

A Mathematical Model For Vibration Based Prognosis For Effect Of Unbalance On Journal Bearing

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Abstract

Now a days vibration based condition monitoring technique is widely used in several core companies. These companies are like - Cement Companies, Thermal Power Stations, Rolling Mills etc.

This technique prevents excessive failure of the machine component. Hence in such a companies special departments are there, which handles the problem related to the health of machine. Some times, maintenance department of the company has this responsibility. There are so many process machines used in the industries. Amongst such a machine some machines have rotor system. Even in some machine the journal bearings bear the load of different rotor. In this present work a one possible approach is presented which provides the prediction of unbalance through mathematical model and the effect of unbalance on journal bearing is discussed.

Keywords: Vibration based Condition Monitoring, Journal Bearing, Mathematical Model.

1. Background of the Work

Literature review indicates the method of identification of imbalance condition through signature analysis. If unbalance force is increased, it means the amplitude at 1 x frequency of rotor will be increased. If the rotor is placed in between journal bearings, then the effect of unbalance on the performance of this bearing will also be considerable. Hence, in this present work, a simulation of this phenomenon, through experimental test rig is executed. Next to this a mathematical model is developed. This mathematical model is useful for the prediction of

amplitude at imbalance frequency of rotor (i.e. 1 x frequency). The dimensional analysis is thus useful for reducing group of π terms. These π terms are used for making this model.

With this experimental test rig the effect of unbalance mass on journal bearing performance is evaluated.

2. Present State of Art

Literature review [3] indicates the method of identification of imbalance condition through signature analysis. This method is discussed below (1) Keep an accelerometer on the bearing cap B1 (2) Feed the data through this accelerometer to FFT analyzer. (3) Then FFT analyzer displays the signature in time domain or frequency domain. The amplitude at 1x frequency will be the confirmation of the amplitude at unbalance condition. The increase of unbalance force will definitely increases the amplitude. But the mathematical model for prediction of such a phenomena is not seen. Even the effect of imbalance on journal bearing is also seen partially. These two important aspects are included in this present work.

3. Concept of Solution

Refer to Figure 1, a rotor R is placed on shaft 'S'. The load of shaft 'S' and rotor 'R' is bared by bearing B1 & B2. Let the mass of rotor R is concentrated apart from geometrical centre. This will induced additional centrifugal force and the force is having certain magnitude. This force imposes additional reactions at bearing B1 & B2. Hence bearing cap of B1 & B2 are sets into harmonics. These harmonics are sense by an accelerometer and it is fed up to FFT analyzer. Thus FFT analyzers display the signature of imbalance condition.

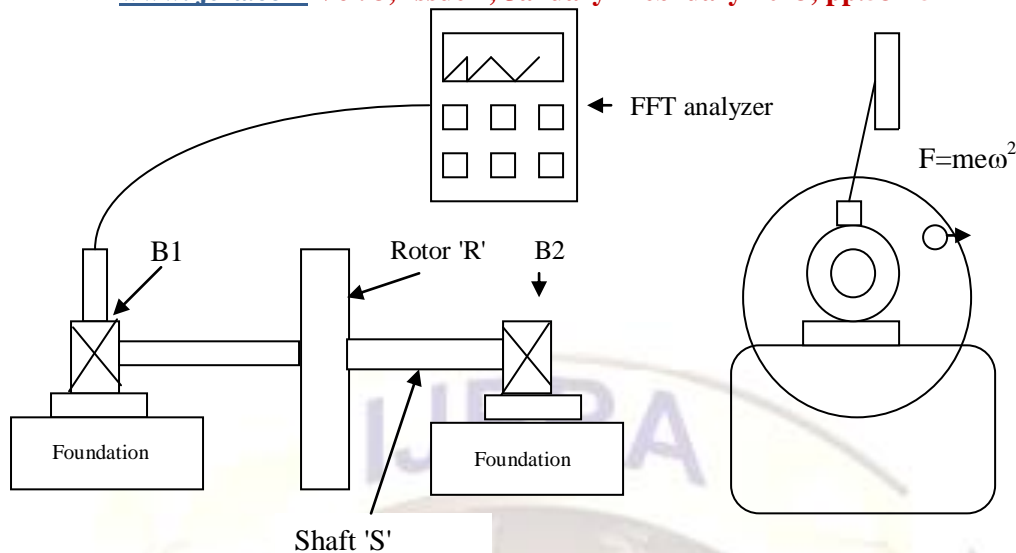


Figure 1: schematic of a rotor unit placed on two bearings B1 and B2

In the mean time, if the unbalance force is further increased, this may create additional load on the journal bearings, which in turn increases the amplitude at 1x frequency of rotor. While the increase in load generates additional heat, in the oil of journal bearing if the machine is kept still in operation. Thus increase in heat will be responsible for decrease in the absolute viscosity (μ). Conversely this may affect the performance of the bearing.

3.1 Salient Details of the Solution

The simulation of experimental test rig is not a scaled model of any machine. But this shows a general concept which can be applied to any machine for the diagnosis purpose. The experimental test rig is established by the two rotors placed on two journal bearings. This experimental test rig is designed and fabricated.

Artificial unbalance are created by placing addition masses at the rotor in the step of 5gm. Then, Data is gathered for every step like vibration amplitude at 1 x frequency of the rotor, speed of the rotor, load torque, time between two spectrum readings, temperature of bearing body etc. Conversely dimensional analysis is formed. This data is then useful to form a mathematical model for the prediction of unbalance at 1x frequency of rotor.

4. Formulation of Experimental Data Based Model

Mathematical model is nothing but the algebraic relationship amongst the response variable and independent variable. Here, response variable is called dependant variable. Any phenomena can be presented mathematically by knowing the physics involve in it. These mathematical models are of three types 1. Logic based 2. Field data based 3. Experimental data based. Some phenomena can be presented by application of basic balances.

In certain situations, it is not possible to formulate a mathematical model for complex phenomena on the basis of application of the basic balances of mechanics. In such situations, it becomes inevitable to collect experimental data for the process and to utilize the generated experimental data to formulate the generalized algebraic relationship amongst the various physical quantities involved in the process.

4.1 Process of Model Formulation

The process of model formulation includes the following sequential steps:

4.1.1 Process Variables

Any physical quantity prevailing in the process under study is designated as process variable. Process variables are categorized as: 1) Independent variables, 2) Dependent variables and 3) Extraneous variables. The following process variables are identified

Table 1 : Type of variable, Name of variable and Designation of variable.

Sr.No.	Type of variable	Name of variable	Designation
i	Dependent	Vibration Amplitude	Y
ii	Independent	Weight of Rotor 1	W1
iii	Independent	Weight of Rotor 2	W2
iv	Independent	Weight of equivalent shaft	WS
v	Independent	Mass moment of Inertia of Rotor 1	IR1
vi	Independent	Mass moment of Inertia of Rotor 2	IR2
vii	Independent	Mass moment of Inertia of Shaft	IS
viii	Independent	Weight of unbalance mass	Wum
ix	Independent	Viscosity of oil	V
x	Independent	Radial clearance of Bearing	RcB
xi	Independent	Length of Bush	LB
xii	Independent	Diameter of Bush	DB
xiii	Independent	Thickness of Bush	tB
xiv	Independent	Density of Bush	ρB1
xv	Independent	Modulus of Elasticity of Rotor 1	ER1
xvi	Independent	Modulus of Elasticity of Rotor 2	ER2
xvii	Independent	Modulus of Elasticity of Shaft	ES
xviii	Independent	Modulus of Elasticity of Bearing Cap	EBM
xix	Independent	Modulus of Elasticity of Bush	EBG
xx	Independent	Equivalent length of Rotor 1	L1
xxi	Independent	Equivalent length of Rotor 2	L2
xxii	Independent	Equivalent length of Shaft	LS
xxiii	Independent	Second moment of inertia	IA
xxiv	Independent	Polar moment of inertia	IP
xxv	Independent	Radius of Bearing Cap	RB
xxvi	Independent	Width of Bearing Cap	WB
xxvii	Independent	Thickness of Bearing Cap	TB
xxviii	Independent	Center dist. between the Bearing Bolt	CDB
xix	Independent	Material density of bearing cap	ρB2
xxx	Independent	Speed of rotor	N
xxxii	Independent	Load Torque	TL
xxxii	Independent	Time	T
xxxiii	Independent	Acceleration due to gravity	g
xxxiv	Independent	Pressure in the Bearing	PB

4.1.2 Reduction of variables – Dimensional analysis

Dimensional analysis is the best known and the most powerful technique of reducing the number of variables and making the experimental plan compact without loss of generality or control. Dimensional analysis, basically, helps in deciding algebraic relationship amongst the various physical quantities encountered in the process. Using Buckingham π theorem following dimensional equation is formed.

1. The general equation form of dependent and independent variables are as follows.

$$Y = \Phi (W1^a, W2^b, Ws^c, IR1^d, IR2^e, IS^f, Wum^g, v^h, RcB^i, LB^j, DB^k, tB^l, \rho B1^m, ER1^n, ER2^o, ES^p, EBM^q, EBG^r, L1^s, L2^t, LS^u, IA^v, Ip^w, RB^x, WB^y, TB^z, CDB^{a1}, \rho B2^{a2}, N^{a3}, TL^{a4}, T^{a5}, g^{a6}, PB^{a7})$$

2. The MLT form of the above equation is –

$$L = \Phi ((M)^a, (M)^b, (M)^c, (ML^2)^d, (ML^2)^e, (ML^2)^f, (M)^g, (ML^{-1}T^{-1})^h, (L)^i, (L)^j, (L)^k, (L)^l, (ML^{-3})^m, (ML^{-1}T^{-2})^n, (ML^{-1}T^{-2})^o, (ML^{-1}T^{-2})^p, (ML^{-1}T^{-2})^q, (ML^{-1}T^{-2})^r, (L)^s, (L)^t, (L)^u, (L)^v, (L)^w, (L)^x, (L)^y, (L)^z, (L)^{a1}, (ML^{-3})^{a2}, (T^{-1})^{a3}, (ML^{+2}T^{-2})^{a4}, (T)^{a5}, (LT^{-2})^{a6}, (ML^{-1}T^{-2})^{a7})$$

4.1.3 Plan of Experimentation

1) Test point 2) Test envelop and 3) Test Sequence

Detailed experimental plans are made at this stage to give speed of testing, minimization of errors, maximization of useful data, and maximum control of extraneous and outside influence. This detailed planning includes the decisions about 1) **Test envelope:** The test envelope is the range in which independent variables is to be varied during experimentation.

2) **Test points:** Test points are discrete values of an independent variable at which experiments are conducted.

3) **Test sequence:** In this work partly reversible experiments are used. Following table 2 and 3 shows test envelop while table 4 and 5 shows test point.

Table 2 : Test envelop for bearing B1

Sr.No.	Values	π_1	π_2	π_3
1	Minimum	9.82081E-07	6.39393E-06	9.74914E-07
2	Maximum	9.82081E-07	6.39393E-06	3.89966E-05

Table 3 : Test envelop for bearing B2

Sr.No.	Values	π_1	π_2	π_3
1	Minimum	9.82081E-07	6.39393E-06	1.07241E-05
2	Maximum	9.82081E-07	6.39393E-06	7.79931E-05

Table 4 List of test point for different π terms of bearing B1

Sr.No.	π_1	π_2	π_3
1	9.82081E-07	6.39393E-06	9.74914E-07
2	9.82081E-07	6.39393E-06	2.59977E-06
3	9.82081E-07	6.39393E-06	4.87457E-06
4	9.82081E-07	6.39393E-06	7.79931E-06
5	9.82081E-07	6.39393E-06	1.1374E-05
6	9.82081E-07	6.39393E-06	1.55986E-05
7	9.82081E-07	6.39393E-06	2.04732E-05
8	9.82081E-07	6.39393E-06	2.59977E-05
9	9.82081E-07	6.39393E-06	3.21722E-05
10	9.82081E-07	6.39393E-06	3.89966E-05

Table 5 List of test point for different π terms of bearing B2

Sr.No.	π_1	π_2	π_3
1	9.82081E-07	6.39393E-06	1.07241E-05
2	9.82081E-07	6.39393E-06	1.55986E-05
3	9.82081E-07	6.39393E-06	2.11231E-05
4	9.82081E-07	6.39393E-06	2.72976E-05
5	9.82081E-07	6.39393E-06	3.4122E-05
6	9.82081E-07	6.39393E-06	4.15963E-05
7	9.82081E-07	6.39393E-06	4.97206E-05
8	9.82081E-07	6.39393E-06	5.84949E-05
9	9.82081E-07	6.39393E-06	6.7919E-05
10	9.82081E-07	6.39393E-06	7.79931E-05

4.1.4 Experimental procedure

The total ten readings of vibration amplitudes at 1x frequency of rotor of bearing B1 and ten readings of bearing B2 are taken. By keeping load torque constant the unbalance mass is change in the step of 5gm. Thus a reading of vibration amplitude at 1x frequency of rotor is noted down in Table 6 and 7.

The experimental procedure is evaluated like as under -

For first reading, a 15 gm mass is placed on rotor R1. Then, keeping accelerometer on bearing B1, the vibration spectra is taken. Conversely the amplitude at 1x frequency is measured. Thus for every reading, the mass is

increased in the step of 5 grams, and measured the amplitude at 1x frequency from vibration spectrum. In the same way the procedure is also done for bearing B2. While the reading was taken measurement of bearing body temperature is also done for every reading.

4.1.5 Measurement of amplitude, temperature and speed.

In this experimentation, the speed of shaft is measured with the help of tachometer, vibration amplitude is measured with the help of FFT analyzer and the temperature is measured with the help of thermocouple. These values are shown in the Table 6 and 7.

Table 6: Different values of amplitudes, speed, load torque, time, imbalance mass for ten readings on bearing B1

Sr.No.	Y (mm)	N (rpm)	TL (N-mm)	T (sec)(Time)	Wum (gm)
1	1.23E-04	1430	6900	120	15
2	1.39E-04	1430	6900	240	20
3	2.56E-04	1430	6900	360	25
4	2.97E-04	1430	6900	480	30
5	3.37E-04	1430	6900	600	35
6	3.90E-04	1430	6900	720	40
7	4.10E-04	1430	6900	840	45
8	4.63E-04	1430	6900	960	50
9	5.03E-04	1430	6900	1080	55
10	6.69E-04	1430	6900	1200	60

Table 7: Different Values of amplitudes, speed, load torque, time, imbalance mass for ten readings on bearing B2

Sr.No.	Y (mm)	N (rpm)	TL (N-mm)	T (sec)	Wum (gm)
1	1.00E-06	1430	6900	1320	15
2	1.23E-04	1430	6900	1440	20
3	1.39E-04	1430	6900	1560	25
4	1.39E-04	1430	6900	1680	30
5	2.31E-04	1430	6900	1800	35
6	2.56E-04	1430	6900	1920	40
7	2.97E-04	1430	6900	2040	45
8	3.37E-04	1430	6900	2160	50
9	3.90E-04	1430	6900	2280	55
10	4.10E-04	1430	6900	2400	60

4.6 Calculate different indices of π terms

The general form of mathematical model to pertaining prediction of amplitudes of unbalance as under.

$$(Y/RB) = K (\pi_1)^a \times (\pi_2)^b (\pi_3)^c \dots\dots\dots(1)$$

Where, K = constant, a, b, c are Indices.

The value of different indices of the above equation is found by using regression equation which is as follows.

Regression Equations:

$$\begin{aligned} \sum Y &= 10K + a\sum A + b\sum B + c\sum C \\ \sum YA &= K \sum A + a\sum A^2 + b\sum AB + c\sum AC \\ \sum YB &= K \sum B + a\sum AB + b\sum B^2 + c\sum BC \\ \sum YC &= K \sum C + a\sum AC + b\sum BC + c\sum C^2 \dots\dots\dots (2) \end{aligned}$$

Where y = dependent π term

A, B,C are independent π term of $\pi_1 \pi_2 \pi_3$, n is number of readings.

By using above equation and placing the values of different independent and dependent π terms for bearing B1, the following matrix is obtained.

$$\begin{pmatrix} -5.19517 \\ 31.21183 \\ 26.98493 \\ 26.053221 \end{pmatrix} = \begin{pmatrix} 10 & -6.00785276 & -5.19423191 & -4.99425 \\ -6.00785278 & 36.09429 & 31.20618 & 30.00471 \\ -5.19423191 & 31.20618 & 26.98005 & 25.94128 \\ -4.99425 & 30.00471 & 25.94128 & 25.18146 \end{pmatrix} \times \begin{pmatrix} K \\ a \\ b \\ c \end{pmatrix}$$

Solving above matrix one can get the values of K, a, b and c for the bearing B1 which is as follows.

$$K = 0.923209, a = -0.2778, b = 0.8015, c = 0.5339$$

Similarly for Bearing B2 the values of K, a, b, c are -

$$K = 1, a = 1.4742, b = -2.4821, c = 2.1487$$

Thus the mathematical model for bearing B1 will be as below:

$$[Y/RB] = 0.923209 \times (\pi_1)^{-0.2778} \times (\pi_2)^{0.8015} \times (\pi_3)^{0.5339} \dots\dots\dots(3)$$

Also the mathematical model for bearing B2 will be as below :

$$[Y/RB] = 1 \times (\pi_1)^{1.4742} \times (\pi_2)^{-2.4821} \times (\pi_3)^{2.1487} \dots\dots\dots(4)$$

5. Result and Discussion

The experimentation is done on experimental test rig. Following results are obtained, which is discussed into two parts (1) Discussion about the Mathematical Model for the Prediction of Unbalance. (2) Discussion about the effect of Unbalance on Journal bearing performance.

1) Discussion about the mathematical model for the prediction of unbalance mass:

The mathematical model for bearing B1 & B2 are reproduced here for the sake of explanation which are detailed in equation 5 & 6.

$$[Y/RB] = 0.923209 \times (\pi_1)^{-0.2778} \times (\pi_2)^{0.8015} \times (\pi_3)^{0.5339} \dots\dots\dots(5)$$

$$[Y/RB] = 1 \times (\pi_1)^{1.4742} \times (\pi_2)^{-2.4821} \times (\pi_3)^{2.1487} \dots\dots\dots(6)$$

The influence of independent π term over dependent π term depicted in equation 5 & 6 discussed below.

The model pertaining to B1 :

In this model π_2 term has the highest influence on dependent variable because the value of index of π_2 term is greater. Also the value of index of π_1 term is less so one can conclude that the π_1 term has less influence on dependent variable.

The model pertaining to B2 :

In this model π_3 term has the highest influence on dependent variable because the value of index of π_3 term is greater. Also the value of index of π_2 term is less so one can conclude that the π_2 term has less influence on dependent variable.

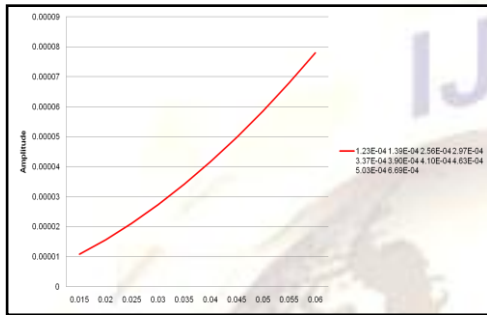
2) Discussion about the performance of journal bearing:

During experimentation, the unbalance mass is attached to the rotor. This unbalance mass is then progressively added in the step of 5gm. Thus increased unbalance mass will be produced additional centrifugal force to the rotor. The increase in centrifugal force is thus added to the load bared by the bearing. Now as load is changed and it is thought that this may increase additional heat generation.

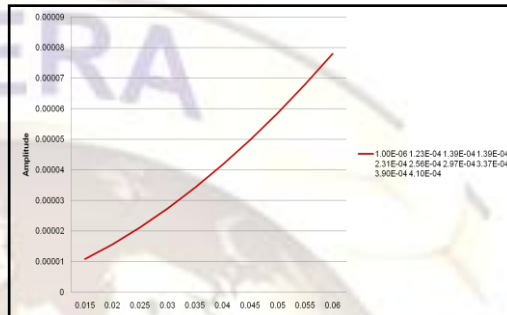
In this context, the temperature of bearing body is noted down. This temperature shows increment in its value. Some important points regarding how a unbalance force affect the parameters referred to the journal bearing is discussed through following points by studying the graphs shown below.

- 1) With reference to Figure 2. This figure is a graph of unbalance mass versus coefficient of friction. From this graph it is quite confirmed that, as the load on journal increases it may in turns increase the friction in between journal and bearing.
- 2) Figure 7 gives an idea about, how the change in oil temperature affects the viscosity (μ) of the oil. The probable reason in increasing of temperature is nothing but the increase in friction.

- 3) Figure 8 details about the relation between unbalance mass and viscosity of oil. From this graph it is quite confirmed that. If the imbalance is increased in the rotor. It may affect the viscosity of oil.
- 4) If unbalance force increases over a period of time, then it will affect the viscosity of oil which will in turn affect the performance of the bearing. Sometime it may lead the bearing towards seizing condition

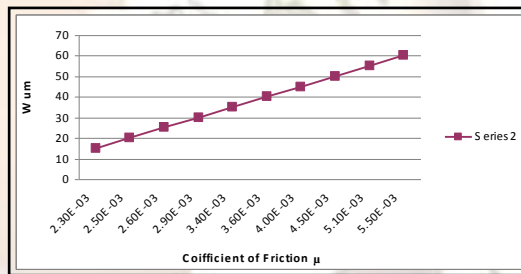
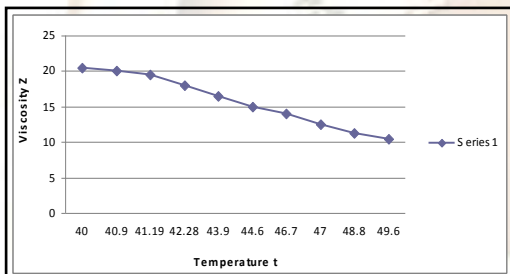


Unbalance mass



Unbalance ass

Figure 2 : Unbalance mass versus vibration amplitude
Figure 3 : Unbalance mass versus vibration amplitude



5.3 : Figure 5.4 :

Figure 4 : Temperature v/s Viscosity
Figure 5 : Coefficient of Friction v/s unbalance mas

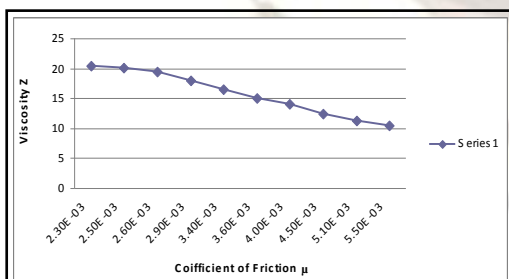


Figure 5.5 :

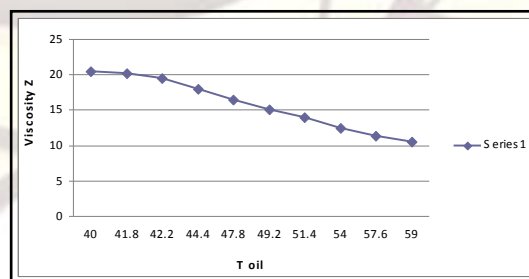


Figure 6 : Coefficient of friction v/s Viscosity
Figure 7 : Temperature v/s Viscosity

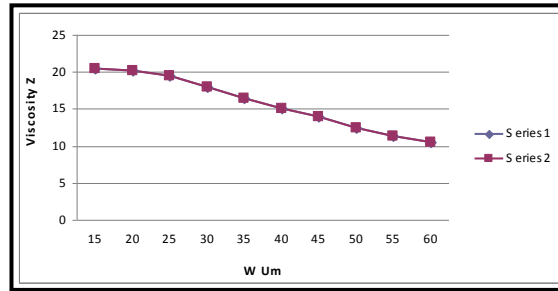


Figure 8 : Unbalance mass v/s viscosity z

Conclusions

In this investigation following some important conclusion are made.

- 1) As there is increase in unbalance mass, amplitude at 1 x frequency gets increased.
- 2) For this phenomenon of unbalance the mathematical model for prognosis of amplitude at 1x frequency of rotor is established for the individual bearing.
- 3) As unbalance mass is increased, this will increase coefficient of friction between journal and oil film. This increase in friction enhances high temperature to the oil. Conversely the viscosity of oil may get changed. If the viscosity of oil changes continuously, then there will be a chance of oil film breakage. Thus bearing may get seize.
- 4) Hence, if it is observed that the bearing body temperature increases. It means there would be a one possibility of unbalance present in the rotor.

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