Enhanced Signal Denoising Performance by EMD-based Techniques

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Abstract

Empirical mode decomposition (EMD) is one of the most efficient methods used for non-parametric signal denoising. In this study wavelet thresholding principle is used in the decomposition modes resulting from applying EMD to a signal. The principles of hard and soft wavelet thresholding including translation invariant denoising were appropriately modified to develop denoising methods suited for thresholding EMD modes. We demonstrated that, although a direct application of this principle is not feasible in the EMD case, it can be appropriately adapted by exploiting the special characteristics of the EMD decomposition modes. In the same manner, inspired by the translation invariant wavelet thresholding, a similar technique adapted to EMD is developed, leading to enhanced denoising performance.

Keywords: Denoising, EMD, Wavelet thresholding

I Introduction

Denoising aims to remove the noise and to recover the original signal regardless of the signal's frequency content. Traditional denoising schemes are based on linear methods, where the most common choice is the Wiener filtering and they have their own limitations(Kopsinis and McLaughlin, 2008).Recently, a new data-driven technique, referred to as empirical mode decomposition (EMD) has been introduced by Huang et al. (1998) for analyzing data from nonstationary and nonlinear processes. The EMD has received more attention in terms of applications, interpretation, and improvement. Empirical mode decomposition (EMD) method is an algorithm for the analysis of multicomponent signals that breaks them down into a number of amplitude and frequency modulated (AM/FM) zero-mean signals, termed intrinsic mode functions (IMFs). In contrast to conventional decomposition methods such as wavelets, which perform the analysis by projecting the signal under consideration onto a number of predefined basis vectors, EMD expresses the signal as an expansion of basis functions that are signal-dependent and are estimated via an iterative procedure called sifting. Although many attempts have been made to improve the understanding of the way EMD operates or to enhance its performance, EMD still lacks a sound mathematical theory and is essentially described by an algorithm. However, partly due to the fact that it is easily and directly applicable and partly because it often results in interesting and useful decomposition outcomes, it has found a vast number of diverse applications such as biomedical, watermarking and audio processing to name a few.

In this study, inspired by standard wavelet thresholding and translation invariant thresholding, a few EMD-based denoising techniques are developed and tested in different signal scenarios and white Gaussian noise. It is shown that although the main principles between wavelet and EMD thresholding are the same, in the case of EMD, the thresholding operation has to be properly adapted in order to be consistent with the special characteristics of the signal modes resulting from EMD. The possibility of adapting the wavelet thresholding principles in thresholding the decomposition modes of EMD directly, is explored. Consequently, three novel EMD-based hard and soft thresholding strategies are presented.

II EMD: A Brief description and notation

EMD[3] adaptively decomposes a multicomponent signal [4] x(t) into a number of the so-called IMFs, $h^{(i)}(t), 1 \le i \le L$

$$x(t) = \sum_{i=1}^{L} h^{(i)}(t) + d(t) \qquad (1)$$

where *d(t)* is a remainder which is a non-zero-mean slowly varying function with only few extrema. Each one of the IMFs, say, the (i) th one , is estimated with the aid of an iterative process, called sifting, applied to the residual multicomponent signal.

$$x^{(i)}(t) = \begin{cases} x(t), & i = 1 \\ x(t) - \sum_{j=1}^{i-1} h^{(i)}(t), & i \ge 2. \end{cases} \qquad (2)$$

The sifting process used in this paper is the standard one [3]. According to this, during the (n+1) th sifting iteration, the temporary IMF estimate

$h_n^{(i)}(t)$ is improving according to the following steps.2

1. Find the local maxima and minima of $h_n^{(i)}(t)$.
2. Interpolate, using natural cubic splines, along the points of $h_n^{(i)}(t)$ estimated in the first step in order to form an upper and a lower envelope.
3. Compute the mean of two envelopes.
4. Obtain the refined estimate $h_{n+1}^{(i)}(t)$ of the IMF by subtracting the mean found in the previous step from the current IMF estimate $h_n^{(i)}(t)$.
5. Proceed from step 1) again unless a stopping criterion has been fulfilled.

The sifting process is effectively an empirical but powerful technique for the estimation of the mean $m^{(i)}(t)$ of the residual multicomponent signal $x^{(i)}(t)$ localy, a quantity that we term total local mean. Although the notion of the total local mean is somewhat vague, especially for multicomponent signals, in the EMD context it means that its subtraction from $x^{(i)}(t)$ will lead to a signal, which is actually the corresponding IMF, i.e., $h^{(i)}(t) = x^{(i)}(t) - m^{(i)}(t)$, that is going to have the following properties.

1) Zero mean.
2) All the maxima and all the minima of $h^{(i)}(t)$ will be positive and negative, respectively. Often, but not always, the IMFs resemble sinusoids that are both amplitude and (frequency modulated (AM /FM).

By construction, the number of say $N(i)$ extrema of $h^{(i)}(t)$ positioned in time instances $r_j^{(i)} = [r_1^{(i)}, r_2^{(i)}, \ldots\ldots, r_{N(i)}^{(i)}]$ and the corresponding IMF points $h^{(i)}(r_j^{(i)}), j = 1, \ldots\ldots, N(i)$, will alternate between maxima and minima, i.e,. positive and negative values. As a result, in any pair of extrema $r_j^{(i)} = [h^{(i)}(r_j^{(i)}), h^{(i)}(r_{j+1}^{(i)})]$, corresponds to a single zero-crossing $z_j^{(i)}$. Depending on the IMF shape, the number of zero-crossings can be either [4] $N(i)$ or $N(i)$ -1. Moreover, each IMF lets say the one of the order, *I*, have fewer extrema than all over the lower order of IMFs, *j=1, i-1*, leading to fewer and fewer oscillations as the IMF order increases. In other words, each IMF occupies lower frequencies locally in the time-frequency domain than its preceeding ones.

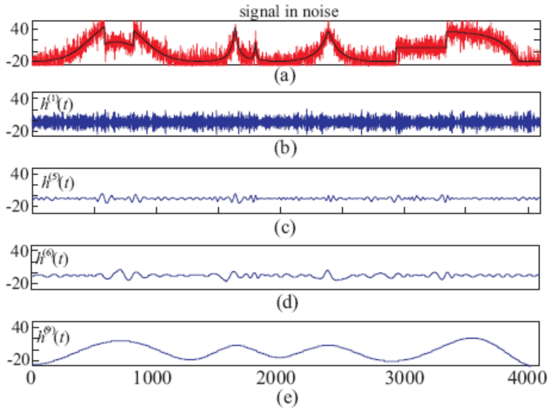

Figure 1.Empirical mode decomposition of the noisy signal shown in (a).

III Signal Denoising

Thresholding is a technique used for signal and image denoising. The discrete wavelet transform uses two types of filters: (1) averaging filters, and (2) detail filters. When we decompose a signal using the wavelet transform, we are left with a set of wavelet coefficients that correlates to the high frequency subbands. These high frequency subbands consist of the details in the data set.

If these details are small enough, they might be omitted without substantially affecting the main features of the data set. Additionally, these small details are often those associated with noise; therefore, by setting these coefficients to zero, we are essentially killing the noise. This becomes the basic concept behind thresholding-set all frequency subband coefficients that are less than a particular threshold to zero and use these coefficients in an inverse wavelet transformation to reconstruct the data set.

3.1. IMF Thresholding based Denoising

An alternative denoising procedure inspired by wavelet thresholding is proposed. EMD thresholding can exceed the performance achieved by wavelet thresholding only by adapting the thresholding function to the special nature of IMFs. EMD can be interpreted as a subband-like filtering procedure resulting in essentially uncorrelated IMFs. Although the equivalent filter-bank structure is by no means predetermined and fixed as in wavelet decomposition, one can in principle perform thresholding in each IMF in order to locally exclude low-energy IMF parts that are expected to be significantly corrupted by noise.

A direct application of wavelet thresholding in the EMD case translates to

$$\tilde{h}^{(i)}(t) = \begin{cases} h^{(i)}(t), & \left|h^{(i)}(t)\right| > T_i \\ 0, & \left|h^{(i)}(t)\right| \le T_i \end{cases}$$

for hard thresholding and to

$$\tilde{h}^{(i)}(t) = \begin{cases} \operatorname{sgn}(h^{(i)}(t))(\left|h^{(i)}(t)\right| - T_i), & \left|h^{(i)}(t)\right| > T_i \\ 0, & \left|h^{(i)}(t)\right| \le T_i \end{cases}$$

for soft thresholding, where, in both thresholding cases, indicates the thresholded IMF. The reason for adopting different thresholds per mode will become clearer in the sequel.
A generalized reconstruction of the denoised signal is given by

$$\hat{x}(t) = \sum_{k=M_1}^{M_2} \tilde{h}^{(i)}(t) + \sum_{k=M_2+1}^{L} h^{(i)}(t)$$

Where the introduction of parameters gives us flexibility on the exclusion of the noisy low-order IMFs and on the optional thresholding of the high-order ones, which in white Gaussian noise conditions contain little noise energy. There are two major differences, which are interconnected, between wavelet and direct EMD thresholding (EMD-DT) shown above.

First, in contrast to wavelet denoising where thresholding is applied to the wavelet components, in the EMD case, thresholding is applied to the samples of each IMF, which are basically the signal portion contained in each adaptive subband. An equivalent procedure in the wavelet method would be to perform thresholding on the reconstructed signals after performing the synthesis function on each scale separately. Secondly, as a consequence of the first difference,the IMF samples are not Gaussian distributed with variance equal to the noise variance, as the wavelet components are irrespective of scale.

In our study of thresholds, multiples of the IMF-dependent universal threshold, i.e, where is a constant, are used. Moreover, the IMF energies can be computed directly based on the variance estimate of the first IMF.

3.2 Conventional EMD Denoising

The first attempt at using EMD as a denoising tool emerged from the need to know whether a specific IMF contains useful information or primarily noise. Thus, significance IMF test procedures were simultaneously developed based on the statistical analysis of modes resulting from the decomposition of signals solely consisting of fractional Gaussian noise and white Gaussian noise, respectively.

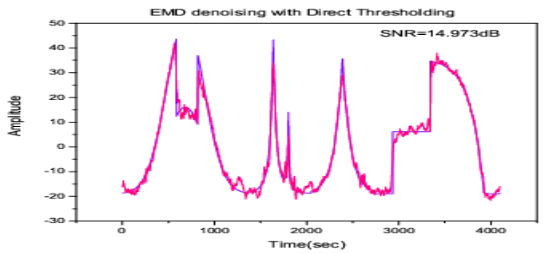

Figure 2. EMD Direct Thresholding.The top-right numbers are the SNR values after denoising.

The reasoning underlying the significance test procedure above is fairly simple but strong. If the energy of the IMFs resulting from the decomposition of a noise-only signal with certain characteristics is known, then in actual cases of signals comprising both information and noise following the specific characteristics, a significant discrepancy between the energy of a noise-only IMF and the corresponding noisy-signal IMF indicates the presence of useful information. In a denoising scenario, this translates to partially reconstructing the signal using only the IMFs that contain useful information and discarding those that carry primarily noise, i.e., the IMFs that share similar amounts of energy with the noise-only case.

In practice, the noise-only signal is never available in order to apply EMD and estimate the IMF energies, so the usefulness of the above technique relies on whether or not the energies of the noise-only IMFs can be estimated directly based on the actual noisy signal. The latter is usually the case due to a striking feature of EMD. Apart from the first noise-only IMF, the power spectra of the other IMFs exhibit self-similar characteristics akin to those that appear in any dyadic filter structure. As a result, the IMF energies should linearly decrease in a semilog diagram of, e.g., with respect to for. It also turns out that the first IMF carries the highest amounts of energy

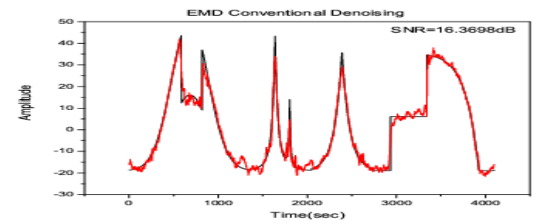

Figure 3. EMD Conventional Denoising. The top-right numbers are the SNR values after denoising.

IV Wavelet based Denoising

Employing a chosen orthonormal wavelet basis, an orthogonal N× N matrix W is appropriately built. Discrete wavelet transform (DWT) c =Wx where, x=[x(1), x(2), . . . , x(N)] is the vector of the signal samples and c = [c1, c2, . . . , cN] contains the resultant wavelet coefficients. Under white Gaussian noise conditions and due to the orthogonality of matrix W, any wavelet coefficient c_i follows normal distribution with variance the noise variance σ and mean the corresponding coefficient value c_i of the DWT of the noiseless signal x(t).

Provided that the signal under consideration is sparse in the wavelet domain the DWT is expected to distribute the total energy of x(t) in only a few wavelet components lending themselves to high amplitudes. As a result, the amplitude of most of the wavelet components is attributed to noise only. The fundamental reasoning of wavelet soft thresholding is to set to zero all the components which are lower than a threshold related to noise level and appropriately shrink the rest of the components

There has recently been a great deal of research interest in wavelet thresholding techniques for signal and image denoising developed wavelet shrinkage and thresholding methods for reconstructing signals from noisy data, where the noise is assumed to be white and Gaussian. They have also shown that the resulting estimates of the unknown function are nearly minimax over a large class of function spaces and for a wide range of loss functions.

The model generally adopted for the observed process is

$$y(i) = f(i) + \xi(i), i \in \{1,.....,K = 2^J\} (J \in N^*)$$

Where ξ(i)} is usually assumed to be a random noise vector with independent and identically distributed (i.i.d.) Gaussian components with zero mean and variance σ^2. Note however that the assumption of Gaussianity is alleviated in the sequel. Estimation of the underlying unknown signal f is of interest. We subsequently consider a (periodic) discrete wavelet expansion of the observation signal, leading to the following additive model:

$$W_y^{j,k} = W_f^{j,k} + W_\xi^{j,k}, k\varepsilon\{1.......2^{-j}K\}$$

Under the Gaussian noise assumption, thresholding techniques successfully utilize the unitary transform property of the wavelet decomposition to distinguish statistically the signal components from those of the noise. In order to fix terminology, we recall that a thresholding rule sets to zero all coefficients with an absolute value below a certain threshold $\chi_j > 0$.

We exhibit close connections between wavelet thresholding and Maximum A Posteriori (MAP) estimation (or, equivalently, wavelet regularization) using exponential power prior distributions. Our approach differs from those previously mentioned by using a different prior and also different loss functions. One of the main advantages of our approach is to naturally provide a thresholding rule, and consequently, a threshold value adapted to the signal/noise under study.

Moreover, we will also show that the MAP estimation is also closely related to wavelet regularization of ill-posed inverse stochastic problems with appropriate penalties and loss functions, that parallel Bridge estimation techniques for nonparametric regression as introduced by the sake of simplicity (but see our discussion later), we assume in the sequel that the wavelet coefficients of the signal and the noise are two independent sequences of i.i.d. random variables.

Hard thresholding zeroes out all values in the frequency band that is found to be Gaussian. The hard thresholding coefficient is

$$c_h = \begin{cases} 0, & \text{if the band is Gaussian} \\ 1, & \text{if the band is not Gaussian.} \end{cases}$$

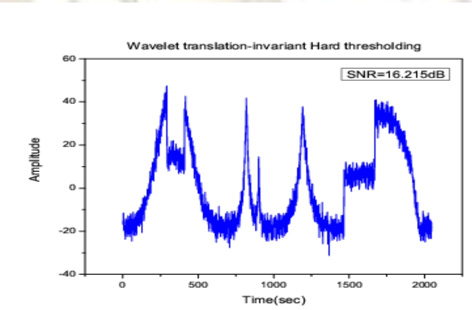

Figure 4. Wavelet Translation invariant Hard thresholding.

Soft Thresholding is obtained by multiplying values in the specific frequency band that is found to be Gaussian, by a coefficient between 0 and 1. Using a coefficient of 0 is the same as hard thresholding, whereas using a coefficient of 1 is the same as leaving the frequency band undisturbed. The soft thresholding coefficient is calculated using

$$c_\varsigma = \frac{|3 - \hat{\gamma}_4|}{1.5},$$

Where µ4 *g* is the bootstrapped kurtosis of the particular frequency band, and denotes the absolute value. The bootstrapped kurtosis value µ4 *g* is limited not to exceed 4.5, which will be explained later. The bootstrapped kurtosis value µ4 *g* is obtained by using the Bootstrap principle, which calls for resamplings from the data set many times with replacement, to obtain *N* resampled sets of the same length as the original data set. Next, the kurtosis value for each resample is found. Finally, the Bootstrapped kurtosis µ4 *g* is defined as the estimated mean obtained from *N* kurtosis values. It should be noted that the thresholding coefficient *c* is a function of the frequency band's degree of Gaussianity. Equation above shows that the coefficient *c* gets closer to 0 as the bootstrapped kurtosis value µ4 *g* for a specific frequency band gets closer to the theoretical value 3, and vice versa. Therefore, the closer a frequency band gets to being Gaussian, the smaller contribution it has after soft thresholding.

In order to simplify the presentation of our result further, we will drop the dependence on the resolution level j and the time index k of the quantities involved subsequently.

4.1. Iterative EMD Interval-Thresholding

Direct application of translation invariant denoising to the EMD case will not work. This arises from the fact that the wavelet components of the circularly shifted versions of the signal correspond to atoms centered on different signal instances. In the case of the data-driven EMD decomposition, the major processing components, which are the extrema, are signal dependent, leading to fixed relative extrema positions with respect to the signal when the latter is shifted. As a result, the EMD of shifted versions of the noisy signal corresponds to identical IMFs sifted by the same amount. Consequently, noise averaging cannot be achieved in this way**.** The different denoised versions of the noisy signal in the case of EMD can only be constructed from different IMF versions after being thresholded. Inevitably, this is possible only by decomposing different noisy versions of the signal under consideration itself.

We know that in white Gaussian noise conditions, the first IMF is mainly noise, and more specifically comprises the larger amount of noise compared to the rest of the IMFs. By altering in a random way the positions of the samples of the first IMF and then adding the resulting noise signal to the sum of the rest of the IMFs, we can obtain a different noisy version of the original signal.

Algorithm

1. Perform an EMD expansion of the original noisy signal x.
2. Perform a partial reconstruction using the last L-1 IMFs only, x_p(t) $=\sum_i L = 2h_t^i$
3. Randomly alter the sample position of the first IMF $h_a^{(1)}(t) = ALTER(h^{(1)}(t))$.
4. Construct a different noisy version of the original signal $x_a(t) = x_p(t) + h_a^{(1)}(t).$
5. Perform EMD on the new altered noisy signal $x_a(t).$
6. Perform EMD-IT denoising (12)or(13) on the IMFs of $x_a(t)$ to obtain a denoised version $\tilde{x}_1(t)\, of\ x.$
7. Iterate K-1 time between steps 3)-6),where K is the number of averaging iteration in order to obtain k denoised versions of $x, ie., \tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_k.$
8. Average the resulted denoised signals $$\tilde{x}(t) = (1/k)\sum_{k=1}^{k} \tilde{x}_k(t).$$

4.2. Clear iterative interval thresholding

When the noise is relatively low, enhanced performance compared to EMD-IIT denoising can be achieved with a variant called clear iterative interval-thresholding (EMD-CIIT). The need for such a modification comes from the fact that the first IMF, especially when the signal SNR is high, is likely to contain some signal portions as well. If this is the case, then by randomly altering its sample positions, the information signal carried on the first IMF will spread out contaminating the rest of the signal along its length. In such an unfortunate situation, the denoising performance will decline. In order to overcome this disadvantage of EMD-IIT it is not the first IMF that is altered directly but the first IMF after having all the parts of the useful information signal that it contains removed. The "extraction" of the information signal from the first IMF can be realized with any thresholding method, either EMD-based or wavelet-based. It is important to note that any useful signal resulting from the thresholding operation of the first IMF has to be summed with the partial reconstruction of the last 1 IMFs.

Algorithm

1. Perform an EMD expansion of the original noisy signal x

2. Perform a thresholding operation to the first IMF of $x(t)$ to obtain a denoised version $\tilde{h}^{(1)}(t)\ of\ h^{(1)}(t)$.

3. Compute the actual noise signal that existed in $h^{(1)}(t),\quad h_n^{(1)}(t) = h^{(1)}(t) - \tilde{h}^{(1)}(t)$.

4.Perform a partial reconstruction using the last L-1 IMFs plus the information signal contained in the first IMF, $x_p(t) = \sum_{i=2}^{L} h^{(i)}(t) + \tilde{h}^{(1)}(t).$

5. Randomly alter the sample positions of the noise-only part of the first IMF, $h_a^{(1)}(t) = ALTER(h_n^{(1)}(t)).$

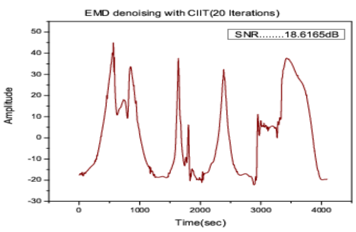

Figure 5. Result of the EMD-based Denoising with Clear Iterative Interval Thresholding method using twenty Iterations.

5. Simulation results and discussion

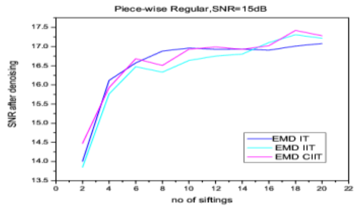

Figure 6. SNR after denoising with respect to the number of shifting iterations

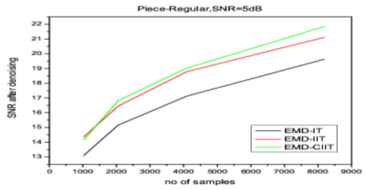

Figure 7. Performance evaluation of the piece-regular signal using EMD-based denoising methods

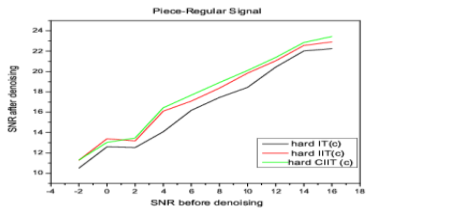

Figure 8. Performance evaluation of EMD-based hard thresholding techniques.

SNR performance and variance of EMD-Based Denoising methods applied on Doppler and Piece-Regular signal. Table 1 & 2

	SNR db							
Piece Regular signal 4096 samples	Methods	-10	-5	-2	0	2	5	10
	EMD-CIIT H(P)	7.63	10.96	14.27	15.95	17.11	18.88	22.22
	EMD-CIIT H(c)	8.26	10.61	13.53	14.42	17.44	19.21	23.06
	EMD-IIT H(c)	8.28	10.13	12.97	14.19	17.18	18.85	22.81
	EMD-IIT H(P)	6.78	10.69	13.54	15.22	16.51	18.51	21.93

	SNR db							
Piece Regular signal 4096 samples	Methods	-10	-5	-2	0	2	5	10
	EMD-CIIT H(P)	7.63	10.96	14.27	15.95	17.11	18.88	22.22
	EMD-CIIT H(c)	8.26	10.61	13.53	14.42	17.44	19.21	23.06
	EMD-IIT H(c)	8.28	10.13	12.97	14.19	17.18	18.85	22.81
	EMD-IIT H(P)	6.78	10.69	13.54	15.22	16.51	18.51	21.93

V Conclusions

In the present paper, the principles of hard and soft wavelet thresholding, including translation invariant denoising, were appropriately modified to develop denoising methods suited for thresholding EMD modes. The novel techniques presented exhibit an enhanced performance compared to wavelet denoising in the cases where the signal SNR is low and/ or the sampling frequency is high. These preliminary results suggest further efforts for improvement of EMD-based denoising when denoising of signals with moderate to high SNR would be appropriate.

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