Maximum Likelihood and Bayes Estimators of the Unknown Parameters For Exponentiated Exponential Distribution Using Ranked Set Sampling

Amal S. Hassan*
*(Department of Mathematical Statistics in Institute of Statistical Studies and Research (ISSR).Cairo University, Egypt)

ABSTRACT

Estimation of the shape and scale parameters of exponentiated exponential distribution is considered based on simple random sample and ranked set sample. Bayesian method of estimation under squared error loss function and maximum likelihood method will be used. Comparison between estimators is made through simulation via their absolute relative biases, mean square errors, and efficiencies. Comparison study revealed that the Bayes estimator is better than maximum likelihood estimator under both sampling schemes. The results show that the estimators based on ranked set sample are more efficient than that from simple random sample at the same sample size.

***Keywords* -** Bayes; Estimation; Ranked Set Sampling; Simple Random Sample; Exponentiated exponential distribution.

I. Introduction

A method of sampling based on ranked sets is an efficient alternative to simple random sampling that has been shown to outperform simple random sampling in many situations by reducing the variance of an estimator, thereby providing the same accuracy with a smaller sample size than is needed in simple random sampling. Ranked set sample (RSS) can be applied in many studies where the exact measurement of an element is very difficult (in terms of money, time, labour and organization) but the variable of interest, although not easily measurable, can be relatively easily ranked (order) at no cost or very little additional cost. The ranking can be done on the basis of visual inspection, prior information, earlier sampling episodes or other rough methods not requiring actual measurement.

RSS was first suggested by McIntyre [1] and was supported by Takahasi and Wakimoto [2] by mathematical theory. Dell and Clutter [3] showed that RSS is more efficient than simple random sampling even with an error in ranking. Samawi et al. [4] suggested using extreme ranked set samples for estimating a population mean. Muttlak [5] introduced median ranked set sampling to estimate the population mean. Al-Saleh and Al-Kadiri [6] considered double ranked set sample, as a procedure that increases the efficiency of the RSS estimator without increasing the set size *m*. It was shown that the double ranked set sample estimator of the mean is more efficient than that using RSS. Al-Saleh and Muttlak [7] used RSS in Bayesian estimation for exponential and normal distributions to reduce Bayes risk. Al-Hadhrami [8] studied the estimation problem of the unknown parameters for the modified Weibull distribution. Maximum likelihood estimators for the parameters were also investigated mathematically and numerically. The numerical results show that the estimators based on RSS are more efficient than that from SRS. Helui et al [9] studied the estimation of the unknown parameter for Weibull distribution under different sampling. Methods of estimation used are ML, moments and Bayes. They concluded that estimators based on RSS and modified RSS have many advantages over those that are based on simple random sample (SRS).

This article is concerned with the maximum likelihood (ML) and Bayes estimators of the shape and scale parameters of exponentiated exponential based on SRS and RSS. Bayes estimators under squared error loss function are discussed assuming gamma prior distribution for both parameters. The performance of the obtained estimators is investigated in terms of their absolute relative biases (ARBs) and mean square errors (MSEs). Relative efficiency of the estimators is also calculated.

The rest of the article is organized as follows. In Section 2, ML and Bayesian methods of estimation of unknown parameters are discussed under SRS. In Section 3, the same methods of estimation are discussed based on RSS.

Numerical illustration is carried out to illustrate theoretical results in Section 4. Simulation results are displayed in Section 5. Finally, conclusions are presented in Section 6.

II. Estimation of Parameters Using SRS

This Section discusses the ML and Bayes estimators under squared error loss function for the unknown parameters of EE distribution under SRS.

2.1 Maximum Likelihood Estimation

Gupta and Kundu [10] proposed an exponentiated exponential (EE) distribution as an alternative to the gamma and Weibull distributions and studied its different properties. EE has the following probability density function (PDF)

$$f(x,\theta,\lambda)=\theta\lambda(1-e^{-\theta x})^{\lambda-1}e^{-\theta x}, \qquad \theta,\lambda,x>0$$

Let $X_1, X_2,..., X_n$ be independent and identically distributed random variables from EE, then the log-likelihood function, $L(\theta,\lambda)$, is

$$L(\theta,\lambda)=n\ln(\theta)+n\ln(\lambda)-\theta\sum_{i=1}^{n}x_i$$
$$+(\lambda-1)\sum_{i=1}^{n}\ln(1-e^{-\theta x_i}).$$

The normal equations become:

$$\frac{\partial L}{\partial\theta}=\frac{n}{\theta}+(\lambda-1)\sum_{i=1}^{n}\frac{x_i}{e^{\theta x_i}-1}-\sum_{i=1}^{n}x_i=0, \qquad (1)$$

$$\frac{\partial L}{\partial\lambda}=\frac{n}{\lambda}+\sum_{i=1}^{n}\ln(1-e^{-\theta x_i})=0. \qquad (2)$$

Obviously, it is not easy to obtain a closed form solution for the two non-linear equations (1) and (2). So, iterative procedure must be applied to solve these equations numerically. Newton Raphson method is used to obtain ML estimates of θ and λ, say, $\tilde{\theta}_{MLE}$ and $\tilde{\lambda}_{MLE}$.

2.2 Bayesian Estimation

Assume that θ and λ are independent random variables. Following Kundu and Gupta [11], it is assumed that θ and λ have the following gamma prior distributions;

$$\pi_1(\lambda)\,\alpha\,\lambda^{b-1}e^{-a\lambda}, \quad \lambda>0;$$
$$\pi_2(\theta)\,\alpha\,\theta^{d-1}e^{-c\theta}, \quad \theta>0; \qquad (3)$$

where, all the hyper parameters a; b; c; d are assumed to be known and non-negative. Suppose $\underline{x}=(x_1,x_2,...,x_n)$ is a random sample from EE, then based on the likelihood function of the observed data;

$$L(\underline{x}|\lambda,\theta)=\theta^n\lambda^n e^{-\theta\sum_{i=1}^{n}x_i}\prod_{i=1}^{n}(1-e^{-\theta x_i})^{\lambda-1}.$$

Using the priors given in equations (3), then the joint posterior density function of θ and λ can be written as;

$$\pi(\theta,\lambda|\underline{x})=\frac{L(\underline{x}|\theta,\lambda)\pi_1(\lambda)\pi_2(\theta)}{\int_0^\infty\int_0^\infty L(\underline{x}|\theta,\lambda)\pi_1(\lambda)\pi_2(\theta)\,d\lambda\,d\theta}.$$

Marginal posterior of a parameter is obtained by integrating the joint posterior distribution with respect to the other parameter and hence the marginal posterior of λ can be written as

$$\pi(\lambda|\underline{x})=\frac{\int_0^\infty\theta^{n+d-1}\lambda^{n+b-1}e^{-(a\lambda+c\theta+\theta\sum_{i=1}^{n}x_i)}\prod_{i=1}^{n}(1-e^{-\theta x_i})^{\lambda-1}d\theta}{H}, \qquad (4)$$

where,

$$H=\int_0^\infty\int_0^\infty\theta^{n+d-1}\lambda^{n+b-1}e^{-(a\lambda+c\theta+\theta\sum_{i=1}^{n}x_i)}\prod_{i=1}^{n}(1-e^{-\theta x_i})^{\lambda-1}d\lambda\,d\theta$$

Similarly integrating the joint posterior with respect to θ, the marginal posterior θ can be obtained as

$$\pi(\theta|\underline{x})=\frac{\int_0^\infty\theta^{n+d-1}\lambda^{n+b-1}e^{-(a\lambda+c\theta+\theta\sum_{i=1}^{n}x_i)}\prod_{i=1}^{n}(1-e^{-\theta x_i})^{\lambda-1}d\lambda}{H}. \qquad (5)$$

The Bayes estimators for parameters λ and θ of EE distribution under squared error loss function may be defined, respectively, as

$$\tilde{\lambda}_{bays}=E(\lambda|\underline{x})=\int_0^\infty\lambda\,\pi(\lambda|\underline{x})\,d\lambda,$$
$$\tilde{\theta}_{bays}=E(\theta|\underline{x})=\int_0^\infty\theta\,\pi(\theta|\underline{x})\,d\theta. \qquad (6)$$

There are no explicit forms for obtaining estimators for the EE, therefore, numerical solution and computer facilities are needed.

III. Estimation of Parameters Using RSS

The aim of this Section is to find the ML and Bayes estimators for the unknown parameters of EE distribution under RSS.

3.1Maximum Likelihood Estimation

RSS is recognized as a useful sampling technique for improving the precision and increasing the efficiency of estimation when the variable under consideration is expensive to measure or difficult to obtain but cheap and easy to rank.

The procedure of using RSS is as follows:

Step1: Randomly draw m random sets with m elements in each sample and ranked them (without actual measuring) with respect to the variable of interest.

Step 2: From the first set, the element with the smallest rank was chosen. From the second set, the element with the second smallest rank was chosen. The procedure was continued until the element with the largest rank from the m[th] sample was chosen. This procedure yielded a total number of m elements chosen to be measured, one from each sample.

Step 3: Repeat the above steps k times (k cycles) until the desired sample size ($n=mk$) is obtained for analysis.

Suppose that $X_{(i)ic}$., $i=1...m, c=1...k$ is a ranked set sample from EE distribution, with sample size $n=mk$, m is the set size and k is the number of cycles. For simplicity, let $Y_{ic} = X_{(i)ic}$, then for fixed c, Y_{ic} are independent with PDF equal to PDF of i[th] order statistics and given by

$$g(y_{ic}) = \frac{m!}{(i-1)!(m-i)!}[F(y_{ic})]^{i-1} f(y_{ic})[1-F(y_{ic})]^{m-i}$$

$$y_{ic} > 0,$$

The likelihood function of the sample $y_{1c}, y_{2c}, ..., y_{mc}$

$$L(\underline{y}|\theta,\lambda) = \prod_{c=1}^{k}\prod_{i=1}^{m} \frac{m!\theta\lambda e^{-\theta y_{ic}}(1-e^{-\theta y_{ic}})^{\lambda i-1}[1-(1-e^{-\theta y_{ic}})^{\lambda}]^{m-i}}{(i-1)!(m-i)!} \qquad (7)$$

The log-likelihood function, is given by

$$Log\,L = C + mk(\ln\lambda + \ln\theta) + \sum_{c=1}^{k}\sum_{i=1}^{m}(\lambda i-1)\ln(1-e^{-\theta y_{ic}})$$

$$-\theta\sum_{c=1}^{k}\sum_{i=1}^{m} y_{ic} + \sum_{c=1}^{k}\sum_{i=1}^{m}(m-i)\ln[1-(1-e^{-\theta y_{ic}})^{\lambda}],$$

where, c is a constant.

Differentiate the log likelihood with respect to λ and θ and equating to zero

$$\frac{\partial Log\,L}{\partial\theta} = \frac{mk}{\theta} - \sum_{c=1}^{k}\sum_{i=1}^{m} y_{ic} + \sum_{c=1}^{k}\sum_{i=1}^{m}\frac{(\lambda i-1)y_{ic}}{e^{\theta y_{ic}}-1}$$

$$-\lambda\sum_{c=1}^{k}\sum_{i=1}^{m}\frac{(m-i)(1-e^{-\theta y_{ic}})^{\lambda-1}e^{-\theta y_{ic}}y_{ic}}{1-(1-e^{-\theta y_{ic}})^{\lambda}}, \qquad (8)$$

$$\frac{\partial Log\,L}{\partial\lambda} = \frac{mk}{\lambda} - \sum_{c=1}^{k}\sum_{i=1}^{m}\frac{(m-i)\ln(1-e^{-\theta y_{ic}})}{(1-e^{-\theta y_{ic}})^{-\lambda}-1} \qquad (9)$$

$$+\sum_{c=1}^{k}\sum_{i=1}^{m} i\ln(1-e^{-\theta y_{ic}}).$$

The ML estimators of θ and λ, say $\bar{\theta}_{MLE}$, $\bar{\lambda}_{MLE}$, are the solution of the two nonlinear equations (8) and (9). Since it is difficult to find a closed form solution for the parameters, numerical technique is needed to solve them.

3.2 **Bayesian Estimation**

Using the priors defined in equations (3) and the likelihood function given in equation (7), then the joint posterior density function of θ and λ under RSS can be written as;

$$\pi(\theta,\lambda|\underline{y}) = \frac{L(\underline{y}|\theta,\lambda)\pi_1(\lambda)\pi_2(\theta)}{\int_0^{\infty}\int_0^{\infty} L(\underline{y}|\theta,\lambda)\pi_1(\lambda)\pi_2(\theta)\,d\lambda\,d\theta}.$$

Marginal posterior of a parameter is obtained by integrating the joint posterior distribution with respect to the other parameter and hence the marginal posterior of λ can be written, as

$$\pi(\lambda|\underline{y}) = \frac{H_2}{H_1}, \qquad (10)$$

where,

$$H_2 = \int_0^{\infty}(m!)^{mk}\theta^{mk+d-1}\lambda^{mk+b-1}e^{-(a\lambda+c\theta+\theta\sum_{c=1}^{k}\sum_{i=1}^{m}y_{ic})} \times$$

$$\prod_{c=1}^{k}\prod_{i=1}^{m}\frac{(1-e^{-\theta y_{ic}})^{\lambda i-1}[1-(1-e^{-\theta y_{ic}})^{\lambda}]^{m-i}}{(i-1)!(m-i)!}d\theta,$$

and,

$$H_1 = \int_0^{\infty}\int_0^{\infty}\frac{(m!)^{mk}(\theta)^{mk+d-1}(\lambda)^{mk+b-1}e^{-(a\theta+c\lambda+\theta\sum_{c=1}^{k}\sum_{i=1}^{m}y_{ic})} \times}{\prod_{c=1}^{k}\prod_{i=1}^{m}\frac{(1-e^{-\theta y_{ic}})^{\lambda i-1}[1-(1-e^{-\theta y_{ic}})^{\lambda}]^{m-i}}{(i-1)!(m-i)!}d\theta\,d\lambda}.$$

Similarly integrating the joint posterior with respect to θ, the marginal posterior θ can be obtained as

$$\pi(\theta|\underline{y}) = \frac{H_3}{H_1}, \qquad (11)$$

$$H_3 = \int_0^{\infty}(m!)^{mk}\theta^{mk+d-1}\lambda^{mk+b-1}e^{-(a\lambda+c\theta+\theta\sum_{c=1}^{k}\sum_{i=1}^{m}y_{ic})} \times$$

$$\prod_{c=1}^{k}\prod_{i=1}^{m}\frac{(1+e^{-\theta y_{ic}})^{\lambda i-1}[1-(1-e^{-\theta y_{ic}})^{\lambda}]^{m-i}}{(i-1)!(m-i)!}d\lambda$$

The Bayes estimators for parameters λ and θ of EE distribution under squared error loss function may be defined, respectively, as

$$\bar{\lambda}_{bays} = E(\lambda|\underline{y}) = \int_0^{\infty}\lambda\,\pi(\lambda|\underline{y})\,d\lambda,$$

$$\bar{\theta}_{bays} = E(\theta|\underline{y}) = \int_0^{\infty}\theta\,\pi(\theta|\underline{y})\,d\theta. \qquad (12)$$

These integrals cannot be obtained in a simple closed form; therefore, numerical solution is applied.

IV. Simulation Procedure

It is very difficult to compare the theoretical performances of the different estimators proposed in the previous Sections. Therefore, this Section presents the numerical solutions to obtain the ML and Bayes estimators of the unknown parameters θ and λ for the EE distribution based on RSS and SRS. A comparison studies between these estimators will be carried out through MSEs, ARBs and relative efficiency. Monte Carlo simulation is applied for different set sizes, different number of cycles and different parameter values. The simulation procedures are described through the following algorithm

Step 1: A random sample of size n = 10, 15, 16, 20, 24 and 30 with set size m=(2,3), number of cycles k = (5,8,10), where $n = m \times k$ are generated from EE distribution.

Step 2: The parameters values are selected as $\lambda = 0.5(0.3)1.1$ for $\theta = 1$ in estimation procedure.

Step 3: For the chosen set of parameters and each sample of size n, eight estimators ($\tilde{\theta}_{MLE}$, $\tilde{\lambda}_{MLE}$,

$\tilde{\theta}_{Bayes}$ $\tilde{\lambda}_{Bayes}$, $\bar{\theta}_{MLE}$, $\bar{\lambda}_{MLE}$, $\bar{\lambda}_{Bayes}$ and $\bar{\lambda}_{Bayes}$) are computed under SRS and RSS.

Step 4: Repeat the pervious steps from 1 to 3 N times representing different samples, where N=1000. Then, the ARBs and MSEs of the estimates are computed.

Step 5: Compute the efficiency of estimators, that defined as, Efficiency = MSE(SRS)/MSE(RSS).

V. Simulation Results

All simulated studies presented here are obtained via MathCAD (14). The results are reported in Tables 1 and 2. Table 1 contains the estimates of parameters for EE under SRS and RSS for different value of sample sizes and different value of parameters. Table 2 contains the efficiency of estimators for SRS relative to RSS. From Tables 1 and 2 many conclusions can be made on the performance of both method of estimation based on RSS and SRS. These conclusions are summarized as follows:

1. Based on SRS, ARBs and MSEs for the estimates of θ and λ are greater than the corresponding in RSS.

2. For both method of estimation, it is clear that ARBs and MSEs decrease as set sizes increase for fixed value of λ.

3. As the value of λ increases, the ARBs and MSEs increase in almost all of the cases.

4- The MSEs for the Bayes estimates of both parameters θ and λ is smaller than the MSEs for the MLEs of θ and λ in almost all the cases under both SRS and RSS, in sense that,

$\bar{\theta}_{Bayes}$ is better than $\bar{\theta}_{MLE}$ based on SRS

$\bar{\lambda}_{Bayes}$ is better than $\bar{\lambda}_{MLE}$ based on SRS

$\tilde{\theta}_{Bayes}$ is better than $\tilde{\theta}_{MLE}$ based on RSS

$\tilde{\lambda}_{Bayes}$ is better than $\tilde{\lambda}_{MLE}$ based on RSS

5- Comparing the biases of estimators, it is noted that the Bayes estimates have the minimum biases in almost all of the cases expect few cases.

6. It is clear from Table 2 that the efficiency of estimators increases as the sample sizes increase. The estimators based on RSS have smaller MSE than the corresponding ones based on SRS. The efficiency of RSS estimators with respect to SRS estimator is greater than one and increases when the sample size increases.

VI. Conclusions

This article deals with the estimation problem of unknown parameters of EE distribution based on RSS. ML and Bayesian methods of estimation are used. Bayes estimates are obtained under squared error loss function. Comparing the performance of estimators, it is observed that the Bayes performs better than ML relative to their ARBs and MSE's. Furthermore, ARBs and MSEs of the estimates for both shape and scale parameters relative to RSS are smaller than the corresponding SRS. This study revealed that the estimators based on RSS are more efficient than the corresponding SRS.

Table 1: Results of simulation study of ARBs and MSEs of estimates for different values of parameters (λ, θ) for EE distribution under SRS and RSS.

	RSS $(\lambda=0.5, \theta=1)$				SRS $(\lambda=0.5, \theta=1)$			
(m,k)	$\bar{\lambda}_{MLE}$	$\bar{\lambda}_{Bayes}$	$\bar{\theta}_{MLE}$	$\bar{\theta}_{Bayes}$	$\tilde{\lambda}_{MLE}$	$\tilde{\lambda}_{Bayes}$	$\tilde{\theta}_{MLE}$	$\tilde{\theta}_{Bayes}$
(2,5)	0.288 0.043	0.123 0.039	0.370 0.163	0.439 0.217	0.650 0.054	0.332 0.052	0.656 0.172	0.461 0.280
(2,8)	0.173 0.033	0.096 0.031	0.173 0.158	0.268 0.165	0.557 0.044	0.197 0.044	0.650 0.169	0.216 0.215
(2,10)	0.135 0.020	0.081 0.028	0.173 0.154	0.083 0.129	0.392 0.038	0.109 0.040	0.649 0.165	0.192 0.184
(3,5)	0.195 0.036	0.107 0.037	0.272 0.162	0.382 0.168	0.574 0.046	0.297 0.050	0.656 0.171	0.358 0.215
(3,8)	0.115 0.015	0.080 0.028	0.147 0.116	0.082 0.116	0.301 0.029	0.082 0.039	0.574 0.130	0.250 0.171
(3,10)	0.104 0.011	0.062 0.010	0.137 0.029	0.062 0.019	0.240 0.025	0.073 0.022	0.572 0.059	0.096 0.054

Continued Table 1

	RSS $(\lambda=0.8, \theta=1)$				SRS $(\lambda=0.8, \theta=1)$			
(m,k)	$\bar{\lambda}_{MLE}$	$\bar{\lambda}_{Bayes}$	$\bar{\theta}_{MLE}$	$\bar{\theta}_{Bayes}$	$\tilde{\lambda}_{MLE}$	$\tilde{\lambda}_{Bayes}$	$\tilde{\theta}_{MLE}$	$\tilde{\theta}_{Bayes}$
(2,5)	0.343 0.339	0.276 0.089	0.196 0.233	0.301 0.255	0.451 0.345	0.276 0.125	0.540 0.345	0.336 0.266
(2,8)	0.116 0.118	0.127 0.065	0.105 0.118	0.219 0.185	0.293 0.121	0.230 0.101	0.532 0.200	0.239 0.237
(2,10)	0.088 0.079	0.068 0.045	0.093 0.079	0.123 0.114	0.193 0.095	0.130 0.075	0.425 0.135	0.179 0.219
(3,5)	0.254 0.138	0.157 0.085	0.194 0.131	0.236 0.225	0.337 0.146	0.234 0.120	0.535 0.217	0.252 0.256
(3,8)	0.079 0.031	0.048 0.036	0.054 0.032	0.096 0.088	0.169 0.068	0.118 0.063	0.404 0.080	0.135 0.196
(3,10)	0.065 0.023	0.047 0.024	0.036 0.020	0.070 0.066	0.168 0.062	0.011 0.044	0.388 0.060	0.107 0.185
	RSS $(\lambda=1.1, \theta=1)$				SRS $(\lambda=1.1, \theta=1)$			
(m,k)	$\bar{\lambda}_{MLE}$	$\bar{\lambda}_{Bayes}$	$\bar{\theta}_{MLE}$	$\bar{\theta}_{Bayes}$	$\tilde{\lambda}_{MLE}$	$\tilde{\lambda}_{Bayes}$	$\tilde{\theta}_{MLE}$	$\tilde{\theta}_{Bayes}$
(2,5)	0.405 0.311	0.124 0.188	0.341 0.266	0.227 0.280	0.593 0.388	0.365 0.280	0.452 0.310	0.338 0.466
(2,8)	0.230 0.176	0.065 0.111	0.128 0.174	0.113 0.104	0.558 0.228	0.273 0.235	0.445 0.278	0.202 0.198
(2,10)	0.156 0.173	0.058 0.085	0.109 0.102	0.105 0.085	0.478 0.225	0.171 0.199	0.300 0.206	0.155 0.197
(3,5)	0.318 0.183	0.076 0.169	0.220 0.244	0.190 0.167	0.582 0.238	0.316 0.345	0.451 0.294	0.260 0.310
(3,8)	0.144 0.090	0.051 0.064	0.089 0.097	0.070 0.083	0.470 0.180	0.148 0.159	0.288 0.197	0.086 0.195
(3,10)	0.116 0.085	0.049 0.043	0.081 0.038	0.036 0.077	0.456 0.171	0.065 0.150	0.086 0.102	0.083 0.189

Note: The first entry is the simulated about ARBs.
The second entry is the simulated about MSEs.

Table 2: Efficiency of the estimators under SRS with respect to RSS in both estimation methods.

	Efficiency of λ $(\lambda=0.5, \theta=1)$		Efficiency of θ $(\lambda=0.5, \theta=1)$	
(m,k)	ML	**Bayesian**	ML	**Bayesian**
(2,5)	1.256	1.333	1.055	1.290
(2,8)	1.333	1.419	1.070	1.303
(2,10)	1.900	1.429	1.071	1.426
(3,5)	1.278	1.351	1.056	1.297
(3,8)	1.933	1.393	1.121	1.474
(3,10)	2.273	2.200	2.034	2.842
	Efficiency of λ $(\lambda=0.8, \theta=1)$		Efficiency of θ $(\lambda=0.8, \theta=1)$	
(m,k)	ML	**Bayesian**	ML	**Bayesian**
(2,5)	1.018	1.404	1.481	1.043
(2,8)	1.025	1.554	1.695	1.281
(2,10)	1.203	1.666	1.709	1.921
(3,5)	1.057	1.412	1.656	1.138

Continued Table 2

(3,8)	2.194	1.750	2.500	2.227
(3,10)	2.696	1.833	3.000	2.803
	Efficiency of λ $(\lambda = 1.1, \theta = 1)$		Efficiency of θ $(\lambda = 1.1, \theta = 1)$	
(m,k)	ML	**Bayesian**	ML	**Bayesian**
(2,5)	1.248	1.489	1.165	1.664
(2,8)	1.295	2.117	1.597	1.904
(2,10)	1.306	2.341	2.019	2.318
(3,5)	1.301	2.041	1.205	1.856
(3,8)	2.000	2.484	2.031	2.349
(3,10)	2.012	3.488	2.684	2.455

References

[1] G. A. McIntyre, A method of unbiased selective sampling, using ranked sets, *Australian Journal of Agricultural Research*, 3, 1952, pp 385-390.

[2] K. Takahasi and K. Wakimoto, On unbiased estimates of the population mean based on the sample stratified by means of ordering, *Annals of the Institute of Statistical Mathematics* 20, 1968, pp 1-31.

[3] T. R. Dell and J. L. Clutter, Ranked set sampling theory with order statistics background, *Biometrics* 28, 1972, pp 545–555.

[4] H. M. Samawi, M. S. Ahmed, and W. A. Abu-Dayyeh, Estimating the population mean using extreme raked set sampling, *The Biometrical Journal*, 38(5), 1996, pp. 577-86.

[5] H. A. Muttlak, Median ranked set sampling. *Journal of Applied Statistical Science*, 6(4), 1997, pp 245-255.

[6] M. F. Al-Saleh and M. Al-Kadiri, Double ranked set sampling, *Statistics and Probability Letters*. 48, 2000, pp 205–212.

[7] M. F. Al-Saleh and H. A. Muttlak, A note in Bayesian estimation using ranked set sampling, *Pakistan Journal of Statistics* 14, 1998, pp 49–56.

[8] S. A. Al-Hadhram, Parametric estimation on modified Weibull distribution based on ranked set sampling, *European Journal of Scientific Research* 44(1), 2010, pp.73-78.

[9] A. Helu, M. Abu-Salih, and O. Alkami, Bayes estimation of Weibull distribution parameters using ranked set sampling, *Communications in Statistics—Theory and Methods*, 39, 2010, pp 2533–2551.

[10] R. D. Gupta and D Kundu, Generalized exponential distributions, *Australian and New Zealand Journal of Statistics*, 41(2), 1999, pp 173 - 188.

11]D. Kundu and R. D.Gupta, Generalized exponential distribution; Bayesian Inference, *Computational Statistics and Data Analysis*, 52, 2008, pp 1873-1883.

.