

## Twisted Amplitude Phase Shift Keying

### ABSTRACT

Several modulation techniques are defined in digital modulation, Here we are discussing about in four modulation techniques, 8-PSK, 8-TAPSK, 16-TAPSK and 16-QAM. In the paper we derive minimum non coherent distance (dnc) of block coded TAPSK and compare it with different modulation techniques, i.e. 8-PSK and 16-QAM using linear component codes. If the block length N is very small, NBC-16-TAPSK performs best among all non-coherent scheme and NBC-16QAM perform worse due to its small minimum non-coherent distance. However, if the block length N is not short, NBC-16QAM has the best error performance because the code words with small non-coherent distances are rare.

**GENERAL TERMS**-Non-coherent detection, block coded modulation (BCM), multilevel coding (MLC), bit error rate (BER), Signal to noise ratio (SNR).

**KEYWORDS**-AWGN, Rayleigh, Flat fading channel

### I. INTRODUCTION

The main aim of the paper is to develop the efficient (more bits per symbol) and errorless modulation (error immune to noise) coding techniques and compare the TAPSK from other modulation coding techniques, i.e. 8PSK, 16PSK, 8QAM and 16QAM. Non-coherent detection is a useful technique because it does not require carrier phase tracking. In a non-coherent receiver, no assumption of the carrier phase is made by the receiver, while communication is possible without knowing the carrier phase. Sometimes it is not possible to track the carrier phase. For ex. IS95 CDMA reverse link, many simulations superimposed signals from many unsynchronized sources, so we use non-coherent detection. Non-coherent blockcodes using Additive white Gaussian noise (AWGN) channel, were proposed in [2]-[5], including non-coherent block coded MPSK (NBC-MPSK) [4], [5]. AWGN Channel is a wireless channel, we can detect the signal and decode by employing several replicas of the received. So, we consider multilink receiver structure. The performance of data transmission over wireless channels is well captured by observing their BER (Bit error rate), which is a function of SNR at the receiver. SNR is the ratio of the received signal strength over the noise strength in the frequency range of the operation. BER is inversely

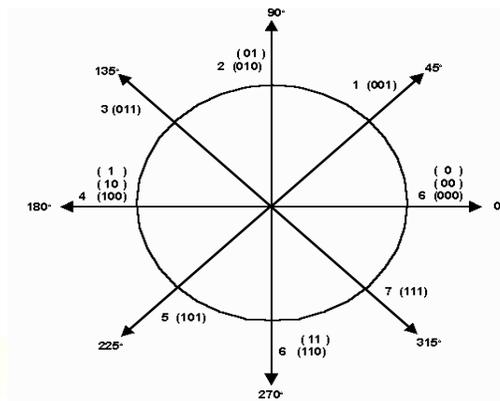
related to SNR, that is high BER causes low SNR. High BER causes increase in packet loss, increase in delay and decrease in throughput in wireless channels, several models have been proposed and investigated to calculate SNR. All the models are a function of the distance between the sender and the receiver, the path loss exponent and the channel gain. The AWGN channel which introduces an unknown carrier phase rotation has been investigated in many works, for ex. [1]-[8]. This channel offers a useful abstraction of the flat fading channel, Fading refers to the distortion that a carrier-modulated telecommunication signal experiences over certain propagation media. When the effects of the phase rotation need to be studied independently of the amplitude variations, block coded modulation (BCM) codes are much easier to construct (or design) than TCM codes. Combining block coding and channel signal sets to construct bandwidth efficient codes is referred to as block coded modulation (BCM). The most powerful method for constructing BCM codes is the multilevel coding (MLC) technique. It is devised by Imai and Hirakawa in 1976. This chapter is devoted to the multistage decoding of these codes. It is a powerful coded modulation scheme capable of achieving both bandwidth and communication. The key idea behind the MLC is to protect the individual bits using different binary codes and use M-ary signal constellation.

A simple model that is commonly used is one where the unknown carrier phase is constant over a block of N symbols, and independent from block to block, [1], [2]. This model is correct for frequency hopping systems. For this non-coherent channel with large N, pilot symbols used for the carrier phase estimation combined with codes designed for coherent decoding perform well. However, for small N, block codes designed for non-coherent decoding outperform this training-based non-coherent code [6], [7].

In [5], the minimum non-coherent distance of block coded MPSK (M-ary phase shift keying) was derived and a new non-coherent block coding scheme called non-coherent block-coded MPSK (NBC-MPSK) was proposed. In [7], a non-coherent distance measure for arbitrary energy signals was derived and non-coherent block coded

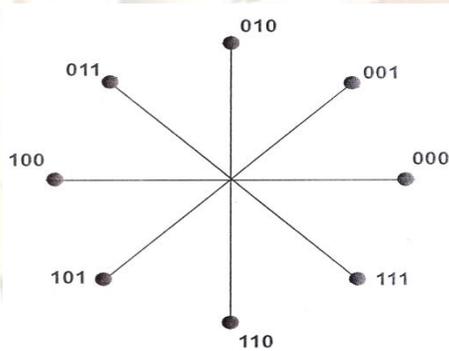
TAPSK (Twisted amplitude and phase shift keying) (NBC-TAPSK) scheme whose component codes is nonlinear was proposed.

In this paper, we consider state constellation diagram.



**LEGEND:**  
 0°...315° = PHASE (DEGREES)  
 0...7 = TRIBIT NUMBERS  
 (000)...(111) = THREE-BIT CHANNEL SYMBOLS  
 (00)...(11) = TWO-BIT CHANNEL SYMBOLS  
 (0)...(1) = ONE-BIT CHANNEL SYMBOLS

Figure 1.State constellation diagram



TRIBIT	PHASE
000	0
001	45
010	90
011	135
100	180
101	225
110	270
111	315

Figure 2. 8 PSK State constellation diagram

Here, we are showing the 8PSK Constellation diagram. In this fig. when we transmit single bit, two possible output comes 0 and 1 and we get 100% error free modulation, when we transmit two bits ,four possible output comes 00,01,10,11 and we get 50% accuracy .when we transmit three bits eight possible output comes 000,001,010,011,100,101,110,111 and get 33% accuracy.

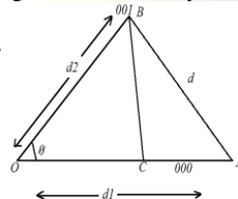


Figure3. 8PSK Distance diagram

From figure,  $OC = OB \cos \phi$

$$AC = OA - OC$$

$$BC = OB \sin \phi$$

$$d^2 = BC^2 + AC^2$$

$$= (OA - OC)^2 + BC^2 \quad (\text{Since, } OA = d_1, OB = d_2)$$

$$= (OA^2 + OC^2 - 2OA \cdot OC) + (OB \sin \phi)^2$$

$$= d_1^2 + (OB \cos \phi)^2 - 2d_1(OB \cos \phi) + (d_2 \sin \phi)^2$$

$$= d_1^2 + d_2^2 \cos^2 \phi - 2d_1 d_2 \cos \phi + d_2^2 \sin^2 \phi$$

$$d = d_1^2 + d_2^2 (\cos^2 \phi + \sin^2 \phi) - 2d_1 d_2 \cos \phi$$

$$d = d_1^2 + d_2^2 - 2d_1 d_2 \cos \phi \quad \text{since, } \cos^2 \phi + \sin^2 \phi = 1$$

1) when  $\phi = \pi/4, d_1 = d_2 = 1$

$$d_0 = 1^2 + 1^2 - 2(1)(1) \cos(\pi/4)$$

$$= 2 - 2(1/\sqrt{2}) = 2 - \sqrt{2} = 2 - 1.414 = 0.586$$

2.) When  $\phi = \pi/2, d_1 = d_2 = 1$

$$d_1 = 1^2 + 1^2 - 2(1)(1) \cos(\pi/2)$$

$$= 2 - 0 = 2$$

3.) When  $\phi = \pi/3, d_1 = d_2 = 1$

$$d_2 = 1^2 + 1^2 - 2(1)(1) \cos(135)$$

$$= 2 - 2(0.707) = 4$$

So, calculated minimum distances  $d_0, d_1$  and  $d_2$  are 0.586, 2 and 4.

Now, we consider block-coded TAPSK (Twisted amplitude phase shift keying) and 16QAM (quadrature-amplitude modulation) for non-coherent detection, both using linear component codes.

#### A. TAPSK

Figure 4 and 5 shows the signal constellations of 8TAPSK ( $\phi = \pi/4$ ) and 16TAPSK ( $\phi = \pi/8$ ) respectively. We generalize TAPSK by setting  $0 \leq \phi \leq \pi/4$  for 8TAPSK and  $0 \leq \phi \leq \pi/8$  for 16TAPSK. Note that if  $\phi = 0$ , generalized-TAPSK becomes APSK (amplitude and phase shift keying). The minimum non-coherent distances of block-coded generalized-TAPSK and 16QAM using linear component codes are derived. A new NBC-TAPSK scheme and non-coherent block-coded 16QAM (NBC-16QAM) are proposed accordingly.

For the transmitted baseband codeword  $\mathbf{x} = (x_1, x_2, \dots, x_M)$ , the received baseband block  $\mathbf{y} = (y_1, y_2, \dots, y_M)$  is given by  $\mathbf{y} = \mathbf{x} \exp\{j\theta\} + \mathbf{n}$  where  $\mathbf{n}$  is a block of independent zero-mean complex Gaussian noise and  $\theta$  is an

arbitrary phase shift which is assumed to be constant for  $\mathbf{y}$ . The squared non-coherent distance between  $\mathbf{x}_1$  and  $\mathbf{x}_2$  is  $d^2_{nc}(\mathbf{x}_1, \mathbf{x}_2) = \frac{||\mathbf{x}_1||^2 + ||\mathbf{x}_2||^2}{2} - |(\mathbf{x}_1, \mathbf{x}_2)|$  where,  $(\mathbf{x}_1, \mathbf{x}_2)$  denotes the complex inner product between  $\mathbf{x}_1$  and  $\mathbf{x}_2$  [7].

Each signal point in the signal constellation of 8TAPSK, shown in Fig. 4, is labeled by  $(a, b, c, d)$  where  $a, b, c, d$  and  $d \in \{0, 1\}$ . Let  $(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2), \dots, (a_N, b_N, c_N, d_N)$  be a block of transmitted signals. If  $\mathbf{ca} = (a_1, a_2, \dots, a_N), \mathbf{cb} = (b_1, b_2, \dots, b_N), \mathbf{cc} = (c_1, c_2, \dots, c_N)$  and  $\mathbf{cd} = (d_1, d_2, \dots, d_N)$  are code words of binary block codes  $\mathcal{C}_a, \mathcal{C}_b, \mathcal{C}_c$  and  $\mathcal{C}_d$ , respectively, a multilevel block-coded 8TAPSK is obtained. For  $i \in \{a, b, c, d\}$ , the minimum non-coherent Hamming distance of  $\mathcal{C}_i$  is defined by  $d_{ncH,i} = \min\{d_{i,\min}, N - d_{i,\max}\}$  where  $d_{i,\min}$  and  $d_{i,\max}$  denote the minimum and maximum values of Hamming distance between any two code words corresponding to different data bits in  $\mathcal{C}_i$ , respectively [5].

For TAPSK with labeling in Fig. 4, the bit in level  $a$  decides the symbol energy. The radii of the inner and outer circles are denoted by  $r_0$  and  $r_1$ , respectively. The values of  $r_0$  and  $r_1$  ( $r_0 \leq r_1$ ) satisfy  $r_0^2 + r_1^2 = 2$  when  $a=0$  has the

same probability as  $a=1$ . Define  $r = r_1/r_0$ .

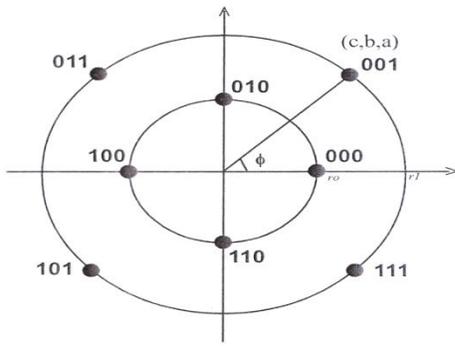


Figure 4. 8TAPSK

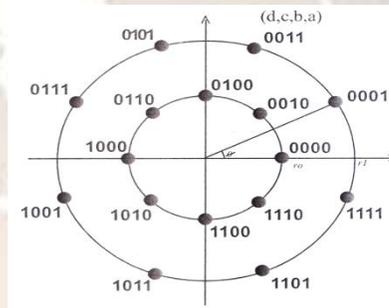


Figure 5. 16TAPSK

From  $d = d_1^2 + d_2^2 - 2d_1d_2\cos\phi$ ,

1.) When  $\phi = \pi/4, d_0=0.5, d_1=1$   
 $d_0 = 0.5^2 + 1^2 - 2(0.5)(1)\cos(\pi/4)$   
 $= 0.25 + 1 - 1(1/\sqrt{2})$   
 $= 0.54$

2.) When  $\phi = \pi/2, d_0=0.5, d_1=1$

$$d_1 = 0.5^2 + 1^2 - 2(0.5)(1)\cos(\pi/2)$$

$$= 0.25 + 1 - 0$$

$$= 1.25$$

3.) When  $\phi = 2\pi/3, d_0=0.5, d_1=1$

$$d_2 = 0.5^2 + 1^2 - 2(0.5)(1)\cos(2\pi/3)$$

$$= 0.25 + 1 - 0(0.9993)$$

$$= 0.251$$

4.) When  $\phi = \pi, d_0=0.5, d_1=1$

$$d_3 = 0.5^2 + 1^2 - 2(0.5)(1)\cos(\pi)$$

$$= 0.25 + 1 - 0.998$$

=0.002

So, calculated minimum distances  $d_0, d_1, d_2$  and  $d_3$  are 0.54, 1.25, 0.251 and 0.002.

For block-coded generalized-8TAPSK C whose component codes are all linear, the minimum squared non-coherent distance is  $d_{nc}^2 = \min\{d_{nc,0}^2, d_{nc,1}^2, d_{nc,2}^2\}$ .

Because  $d_{nc,0}$  reaches its maximum value when  $\phi = \pi/4$  and  $\phi = \pi/8$  for generalized-8TAPSK and generalized-16TAPSK, respectively, we use TAPSK instead of generalized-TAPSK in the remainder of this paper. When  $r = 1$ , i.e. TAPSK becomes MPSK, we have  $f(N) = 0$  and  $f(d) = f(N - d) \forall d$ . Consequently,  $d_{nc,0}^2$  of block-coded MPSK is equal to  $f(d_{ncH,0})$ .

Therefore, for block-coded MPSK,  $C_a$  should be a binary block code with large  $d_{ncH,0}$ . We proposed NBC-MPSK in [5] by setting  $d_{0,max} = N - d_{0,min}$  such that  $d_{ncH,0} = d_{0,min}$  at the price of sacrificing one data bit. But as  $r$  increases,  $(N)$  also increases. For block-coded 8TAPSK where  $r$  is large enough,  $(N) = (r1 - r0)2N/2$  can be larger than  $(d_{0,min})$ . If  $r > 1.61238$ ,  $(N)$  is always larger than  $f(d_{0,min})$  for any value of  $d_{0,min} (d_{0,min} \leq N/2)$ .

In such case, since  $d_{nc,0}^2 = f(d_{0,min})$ ,  $C_a$  could be a normal code with large  $d_{a,min}$ , and thus the one-bit loss is unnecessary. The proposed NBC-TAPSK is defined in the following. The definition of component codes except  $C_a$  is the same as NBC MPSK (Minimum phase shift keying). We choose  $r$  and  $d_{a,min}$  to satisfy  $f(N) \geq f(d_{a,min})$ , and let  $C_b$  be a code designed for  $d_{a,min}$ . The minimum non-coherent distance of NBC-TAPSK can be easily obtained, which is  $d_{nc}^2 = \min\{d_{a,min}^2, r^2 0(N - \sqrt{(N - d_{b,min})^2 + d_{b,min}^2}), 2r^2 0d_{c,min}\}$  for NBC-8TAPSK.

**B. QAM**

Figure-6 shows the signal constellation diagram for 16 QAM. QAM is the encoding of the information into a carrier wave by variation of the amplitude of both the carrier wave and a “quadrature” carrier that is 90 degrees out of phase with the main carrier accordance with two input signals. That is, the amplitude and the phase of the carrier wave are simultaneously changed according to the information you want to transmit.

In 16-in 16-state Quadrature Amplitude Modulation (16-QAM), there are four I values and four Q values. This results in a total of 16 possible states for the signal. It can transition from any state to any other state at every symbol time. Since  $16 = 2^4$ , four bits per symbol can be sent. This consists of two bits for I and two bits for Q. The symbol rate is one fourth of the bit rate. So this modulation format produces a more spectrally efficient transmission. It is more efficient than BPSK, QPSK or 8PSK. Note that QPSK is the same as 4-QAM.

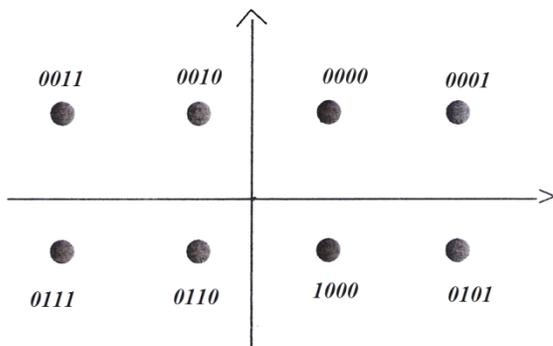


Figure-8 QAM Constellation diagram  
 SIMULATION RESULTS AND DISCUSSION

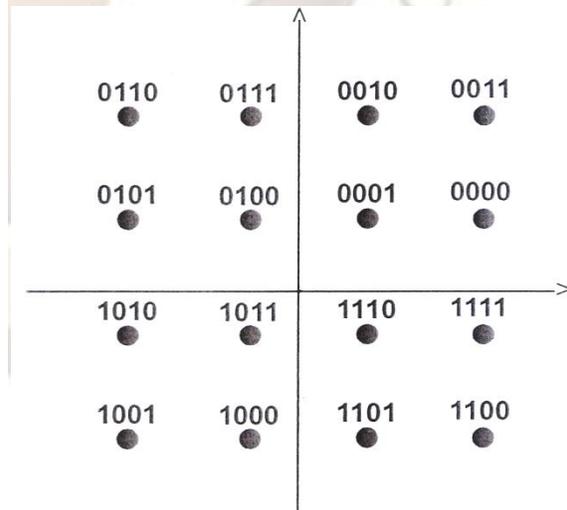


Figure -16 QAM Constellation diagram

The channel used in the simulations is the AWGN channel as [1].NBC-8TAPSK is compared with NBC-8PSK and NBC-16TAPSK is compared with NBC-16PSK.For NBC-TAPSK, we search for the value of r that need the lowest SNR at a BER of 10<sup>-6</sup> according to the simulation results.

TABLE-1

Here,  $r_2/r_1 = 0.5, 1, 1.5, 2$

VALUES OF EFFICIENCY FOR DIFFERENT MODULATION TECHNIQUE

r(r1/r0)	Efficiency						
	8PSK	16PSK	8QAM	16QAM	8TAPSK(L)	8TAPSK(H)	16TAPSK
0.5	2.7244	3.1181	2.4488	3.4488	2.4488	2.6693	2.4488
1.0	2.7244	3.1181	2.4488	3.4488	2.7244	2.7244	2.7244
1.5	2.7244	3.1181	2.4488	3.4488	2.7795	2.7795	2.7795
2.0	2.7244	3.1181	2.4488	3.4488	2.8346	2.8346	2.8346

For NBC-16TAPSK, Table 1 compares the best values of r for simulation with the theoretical best values of r that maximize dnc (mimumnon-coherent distance).From figure1, 8QAM has better BER than 8TAPSK(L) and 16TAPSK.

In table 1,whenr=0.5,efficiency for 8PSK is 2.7244,efficiency for 16PSK is 3.1181,efficiency for 8QAM,8TAPSK(L) and 16TAPSK are 2.4488,efficiency for 16QAM and 8TAPSK(H) are 3.4488 and 2.6693respectively.when r=1,efficiency for 8PSK,16PSK,8QAM,16QAM are 2.7244,3.1181,2.4488,3.4488 respectively. Efficiency for 8TAPSK(L),8TAPSK(H) and 16TAPSK is 2.7244.when r=1.5,efficiency for 8PSK,16PSK,8QAM and 16QAM are 2.7244,3.1181,2.4488 and 3.4488,efficiency for 8TAPSK(L),8TAPSK(H) and 16TAPSK is 2.7795.when r=2,efficiency for 8PSK,16PSK,8QAM and 16QAM are 2.7244,3.1181,2.4488 and 3.4488,efficiency for 8TAPSK(L),8TAPSK(H) and 16TAPSK is 2.8346.

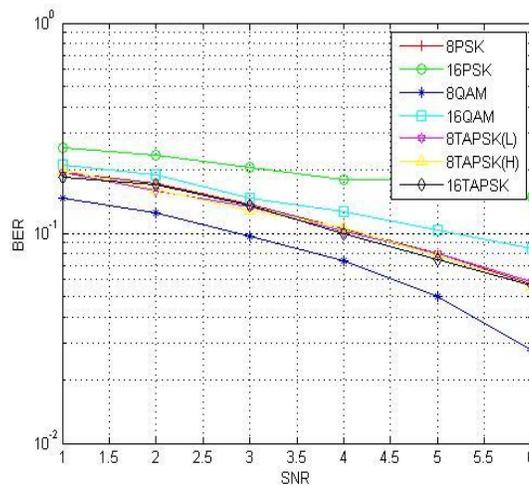


Figure1- Simulation results of non-coherent block codes at r=1

**TABLE -2**

Block length (N) = 15, 31, 63,127

VALUES OF EFFICIENCY FOR DIFFERENT MODULATION TECHNIQUE

Block length(N)	Efficiency						
	8PSK	16PSK	8QAM	16QAM	8TAPSK(L)	8TAPSK(H)	16TAPSK
15	2.7244	3.1181	2.4488	3.4488	2.4488	2.6693	2.4488
31	2.7244	3.1181	2.4488	3.4488	2.4488	2.6693	2.4488
63	2.7244	3.1181	2.4488	3.4488	2.4488	2.6693	2.4488
127	2.7244	3.1181	2.4488	3.4488	2.4488	2.6693	2.4488

In table-2,For block length N=15,31,63 and 127,determine the values of efficiency for different modulation technique. Here, efficiency for 8PSK,16PSK,8QAM,16QAM,8TAPSK(L),8TAPSK(H),16TAPSK are 2.7244,3.1181,2.4488,3.4488,2.4488,2.6693 and 2.4488.

Figure 2,presents the result for N=63,NBC-16TAPSK has better BER than 8QAM and 16QAM, but they all do not decrease exponentially because the average number of code words with small non-coherent distances is little, but not little enough. For ideal coherent decoding, NBC-16TAPSK is worse than NBC 16QAM.

But for non-coherent decoding, NBC-16TAPSK is better than NBC-16QAM at high SNRs which agrees with the minimum non-coherent distance analysis. For NBC-16QAM, the gap between non-coherent decoding and ideal coherent decoding is quite wide. At high SNRs, the pilot-optimized 16QAM outperforms NBC-16QAM, and NBC-16TAPSK is the best among all non-coherent schemes.

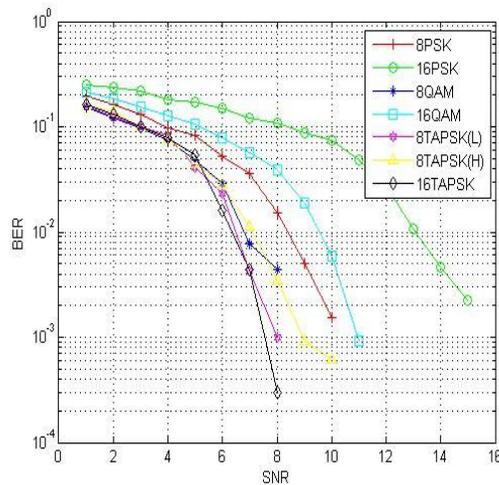


Figure2- Simulation results of non-coherent block codes at N=63

TABLE-3

Here, Minimum distance( $d_{min}$ ) = 4,5,6,7

VALUES OF EFFICIENCY FOR DIFFERENT MODULATION TECHNIQUE

Minimum Distance	Efficiency						
	8PSK	16PSK	8QAM	16QAM	8TAPSK(L)	8TAPSK(H)	16TAPSK
4	2.7244	3.1181	2.4488	3.4488	2.4488	2.6693	2.4488
5	2.6693	2.8976	2.2835	3.2283	2.3386	2.6142	2.3386
6	2.6142	2.8425	2.1181	3.0079	2.1732	2.5039	2.1732
7	2.5039	2.6772	2.4488	3.4488	2.4488	2.6693	2.6772

In table3,we determine the values of efficiency for different modulation technique at different minimum distances i.e.4,5 and 7.

Figure3,presents the result for  $d_{min}(\text{minimum distance})=4$ ,NBC-TAPSK(L) has better BER than 8QAM and 16TAPSK.

In table 1,when  $d_{min}=4$ ,efficiency for 8TAPSK(L) and 16 TAPSK is 2.4488,efficiency for 8TAPSK(H) is 2.6693.when  $d_{min}=5$ ,efficiency for 8TAPSK(L) and 16 TAPSK is 2.3386, efficiency for 8TAPSK(H) is 2.6142.when  $d_{min}=6$ , efficiency for 8TAPSK(L) and 16 TAPSK is 2.1732, efficiency for 8TAPSK(H) is 2.6693.

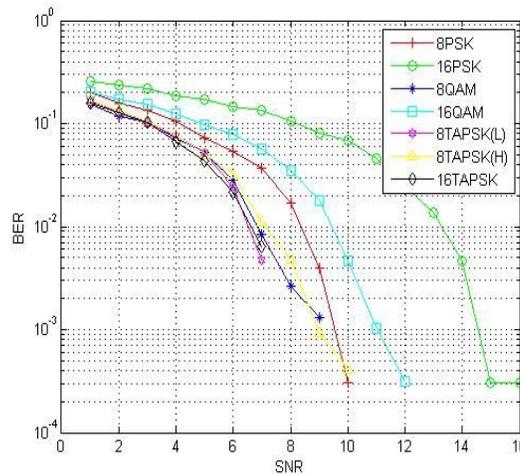


figure3- Simulation results of non-coherent block codes at  $d_{min}=4$

## II. CONCLUSION AND FUTURE WORK

In this paper, we propose the NBC-TAPSK (Non-coherent block coded-Twisted amplitude phase shift keying)scheme and derive the minimum non-coherent distance of block coded TAPSK and 16QAM using linear component codes. The minimum noncoherent distance of block-coded QAM with more signal points can be derived similarly. We find that the minimum non-coherent distance of block-coded MPSK derived in [5] is a special case of the derived minimum non-coherent distance of block-coded TAPSK. According to the derived distances, we propose NBC-TAPSK and NBC-16QAM. The comparison of minimum noncoherent distances

shows the superiority of NBC-TAPSK over NBC-MPSK at high data rates. We compare various non-coherent block codes based on the simulation results. If the block is very short, NBC-16QAM has worse error performance due to its small minimum non-coherent distance, and NBC-16TAPSK has the best error performance. But if the block length is not small, NBC-16QAM has the best error performance because the code words with small non-coherent distances becomes rare.

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