dynamic characteristics. There are two methods in the perturbation,

1) Multistep Perturbation Method (also called as Modal Perturbation Method),
2) Single Step Perturbation Method.

S.S.A. Ravi, T.K. Kundra and B.C.Nakra [13] proposed a single step perturbation method for an Aluminum beam. The experiment was conducted on a simply supported beam for full constrained damping layer treatment. It was found that single step perturbation method is more accurate, has a faster convergence and computationally more efficient as compared to the multi step perturbation. The single step perturbation method gives more accurate results in case of very large modifications also except for partial coverage.

In 2000 A.Sestieri [14] gave a detailed description of SDM. Two different problems were usually considered i.e., 1) direct problem and 2) inverse problem

The direct problem consists in determining the effect of already established modifications.

Inverse problem is a typical designing problem and is complex.

Various relations were derived for the direct problem and inverse problem. Detailed descriptions of the modifications are dealt.

The considered elementary modifications are concentrated masses, springs, viscous dampers, dynamic absorbers and continuous rods.

It was concluded that when dealing with structural modifications either a direct problem or an optimistation problem could be considered Thus both modal and the FRF approaches were successful. But use of FRF data base was particularly appropriate, giving a satisfactory solution.

Later T.K.Kundra [15] gave a detailed description of SDM for an F type structure involving various mathematical models namely mass, stiffness and damping matrices of the equations of motion of a structure. Various mathematical equations were framed for the mass modifications and tuned absorber modifications.

Raleigh's method was widely used for Eigen value reanalysis. The following was the equation for small modifications,

$$d\Omega_r = \Omega_r \left[\frac{\{\Phi_r\}^T\{dK\}\{\Phi_r\}}{2\{\Phi_r\}^T\{K\}\{\Phi_r\}} - \frac{\{\Phi_r\}^T\{dM\}\{\Phi_r\}}{2\{\Phi_r\}^T\{M\}\{\Phi_r\}} \right]$$

For any changes of [dM] and [dK] changes in Ω can be approximately and readily calculated. The optimization aspect of SDM was also highlighted by the author.

B.C. Nakra [16] applied Perturbation method based on FEM to the structural systems in which PVC was used as the damping material. The modulas properties of this PVC increases with frequency and decreases with temperature. Sensitivity analysis can provide the ratio of the change of modal parameters such as natural frequency, mode shape etc with respect to that of design parameter such as mass, stiffness or damping.

In the research carried out by Nobuyuki okubu and Takeshitoi [17], the theoretical background of sensitivity analysis was described followed by verification using simple structures. The different sensitivity parameters considered are natural frequency, mode shape, operational deflection shape, relative motion, transmitted force, servo sensitivity and acoustic sensitivity. The first two sensitivities were described and applied to simple structure for verification of its effectiveness. By these methods the point of modification in the structure was identified to suppress the vibration. Later 5 sensitivities were also analysed and practical examples were illustrated.

R.K.Srivatasava [18] studied the SDM techniques as applied to various applications. The case of undamped and damped was also studied with the help of relevant equations. The different applications under study were SDM using spatial models, modal models, tuned absorber, using FRF and optimal SDM.

The solution for inverse problem for structural modification was studied by Tao Li Jimin [19]. Among the dynamic properties, changing a natural frequency was perhaps the most common objective of structural modification. The theoretical description of structural modification was explained and equations for mass and stiffness modification were presented along with a set of linear simultaneous equations for mass and stiffness; equations for mass under linear modification method (LMM) and stiffness modification by direct modification method (DMM) were also given which derives the answers explicitly by linear transformation. The validity and feasibility of LMM and DMM was verified by the author who revels that the DMM is a convenient approach as it reduces the numerical calculations to minimum than LMM as there was only one input for DMM.

IV. SDM for beams after 2000:

For the uncoupled beam structures the structural optimization method was applied using only FRF matrices by Hwa Park and Youn Sik Park [20]. The optimal structural modification was calculated by combining the results obtained by the Eigen value sensitivities and reanalysis through several iterations. Also a comparison of this was done with perturbation method. The comparison shows an advantage of the proposed experimental method over perturbation method as applied to the complex real structures. Inspite of the advantages extra efforts are needed for the measurements of FRFs to construct the full FRF values

The author gives a key equation for the above conclusions viz

$$H(\omega)f = H^b(\omega) + H^m(\omega)$$

Where H (ω) is the modal force matrix,

H^b (ω) is the FRF matrix of the baseline structure and

H^m (ω) is the FRF matrix of the modification structure.

A comparative study of SDM and sensitivity method approximation was done by Keng C.yap and David C.Zimmerman [21]. A numerical analysis was performed to compare the Eigen solution estimation accuracy of SDM and sensitivity method. The following observations are derived by the authors:

1) SDM may be more feasible than sensitivity method for applications involving Eigen value estimates especially when stiffness perturbation is predominant.

2) The natural frequencies are better with SDM method and mode shapes are better with sensitivity method.

A further research in the above area was enhanced by F.Aryana and H.Bahai`[22] studying sensitivity analysis and modification of Structural dynamic characteristics using second order approximation in Taylor expansion. The method proposed by the authors was based on a matrix treatment procedure for modifying stiffness and mass of the finite elements.

An algorithm was developed and four case studies were conducted. The four case studies are a plane truss structure, a plane stress model of a cantilever, plane stress mode of a bracket and a plane strain model of a dam. For a plane truss structure and plane stress cantilever the second derivative of the first Eigen vector showed good accuracy. Plane Stress Bracket and Plane strain Dam: The comparison between the first order and second order approaches follow the same pattern as the previous studies for these two case studies also.

Large modifications of frequencies can be conducted very efficiently with an acceptable level of accuracy. It was found that the proposed model always yields the exact Eigen frequencies with minimized numerical effort. Model was more computationally economical and optimization was carried in one step with no iterations.

In 2003 a review paper by Peter Avitable [23] was presented giving a detailed textual knowledge of SDM. The authors start from the basis of SDM. Inherent problems that are encountered when using these techniques are discussed with examples.
1) Eigen Value Modification (EM)
2) Local Eigen Value Modification Procedure (LEMP)

With the Eigen value modification approach only single modification could be performed. So this was replaced by LEMP.
The author describes various conditions for Structural Dynamic modification using response functions, rotational DOF, mode shape scaling, rigid body modes and damping conditions

A case study on dynamic design of Drilling Machine was done by Rajiv Singh Bais *etal* [24] Using Direct Method Based Updated F.E. model. Two aspects were studied for modal testing viz tests using instrumentation impact hammer and random noise generator & modal exciter. The updated analytical FE models from both studies had been used for SDM which predicted a fair degree of accuracy to obtain desired modal frequencies. Results show that natural frequencies are close to each other. The mode shape at II mode shows a good level of correlation than at I, III and IV mode.

Along with the above 3 authors A.K.Gupta [25] studied the FE formulation of the drilling machine. It includes the FE model updating using direct and indirect method. In the direct method the results obtained using FEM were compared with the experimental ones using mode shape comparison and MAC values. The indirect method is an iterative method. The updated FE model closely represents the actual machine tool structure. An improvement is seen in the MAC values over initial MAC values. Both studies predict that effect of modifications on the dynamic characteristics of the machine with a fair degree of accuracy and the procedure can be used with confidence in order to obtain desired modal frequencies.

Studies on Dynamic Design using updated models and its subsequent use for predicting the effects of structural modifications were done by S.V.Modak *etal* [26]. The two methods of model updating are:
a) Modal updating using constrained optimization (MUCO) and 2) Iterative method or inverse Eigen sensitivity Method (IESM) is briefly presented. In MUCO, the error between measured and analytical natural frequencies and the mode shapes is minimized.

In IESM the updating parameter corresponding to an analytical model are corrected to bring the analytical modal data close to that of the experimentally derived. Thus is an iterative method. The process is repeated until convergence is obtained. In SDM, the number of finite elements, the number of nodes and consequently the size of the modified model will be higher than that for the unmodified model.

Let $[K_m]$ and $[M_m]$ be the stiffness matrix and mass matrix of the modified model. Eigen values $[\lambda_m]$ and Eigen vectors $[\Phi_m]$ can be obtained by resolving the Eigen value problem.

i.e. $[K_m]\,[\Phi_m] = [M_m]\,[\Phi_m]\,[\lambda_m]$

Models like a fixed-fixed beam and F structure were considered for updation. Comparison of natural frequencies before and after modification showed that the values are very close to the actual changes. To improve the correlation the FE model is updated. First updating is carried out for matching only the first five natural frequencies and updating the three stiffness parameters. It is seen that the correlation of the natural frequencies had significantly improved. It was also seen that there is a very good match between the updated model FRF and the measured FRF. For the case of updating using MUCO, the mode shapes were included in the form of MAC based constraints. The results show that the MUCO updated model had given reasonable predictions of both the natural frequencies and the mode shapes for the case of mass modifications.

A beam modification was introduced by attaching stiffener at the end of the horizontal members. The MUCO updated model gave good reasonable predictions of both the natural frequencies and the mode shapes for this case also.

So it was concluded that selection of updating parameters during updating was very important for making reliable predictions.

Hua Peng Che [28] presented the efficient methods for determining the modified modal parameters in SDM when the modifications were relatively large. An efficient iterative computational procedure was proposed for determining the modified Eigen values and corresponding Eigen vectors for complex structural systems. The iterative procedure gave exact predictions.
relationship for modal updating. A high order approximation approach was also presented without iterative procedures which gave excellent estimates of the modified modal parameters.

Brian Schwarz *etal*i [28] presented FEA model updating using SDM. This paper gives a very detailed basic ideas and definitions on SDM, FEA, Modal updating etc. Two examples of beam structures were considered. In the first example a beam was taken and Modal analysis using FEA (analytical) and EMA (experimental) were performed. The results with FEA were less than its corresponding

Two approach

EMA values. Next modal updating is done by increasing the thickness of a back plate. There is a clear improvement in FEA modal frequencies and MAC values show negligible change in mode shapes. In example 2, an aluminum cantilever beam was taken. FEA and EMA tests were performed and results tabulated. It is evident that both the modal frequencies and shape are more closely matched following model updating.

This tool shows much promise for closing the gap between FEA and EMA results.

Experiments on the dynamic characteristics of a tube collector were conducted using reanalysis by Natasa Trisovic [29]. Reanalysis is a method by which the dynamic behavior is brought about by adding modifications like those of lumped masses, rigid links, dampers, beams etc. The study deals with improving of dynamic characteristics (of the ring cross section) by changing the boundary conditions and geometry. It was proved that the change of boundary conditions was the most efficient way to increase the natural frequencies. The ideal case would be to have both ends of the beam fixed, which was technically impossible. Only one fixed end and the other hinged gives improved dynamic characteristics.

SDM of vibrating system was studied by M.Nad [30]. The design and technological treatments were considered to achieve suitable vibration and acoustical properties of vibrating system. A short summary of the structural dynamic modification techniques and the general mathematical theory of the modification process was presented. Two case studies were also dealt viz i) SDM of a cantilever beam by the constraining visco elastics layers and ii) SDM of circular disc by in plane residual stresses.

The results obtained confirm that SDM is a very effective tool to change the dynamic properties of vibratory system.

Very often natural frequencies and mode shapes (i.e. Eigen values and Eigen vectors) of a FE model do not match very well with experimental measured frequencies and mode shapes obtained from a real life vibration test. Thus the FE model updating problem is how to incorporate the measured modal data into the FE model to produce an adjusted FE model with the modal properties that closely match the experimental modal data. Then the updated model may be considered to be a better dynamic representation of the structure.

Yong Xin Ynan presented a new method for FE model updating problem using minimization theory [31] and [32].

Two problems in the matrices were considered and theorems were developed. An algorithm was also generated. There was no iteration or Eigen analysis. The approach is demonstrated by a 10 DOF cantilever beam. Numerical results are produced which are reasonably good.

Later Vikas Arora *etal* [33] studied Damped FE model Updating using Complex Updating Parameters and its use for dynamic design. Here the F shaped structure which resembles the skeleton of a drilling machine was used to evaluate the effectiveness of complex parameters based on updating methods for accurate prediction of the complex FRFs

The experimental data is valid and more accurate modal updating aims at reducing the inaccuracies present in the analytical model in the light of measured test data. Surprisingly most of the updating method neglect damping. But all structures exhibit some form of damping, but despite a large literature on damping, it still remains one of the least well understood aspects of general vibration analysis. A model updating method should be able to predict the changes in dynamic characteristics of the structure due to potential structural modifications.

The results of the complex parameter based updating were compared with FE model updating with damping identification method and also the complex FRFs were also predicted.

The FRFs for the mass modified structure were then acquired. It was observed that the predicted dynamic characteristics of complex parameter based updated model are closer to the measured characteristics of the modified structure even at resonance and anti resonance frequencies.

A beam modification increased the size of mass and stiffness matrices. The mass and stiffness matrices for the modified structure were obtained assuming there was a little effect of the beam modification. The dynamic characteristics predicted by complex parameters based on updated model were closer to the measured characteristics of the modified structure. It was noticed that:

i) Complex parameter method was able to predict more accurately,

ii) Predicted FRFs for mass modification matched better than beam modification and

iii) Complex parameter based updated model could be used for dynamic design.

V. Conclusion:

Thus critical review on the Structural Dynamic Modification Techniques is done in this paper. The SDM for the beam structures is concentrated implementing it in various applications. All through the review the variables or parameters that are accounted for SDM are lumped masses, springs, viscous dampers, dynamic absorbers and stiffening rods. Though a lot of work is carried in the area of SDM, but research on the structures with damping is much awaited.

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