Analysis Of New Non Traditional Tolerance Stack Up Conditions

Dr. Dinesh Shringi*, Dr. Kamlesh Purohit**

*(Department of Mechanical Engineering, Jai Narain Vyas University, Jodhpur-342001)

** (Department of Mechanical Engineering, Jai Narain Vyas University, Jodhpur-342001)

ABSTRACT

Manufacturing industry is always on the lookout for ways and means to reduce cost and increase profitability. Tolerance stack up is term used for describing the problem solving process in designing and manufacturing to calculate the effect of accumulated variation that is allowed by specified dimensions and tolerances. The stack-up conditions based on worst case (WC) model and RSS model are not realistic in general, though these have been widely used in research because of their simplicity. To account for the realistic nature of the process distribution, a few modifications to these traditional approaches have been proposed.

In this paper some nontraditional stack up condition methods like modified RSS Spott's model and EMS are also analyzed to calculate accumulation of tolerance in assembly. A comparative cost analysis of different stack up models is solved by the combined Simulated Annealing and Pattern Search (SA-PS) algorithm. The application of proposed methodology has been demonstrated through simple shaft bearing examples.

Keywords - Functional Dimensions, Pattern Search Tolerance, Sequential Approach, Simulated Annealing Simultaneous Allocation, Tolerance Synthesis

1. Introduction

Tolerance design is one of the most essential requirements of the part design and manufacturing. In practice, two types of tolerances are often defined: Design tolerance and manufacturing tolerance. The design tolerances are related to the functional requirements of a mechanical assembly or components, whereas manufacturing tolerances are used in process plan, which must respect functional requirements as suggested by design tolerances. A common tolerance synthesis problem is to distribute the specified tolerance among the components of the mechanical assembly. This allocation of design tolerance among the components of a mechanical assembly and manufacturing tolerance to the machining process used in the fabrication of component plays a key role in the cost reduction and quality improvement. Unnecessarily high tolerances lead to higher manufacturing cost while loose tolerance may lead to malfunctioning of the product. Traditionally, this important phase of product design and manufacturing is accomplished intuitively to satisfy design constraints based on past designs, standards, hand books, skills and experience of the designer and process planner. Tolerance design carried out by this approach does not necessarily lead to optimal allocation. Therefore, tolerance allocation has been widely studied in the literature. The review of the research carried by several researcher [1,2,5,6,7,8] presented reveals that in general tolerance design is carried out sequentially in two steps (i) functional (or design) tolerance allocation, and (ii) distribution of these tolerances on different manufacturing operations involving process capability of the machine, machining allowance etc. This sequential approach has serious limitations (i) infeasibility of design tolerance from the point of view of availability of manufacturing facilities, and (ii) process planner may not able to utilize the space provided by design tolerance, which leads to sub-optimal distribution of tolerance.

In this paper an attempt has been made to develop a model for the comparison of individual tolerances and associated costs in the different stack up conditions with the help of *a simple component shaft -bearing assembly example.*

2. Tolerance Stack-up Conditions

In a mechanical assembly, individual components are seldom produced in unique sizes. Their functional dimensions can always be produced within some tolerance due to manufacturing and other limitations. Thus for a given set of tolerances associated with individual dimension, the tolerance accumulated on the assembly dimension needs to be estimated. This is usually called tolerance analysis. The accumulated tolerance on the assembly dimension must be equal to or less than the corresponding assembly tolerance specified by the designer based on the functional and assembly requirements. In the tolerance techniques, the different criteria used for establishing relation between accumulated tolerance on the assembly dimensions and the assembly tolerance is popularly called as tolerance stack-up conditions. Over the years various researcher's [4,9,10] have proposed different models for determining stack-up conditions. Each one has its own merits, demerits and area of applications. A few important models to determine the tolerance stack-up conditions are discussed below:

2.1 Worst Case Model

This model assumes that worst possible conditions for assembly where all the functional dimensions may attain extremities on same side simultaneously. This method results in very tight tolerance and hence a higher manufacturing cost. The worst case (WC) model is favored when all assemblies must be within the allowable variability and a few rejections are not possible.

Let $Y = f(x_i)$ be the assembly response function for a given dimension chain of the assembly.

For small changes in the functional independent dimension (x_i), the assembly response function can be expressed by a Taylor's series expansion as given below:

$$\Delta y = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \Delta x_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j} \Delta x_i \Delta x_j + \ldots. \quad \ldots\ldots.(1)$$

The common practice for tolerance analysis is to substitute tolerance 't' for the delta (Δ) quantities. For a worst case stack-up conditions model, only the first order terms are used and absolute values are placed on these terms. The Eq. (1) for the general worst case stack-up condition is given as:

$$t_y = \left|\frac{\partial f}{\partial x_1}\right| t_1 + \left|\frac{\partial f}{\partial x_2}\right| t_2 + \ldots. + \left|\frac{\partial f}{\partial x_n}\right| t_n$$

(2)

The partial derivatives of the assembly response function represent the sensitivity of the assembly tolerance to the component tolerances. These should be evaluated at the midpoint of the tolerance zone. The tolerance may be unilateral or equal bilateral.

If the assembly response equation is a linear sum of the components, then the first order terms are either positive or negative one and all higher terms are zero. Thus, for a given linear assembly response equation, the worst case tolerance analysis equation states that the assembly tolerance must be greater than or equal to the sum of the component tolerance i.e.

$$t_y = \sum_{i=1}^{n} \left(\left| \frac{\partial f}{\partial x_i} \right| t_i \right) \le T_k \qquad (3)$$

Where T_k is assembly tolerance and t_i is the tolerance on the i^{th} dimension of a tolerance chain.

2.2 Root-sum-square (RSS) Model

The Root sum square (RSS) model is also called as simple statistical method. It is based on the assumption that the tolerance is distributed normally with mean centered at the nominal value. This assumption is most idealistic one. The application of this model gives very loose tolerance and hence a lower manufacturing cost. The tolerance analysis equation for the RSS stack-up condition is given as:

$$t_y = \sqrt{\sum_{i=1}^{n} \left(\left| \frac{\partial f}{\partial x_i} \right|^2 t_i^2 \right)} \le T_k \quad \ldots.(4)$$

$$= \frac{Z}{3} \sqrt{\sum_{i=1}^{n} t_i^2} \le T_k \quad \ldots.(5)$$

Where Z = normal distribution parameter

Since the RSS model assumes that tolerances on the components are distributed normally with a mean at the midpoint of the tolerance zone, the standard deviation (σ) is usually assumed to be equal to one third (1/3) of the equal bilateral tolerance. When the tolerance limits are of $\pm 3\sigma$, there are 2.7 components per thousand, out of the tolerance, when Z = 3. This corresponds to an acceptance rate of 99.73 per cent. The value of Z may be increased for further fewer rejections. The RSS model permits larger component tolerances and hence at the reduced cost. The reduced component cost offsets and justifies the occasional assembly which falls out of tolerance with the resulting scrap or rework cost.

The stack-up conditions based on worst case (WC) model and RSS model are not realistic in general, though these have been widely used in research because of their simplicity. To account for the realistic nature of the process distribution, a few modifications to these traditional approaches have been proposed. These are discussed below:

2.3 Modified RSS Model

When tolerances on the component are not well approximated to normal distribution or the mean is not at the midpoint of the tolerance zone, a modified RSS model with correction factor C_f is used. The tolerance analysis equation for the above stack-up condition is given as:

$$t_y = C_f \frac{Z}{3} \left[\sqrt{\sum_{i=1}^{n} t_i^2} \right] \le T_k \quad \ldots.(6)$$

Where C_f = Correction Factor

= 1.5 recommended by Bender levy.

= 1.48 to 1.8, suggested by Gladman

This method has a limitation which is that when the number of components in an assembly is equal to two, the assembly tolerance predicted by modified RSS model is greater than worst case model.

2.4 Spott's Model

To account for the realistic nature of the process distribution, Spott [10] proposed a method based on the assumption that actual tolerance stack-up condition does neither follows worst case condition nor RSS model. Thus he averaged out the results obtained from the worst case and RSS conditions. Accordingly the tolerance equation is:

$$t_y = \frac{1}{2}\left[\sum_{i=1}^{n}\left|\frac{\partial f}{\partial x_i}\right| t_i + \sqrt{\sum_{i=1}^{n}\left|\frac{\partial f}{\partial x_i}\right|^2 t_i^{\,2}}\right] \le T_k$$

….(7)

Although, Spott's model predicted tolerances near to actual stack-up conditions in some applications. It is not universally acclaimed as it lacks justification about averaging the worst case and RSS stack-up conditions. Further, any percentage shift of the mean away from the tolerance mid point cannot be modeled accurately.

2.5 Estimated Mean Shift (EMS) Model

In general, manufacturing process do have process variability less than the tolerance range specified, otherwise, significant component rejection will occur, resulting in increased cost due to reworking or scrap. The greater the difference between the process variability and the tolerance ranges, the less frequent adjustment such as tool sharpening etc are needed to keep part dimensions within the tolerance limits. In addition, production processes are seldom controlled closely enough to keep the mean dimension exactly centered between the tolerance limits.

When mean shift occurs, the assembly tolerance accumulates and possibly result into an unexpected high assembly rejection rate. Even moderate tolerance means shift can produce significant assembly rejection as reported by Spott [10]. Figure 4.3 shows the effect of percentage mean shift. [7]

Greenwood and Chase [7] suggests a unified model called Estimated Mean Shift (EMS) which permits the inclusion of mean shift on the tolerance analysis. The method is based on resolving the component tolerance into two parts:

(i) Mean shift or bias from the tolerance midpoint (first moment of distributions), and

(ii) The variability about the mean (second moment of the distribution)

This is accomplished by selecting mean shift factor f_i for each component between 0 and 1.

The resulting tolerance sum has the following form:

$$t_y = \sum_{i=1}^{n} f_i t_i + \frac{z}{3}\sqrt{\sum_{i=1}^{n}\left(1-f_i\right)^2 t_i^{\,2}} \le T_k$$

….(8)

The first term of the summation (Eq. 8) is composed of estimated mean shift. It is treated as a worst case model when all shifts are assumed to combine to give the greatest assembly shift. The second term of summation represents the component variability and is treated as the sum of squares. This method is similar to the measurement or analysis done according to ASME power test code where bias and variability are calculated separately. In the second summation, each component variability is reduced by the factor (I – f_i). This assumes that the process variability is small for parts with a large mean shift. The component variability is still assumed to be equal to 3σ from the mean to the nearest tolerance limit.

The EMS model is also called as unified model. In this model, if the percentage mean shift is 100 percent it results into worst case stack-up condition; whereas zero percent mean shift is knows as RSS stack-up conditions.

3. Model Description

Journal bearing is a most common machine element used to take up load and support the shaft. In the present study a shaft bearing assembly having close running fit for running an accurate machine and for accurate location at moderate speed and journal pressure is chosen for tolerance synthesis. For this purpose, the most suitable class of fit, as per IS 919-1963, H_7/f_7 is considered

Let us assume that the diameter of shafts is 40 mm, therefore a shaft bearing assembly of 40 H_7/f_t class of fit is being synthesize for tolerances.

Figure (1) shows a shaft bearing assembly. It is a simple linear assembly involving only two part feature dimensions.

The detailed synthesis model is discussed below:

An objective function based on the minimization of assembly manufacturing cost is formulated which is obtained by summing up the cost of all the processes involved in manufacturing of shaft – bearing assembly. In the present study, the exponential cost function with valid range of tolerances as suggested by Zhang and Wang [11] and Al-Ansary and Deiab [2] has been chosen. The mathematical expression of exponential cost function is given below:

$$C(t) = C_0 e^{-C_1(t-C_2)} + C_3 \qquad \text{….. (9)}$$

where C_0, C_1 and C_2 = Constants of the cost–tolerance function.

The total manufacturing cost of an assembly can be expressed as: Total cost

$C = C_s + C_b$

Where C_s = Cost of manufacturing

$= C_{11}(t_{11}) + C_{12}(t_{12}) + C_{13}(t_{13}) + C_{14}(t_{14})$

4. Functional Dimensions

A mechanical assembly is composed of a number of individual components, which interacts with one another to perform a predefined task. The dimensions of individual components are called functional dimensions. Some functional dimensions affect the performance of the assembly more than others. Thus for the efficient functioning of an assembly, the functional dimensions are required to be controlled within a specified range. For example, the satisfactory functioning of a journal bearing and shaft assembly requires that hole size should be greater than shaft size. However, the clearance between bearing hole and shaft is governed by the theory of hydrodynamic lubrication so that sufficient pressure is built-up to bear external load (Figure 1). In general, the functional dimensions for the components (say shaft size X_i, and hole size X_2) are represented by X_i.

5. Assembly Response Function

The functional dimensions X_i of a mechanical component / assembly are independent variables. Their value depends upon the functional requirement of components and the process capability of the manufacturing process. These dimensions form a dimension chain which result into one or more assembly response function Y. The relation between functional dimensions X_i and the assembly dimension is known as assembly response function and is expressed as:

$Y = f(X_i)$ …..(9)

A mechanical assembly may have one or more than one assembly response functions, depending upon type of dimensional chain formed as per functional requirement of the assembly. Thus, the K^{th} assembly response function of such an assembly can be written as:

$Y_k = f_k(X_i)$ …..(10)

For example, assembly response function of a shaft bearing assembly (Figure 1) is expressed as:

$Y = X_2 - X_1$ ……(11)

Where

Y = Clearance between bearing hole and shaft diameter

X_1 = Diameter of the shaft

X_2 = Bore size of the bearing hole

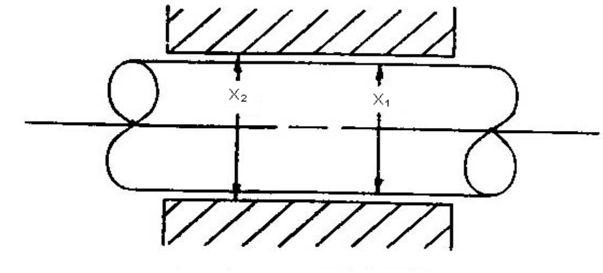

Figure 1 A Shaft Bearing Assembly

Y = Clearance between bearing hole and shaft diameter

X_1 = Diameter of the shaft

X_2 = Bore size of the bearing hole

TABLE 1: Constants for cost-tolerance function for manufacturing shaft- bearing

Manufacturing Operations	Constants C_o	C_1	C_2	C_3	Minimum Tolerance t^L (mm)	Maximum Tolerance t^U (mm)
Shaft						
(i) Rough turning C_{11} (t_{11})	8.5	10.7	0.05	1.5	0.05	0.50
(ii) Finish turning C_{12} (t_{12})	10.6	24.7	0.02	2.3	0.02	0.10
(iii)Rough Grinding C_{13} (t_{13})	10.3	41.3	0.005	3.2	0.005	0.03
(iv)Finish Grinding C_{14} (t_{14})	18.0	161.2	0.002	4.9	0.002	0.01
Bearing						
(i) Drilling C_{21} (t_{21})	6.2	8.4	0.07	2.1	0.07	0.50
(ii) Boring C_{22} (t_{22})	8.6	22.8	0.03	2.8	0.03	0.10
(iii)Finish Boring C_{23} (t_{23})	12.4	36.5	0.006	4.3	0.06	0.05
(iv) Grinding C_{24} (t_{24})	20	57.3	0.003	5.8	0.003	0.02

TABLE 2 Comparison of Optimal Tolerance Allocation and associated cost for Shaft Bearing Assembly for different stack up conditions

Criteria	Shaft		Bearing		Δy and Cost
	Tol. Notation	Tolerances (mm)	Tol. Notation	Tolerances (mm)	
WC	t_{11}	0.4299	t_{21}	0.43498	Δy =0.0249 Rs 58.72
	t_{12}	0.0700	t_{22}	0.0650	
	t_{13}	0.0300	t_{23}	0.0350	
	t_{14}	0.0099	t_{24}	0.0150	
RSS	t_{11}	0.4300	t_{21}	0.4300	Δy =0.0223 Rs 56.68
	t_{12}	0.0700	t_{22}	0.0700	
	t_{13}	0.0300	t_{23}	0.0300	
	t_{14}	0.0099	t_{24}	0.0200	
SM	t_{11}	0.4300	t_{21}	0.4312	Δy =0.0231 Rs 57.10
	t_{12}	0.0700	t_{22}	0.068	
	t_{13}	0.0300	t_{23}	0.0312	
	t_{14}	0.0100	t_{24}	0.0187	
EMS (m_1=0.5 m_2=0.5)	t_{11}	0.429	t_{21}	0.4312	Δy =0.0231 Rs 57.10
	t_{12}	0.0700	t_{22}	0.0680	
	t_{13}	0.0300	t_{23}	0.0315	
	t_{14}	0.0100	t_{24}	0.0185	

6. Conclusion

The objective function is minimized subjected to design, manufacturing and process limit constraints. In the present study a comparison is demonstrated between various stack up conditions like WC and RSS, Spott's criteria and estimated mean shift criteria are evaluated. Optimal tolerance allocation for all eight manufacturing operations (required to manufacture a shaft bearing assembly) are reported in Table 2. A close look to the Table 2 shows that WC criteria give tight tolerances while RSS leads to loose tolerance. Hence the cost of manufacturing a shaft bearing assembly with RSS is less than what required for WC conditions.

Table(2); also shows that cost of manufacturing with Spotts criteria lies between two extreme conditions i.e. WC and RSS criteria. This is due to the fact that Spotts Criteria takes the average value of both WC and RSS condition. The optimal tolerances obtained for estimated mean shift (EMS) criteria, at 50 percent mean shift, are also reported in Table (2) It is found that at 50 percent mean shift the optimization results are same as found in the case of Spotts criteria. This shows that theory of averaging suggests by Spott criteria represents 50 percent mean shift in EMS model. The effect of variation in percent mean shift on the tolerance cost indicates that tolerance cost varies with percent mean shift exponentially. The accumulated design tolerances, for all four conditions are also reported in the Table (2)

References

[1] AL-ANSARY M.D., DEIAB I.M. 1997 "Concurrent Optimization of Design and Machining Tolerance using Genetic Algorithms Method", *International Journal of Machine Tools & Manufacture, vol. 37*, pp. 1721-1731.

[2] CHASE K.W., GREENWOOD W.H., LOOSLI, B.G. and HAUGLUND, L.F., 1990, *Least Cost Tolerance Allocation for Mechanical Assemblies with Automated Process Selection",* Manufacturing Review, vol. 3, pp. 49-59.

[3] CHASE K.W. and GREENWOOD W.H., 1988, "Design Isues in Mechanical Tolerance Analysis", *Manufacturing Review, vol. 1*, pp. 50-59.

[4] BJQRKE O. 1989, "Computer – Aided Tolerancing ", 2nd Ed. (New York : ASME Press)

[5] DONG Z., 1997, "Tolerance Synthesis by Manufacturing Cost Modeling and Design Optimization" H.C. Zhang (ed.), Advanced Tolerancing Techniques (Wiley), pp. 233-260.

[6] FATHI Y., MITTAL R.O., CLINE, J.E. and MARTIN, P.M., 1997, "Alternative Manufacturing Sequences and Tolerance buildup*",* *International Journal of production Research, vol. 35*, pp.123-136.

[7] GREENWOOD, W.H. and CHASE, K.W., 1987, "New Tolerance Analysis Method for Designers and Manufacturers", Journal of Engineering

for Industry, *Transactions of the ASME, vol. 109*, pp. 112-116.

[8] GREENWOOD, W.H. and CHASE, K.W., 1990. "Root Sum Squares Tolerance Analysis with Non-linear Problems", Journal of Engineering for Industry, *Transactions of the ASME, vol. 112*, pp. 382-384

[9] SPECKHART, F. H., 1972, "Calculation of Tolerance Based on a Minimum Cost Approach*", Journal of Engineering for Industry, Transactions of the ASME*, 447-453.

[10] SPOTTS, M.F.,1973, "Allocation of Tolerances to Minimize Cost of Assembly", *Journal of Engineering for Industry, Transactions of the ASME,* 762-764.

[11] ZHANG, C., WANG, H. P. and LI, J.K., 1992, "Simultaneous Optimization of Design and Manufacturing Tolerance with Process (Machine) Selection*", Annals of the CIRP, vol. 41/1*, pp. 569-572.