On Recurrent Hsu-Structure Manifold

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ABSTRACT

In this Paper, we have defined recurrence and symmetry of different kinds in Hsu-structure manifold. Some theorems establishing relationship between different kinds of recurrent Hsu-structure manifold have been stated and proved. Furthermore, theorems on different kinds of recurrent and symmetric Hsu-structure manifold involving equivalent conditions with respect to various curvature tensors have also been discussed.

***Keywords :* Hsu-structure manifold , Curvature tensor ,Recurrent Parameter.**

I. INTRODUCTION

If on an even dimensional manifold V_n, n = 2m of differentiability class C^∞, there exists a vector valued real linear function ϕ , satisfying

$$\phi^2 = a^r I_n, \quad (1.1)a$$

$$\overline{\overline{X}} = a^r X, \text{ for arbitrary vector field X.} \quad (1.1)b$$

Where $\overline{X} = \phi X$, $0 \le r \le n$ and 'r' is an integer and 'a' is a real or imaginary number.

Then {ϕ} is said to give to V_n a Hsu-structure defined by the equations (1.1) and the manifold V_n is called a Hsu-structure manifold.

The equation (1.1)a gives different structure for different values of 'a' and 'r'.

If $r = 0$, it is an almost product structure.

If $a = 0$, it is an almost tangent structure.

If $r = \pm 1$ and $a = +1$, it is an almost product structure.

If $r = \pm 1$ and $a = -1$, it is an almost complex structure.

If r =2 then it is a GF-structure which includes π-structure for $a \neq 0$,

an almost complex structure for $a = \pm i$,

an almost product structure for $a = \pm 1$,

an almost tangent structure for $a = 0$.

Let the Hsu-structure be endowed with a metric tensor g, such that $g(\overline{X}, \overline{Y}) + a^r g(X,Y) = 0$ (1.2)

Then $\{\phi, g\}$ is said to give to V_n - metric Hsu-structure and V_n is called a metric Hsu-structure manifold.

The curvature tensor K, a vector -valued tri-linear function w.r.t. the connexion D is given by

$$K(X,Y,Z) = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z, \quad (1.3)$$

where

$$[X,Y] = D_X Y - D_Y X.$$

The Ricci tensor in V_n is given by

$$Ric(Y,Z) = (C_1^1 K)(Y,Z). \quad (1.4)$$

where by $(C_1^1 K)(Y,Z)$, we mean the contraction of $K(X,Y,Z)$ with respect to first slot.

For Ricci tensor, we also have

$$Ric(Y,Z) = Ric(Z,Y), \quad (1.5)a$$

$$Ric(Y,Z) = g(r(Y),Z) = g(Y,r(Z)), \quad (1.5)b$$

$$(C_1^1 r) = R. \quad (1.5)c$$

Let W, C, L and V be the Projective, conformal, conharmonic and concircular curvature tensors respectively given by

$$W(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-1)}[Ric(Y,Z)X - Ric(X,Z)Y]. \quad (1.6)$$

$$C(X,Y,Z) = K(X,Y,Z) - \frac{1}{n-2}\{Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)r(X) - g(X,Z)r(Y)\} + \frac{R}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]. \quad (1.7)$$

$$L(X,Y,Z)=K(X,Y,Z)-\frac{1}{(n-2)}[Ric(Y,Z)X - Ric(X,Z)Y-g(X,Z)r(Y)+g(Y,Z)r(X)].$$
(1.8)

$$V(X,Y,Z)=K(X,Y,Z)-\frac{R}{n(n-1)}[g(Y,Z)X - g(X,Z)Y].$$
(1.9)

A manifold is said to be recurrent, if

$$(\nabla K)(X,Y,Z,T)=A(T_1)K(X,Y,Z).$$
(1.10)

The recurrent manifold is said to be symmetric, if $A(T_1)=0,$ in the equation (1.10).

The manifold is said to be Ricci-recurrent, if

$$(\nabla Ric)(Y,Z,T)=A(T_1)Ric(Y,Z),$$
(1.11)a

or

$$(\nabla r)(Y,T)=A(T_1)r(Y),$$
(1.11)b

or

$$(\nabla R)(T)=A(T_1)R.$$
(1.11)c

The Ricci-recurrent manifold is said to be symmetric, if $A(T_1)=0,$ in equation (1.11).

II. RECURRENCE AND SYMMETRY OF DIFFERENT KINDS:

Let P, a vector-valued tri-linear function, be any one of the curvature tensors K, W, C, L or V. Then we will define recurrence of different kinds as follows:

Definition (2.1): A Hsu-structure manifold is said to be (1) - recurrent in P, if

$$a^r\nabla P(X,Y,Z,T_1)+P((\nabla\phi)(\overline{X},T_1),Y,Z) = a^r A(T_1)P(X,Y,Z).$$
(2.1)

Where $A(T_1)$ is a non-vanishing C^∞ function.

Definition (2.2): A Hsu-structure manifold is said to be (12)- recurrent in P, if

$$a^r\nabla P(X,\overline{Y},Z,T_1)+P((\nabla\phi)(\overline{X},T_1),\overline{Y},Z)+ a^r P(X,(\nabla\phi)(Y,T_1),Z)=a^r A(T_1)P(X,\overline{Y},Z),$$
(2.2)a

or equivalently

$$a^r\nabla P(\overline{X},Y,Z,T_1)+P(\overline{X},(\nabla\phi)(\overline{Y},T_1),Z)+ a^r P((\nabla\phi)(X,T_1),Y,Z)=a^r A(T_1)P(\overline{X},Y,Z).$$
(2.2)b

Definition (2.3): A Hsu-structure manifold is said to be (123)-recurrent in P, if

$$a^r\nabla P(X,\overline{Y},\overline{Z},T_1)+P((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z})+ a^r P(X,(\nabla\phi)(Y,T_1),\overline{Z})+a^r P(X,\overline{Y},(\nabla\phi)(Z,T_1)) = a^r A(T_1)P(X,\overline{Y},\overline{Z}),$$
(2.3)a

or equivalently

$$a^r\nabla P(\overline{X},Y,\overline{Z},T_1)+a^r P((\nabla\phi)(X,T_1),Y,\overline{Z})+ P(\overline{X},(\nabla\phi)(\overline{Y},T_1),\overline{Z})+a^r P(\overline{X},Y,(\nabla\phi)(Z,T_1)) = a^r A(T_1)P(\overline{X},Y,\overline{Z}),$$
(2.3)b

or equivalently

$$a^r\nabla P(\overline{X},\overline{Y},Z,T_1)+a^r P((\nabla\phi)(X,T_1),\overline{Y},Z)+. a^r P(\overline{X},(\nabla\phi)(Y,T_1),Z)+P(\overline{X},\overline{Y},(\nabla\phi)(\overline{Z},T_1)) = a^r A(T_1)P(\overline{X},\overline{Y},Z).$$
(2.3)c

Note: Similarly (2), (3), (4), (23), (24), (13), (14), (34), (124), (134), (234), and (1234) recurrence in P can also be defined.

Definition (2.4): A Hsu-structure manifold is said to be Ricci-(1)-recurrent, if

$$a^r\nabla Ric(Y,Z,T_1)+Ric((\nabla\phi)(\overline{Y},T_1),Z). = a^r A(T_1)Ric(Y,Z,T_1).$$
(2.4)

Note: Similarly Ricci-(2)-recurrent can also be defined.

Definition (2.5): A recurrent Hsu-structure manifold is said to be P-symmetric and Ricci-symmetric if $A(T_1)=0$.

Theorem (2.1): A P-(1)-recurrent Hsu-structure manifold is P-(2)-recurrent Hsu-structure manifold for the same recurrence parameter iff

$$P\left((\nabla\phi)\left(\overline{X},T_1\right),Y,Z\right)=P\left(X,(\nabla\phi)\left(\overline{Y},T_1\right),Z\right).$$
(2.5)

Necessary condition: If P-(1)-recurrent Hsu-structure manifold is P-(2)-recurrent Hsu-structure manifold then

$$a^r\nabla P(X,Y,Z,T_1)+P\left((\nabla\phi)\left(\overline{X},T_1\right),Y,Z\right)-a^rA(T_1)P(X,Y,Z)=a^r\nabla P(X,Y,Z,T_1)+P\left(X,(\nabla\phi)\left(\overline{Y},T_1\right),Z\right)-a^rA(T_1)P(X,Y,Z).$$

$$\Rightarrow P\left((\nabla\phi)\left(\overline{X},T_1\right),Y,Z\right)=P\left(X,(\nabla\phi)\left(\overline{Y},T_1\right),Z\right).$$

Sufficient condition:

If $P\left((\nabla\phi)\left(\overline{X},T_1\right),Y,Z\right)=P\left(X,(\nabla\phi)\left(\overline{Y},T_1\right),Z\right)$, then P-(1)-recurrent Hsu-structure manifold is P-(2)-recurrent Hsu-structure manifold.

Note: Similarly a P-(1)-recurrent Hsu-structure manifold is P-(3)-recurrent Hsu-structure manifold for the same recurrence parameter iff

$$P\left((\nabla\phi)\left(\overline{X},T_1\right),Y,Z\right)=P\left(X,Y,(\nabla\phi)\left(\overline{Z},T_1\right)\right),$$
(2.6)

and

A P-(2)-recurrent Hsu-structure manifold is P-(3)-recurrent Hsu-structure manifold for the same recurrence parameter iff

$$P\left(X,(\nabla\phi)\left(\overline{Y},T_1\right),Z\right)=P\left(X,Y,(\nabla\phi)\left(\overline{Z},T_1\right)\right).$$
(2.7)

Theorem (2.2): A Ricci-(1)-recurrent Hsu-structure manifold is Ricci-(2)-recurrent Hsu-structure manifold for the same recurrence parameter iff,

$$Ric\left((\nabla\phi)\left(\overline{Y},T_1\right),Z\right)=Ric\left(Y,(\nabla\phi)\left(\overline{Z},T_1\right)\right).$$

Necessary condition: If Ricci-(1)-recurrent Hsu-structure manifold is Ricci-(2)-recurrent Hsu-structure manifold then

$$a^r\nabla Ric(Y,Z,T_1)+Ric\left((\nabla\phi)\left(\overline{Y},T_1\right),Z\right)-a^rA(T_1)Ric(Y,Z)=a^r\nabla Ric(Y,Z,T_1)+Ric\left(Y,(\nabla\phi)\left(\overline{Z},T_1\right)\right)-a^rA(T_1)Ric(Y,Z).$$

$$\Rightarrow Ric\left((\nabla\phi)\left(\overline{Y},T_1\right),Z\right)=Ric\left(Y,(\nabla\phi)\left(\overline{Z},T_1\right)\right).$$

Sufficient condition:

If $Ric\left((\nabla\phi)\left(\overline{Y},T_1\right),Z\right)=Ric\left(Y,(\nabla\phi)\left(\overline{Z},T_1\right)\right)$,
(2.8)

then Ricci-(1)-recurrent Hsu-structure manifold is Ricci-(2)-recurrent Hsu-structure manifold.

Theorem (2.3): A P-(12)-Recurrent Hsu-structure manifold is P-(1)-recurrent for the same recurrence parameter, provided

$$a^rP\left(X,(\nabla\phi)(Y,T_1),Z\right)=0.$$
(2.9)

Proof: Barring Y in equation (2.1) , we get

$$a^r\nabla P\left(X,\overline{Y},Z,T_1\right)+P\left((\nabla\phi)\left(\overline{X},T_1\right),\overline{Y},Z\right)=a^rA(T_1)P\left(X,\overline{Y},Z\right).$$
(2.10)

If the manifold is P-(12)-recurrent then from equation (2.2)a, we have

$$a^r\nabla P\left(X,\overline{Y},Z,T_1\right)+P\left((\nabla\phi)\left(\overline{X},T_1\right),\overline{Y},Z\right)+a^rP\left(X,(\nabla\phi)(Y,T_1),Z\right)=a^rA(T_1)P\left(X,\overline{Y},Z\right).$$
(2.11)

Using equation (2.9) in equation (2.11) , we get the equation (2.10). Hence the theorem.

Note: Similarly it can shown that a P-(12)-recurrent Hsu-structure manifold is P-(2)-recurrent for the same recurrence parameter, provided

$$a^rP\left((\nabla\phi)(X,T_1),Y,Z\right)=0.$$
(2.12)

Theorem (2.4): A P-(123)-recurrent Hsu-structure manifold is P-(1)-recurrent for the same recurrence parameter, provided

$$a^rP\left(X,(\nabla\phi)(Y,T_1),\overline{Z}\right)+a^rP\left(X,\overline{Y},(\nabla\phi)(Z,T_1)\right)=0.$$
(2.13)

Proof: Barring Y and Z in equation (2.1), we get

$$a^r\nabla P\left(X,\overline{Y},\overline{Z},T_1\right)+P\left((\nabla\phi)\left(\overline{X},T_1\right),\overline{Y},\overline{Z}\right)=a^rA(T_1)P\left(X,\overline{Y},\overline{Z}\right).$$
(2.14)

In equation (2.3)a if

$$a^rP\left(X,(\nabla\phi)(Y,T_1),\overline{Z}\right)+a^rP\left(X,\overline{Y},(\nabla\phi)(Z,T_1)\right)=0$$
.

Then we get the equation (2.14). Hence the theorem.

Note: Similarly we can prove that a P-(123)-recurrent Hsu-structure manifold is P-(2)-recurrent, if

$$a^rP\left((\nabla\phi)(X,T_1),Y,\overline{Z}\right)+a^rP\left(\overline{X},Y,(\nabla\phi)(Z,T_1)\right)=0,$$

and P-(3)-recurrent , if

$$a^rP\left((\nabla\phi)(X,T_1),\overline{Y},Z\right)+a^rP\left(\overline{X},(\nabla\phi)(Y,T_1),Z\right)=0$$
,

for the same recurrence parameter.

Theorem (2.5): A P-(123)-recurrent Hsu-structure manifold is P-(12)-recurrent for the same recurrence parameter, provided

$$a^rP\left(X,\overline{Y},(\nabla\phi)(Z,T_1)\right)=0.$$
(2.15)

Proof: Barring Z in equation (2.2)a , we get

$$a^r\nabla P\left(X,\overline{Y},\overline{Z},T_1\right)+P\left((\nabla\phi)\left(\overline{X},T_1\right),\overline{Y},\overline{Z}\right)+a^rP\left(X,(\nabla\phi)(Y,T_1),\overline{Z}\right)=a^rA(T_1)P\left(X,\overline{Y},\overline{Z}\right).$$
(2.16)

Using equation (2.15) in equation (2.3)a , we get equation(2.16). Hence the theorem.

Note: Similarly we can prove that P-(123)-recurrent Hsu structure manifold is P-(13)-recurrent

$$a^rP\left(X,(\nabla\phi)(Y,T_1)\overline{Z}\right)=0,$$

and P-(23)-recurrent , if

$$a^r P((\nabla\phi)(X,T_1),Y,\overline{Z}) = 0,$$

for the same recurrence parameter.

Theorem (2.6): In a (1)-recurrent Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then the third also holds:

a) It is conformal (1)-recurrent,
b) It is conharmonic (1)-recurrent,
c) It is concircular (1)-recurrent.

Proof: From equation (1.6), (1.7) and (1.8), we have

$$C(X,Y,Z) = L(X,Y,Z) + \frac{n}{n-2}\{K(X,Y,Z) - V(X,Y,Z)\}.$$

(2.17)

Barring X in equation (2.17) ,we get

$$C(\overline{X},Y,Z) = L(\overline{X},Y,Z) + \frac{n}{n-2}\{K(\overline{X},Y,Z) - V(\overline{X},Y,Z)\}.$$

(2.18)

Multiplying equation (2.18) by $A(T_1)$, barring X and then using equation (1.1)a in the resulting equation, we get

$$a^r A(T_1)C(X,Y,Z) = a^r A(T_1)L(X,Y,Z) + \frac{na^r A(T_1)}{(n-2)}\{K(X,Y,Z) - V(X,Y,Z)\}.$$

(2.19)

Differentiating equation (2.18) w.r.t. T_1, using equation (2.18) and then barring X in the resulting equation, we get

$$a^r\nabla C(X,Y,Z,T_1) + C((\nabla\phi)(\overline{X},T_1),Y,Z) = a^r\nabla L(X,Y,Z,T_1) + L((\nabla\phi)(\overline{X},T_1),Y,Z) + \frac{n}{(n-2)}\{a^r\nabla K(X,Y,Z,T_1) + K((\nabla\phi)(\overline{X},T_1),Y,Z) - a^r\nabla V(X,Y,Z,T_1) - V((\nabla\phi)(\overline{X},T_1),Y,Z)\}.$$

(2.20)

Subtracting equation (2.19) from (2.20) , we have

$$a^r\nabla C(X,Y,Z,T_1) + C((\nabla\phi)(\overline{X},T_1),Y,Z) - a^r A(T)C(X,Y,Z) = a^r\nabla L(X,Y,Z,T_1) + L((\nabla\phi)(\overline{X},T_1),Y,Z) - a^r A(T_1)L(X,Y,Z) + \frac{n}{(n-2)}\{a^r\nabla K(X,Y,Z,T_1) + K((\nabla\phi)(\overline{X},T_1),Y,Z) - a^r A(T_1)K(X,Y,Z) - a^r\nabla V(X,Y,Z,T_1) - V((\nabla\phi)(\overline{X},T_1),Y,Z) + a^r A(T_1)V(X,Y,Z)\}.$$

(2.21)

If a (1)-recurrent Hsu-structure manifold is conformal-(1) - recurrent and conharmonic (1)-recurrent for the same recurrence parameter then from equation (2.21), we get

$$a^r(\nabla V)(X,Y,Z,T_1) + V((\nabla\phi)(\overline{X},T_1),Y,Z) = a^r A(T_1)V(X,Y,Z).$$

(2.22)

Which shows, that the manifold is concircular-(1)-recurrent.

The proof of the remaining two cases follows similarly.

Theorem (2.7): In a (1)-symmetric Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then third also holds:

a) It is conformal (1)-symmetric,
b) It is conharmonic (1)-symmetric,
c) It is concircular (1)-symmetric.

Proof: The statement follows from the theorem (2.6) and definition (2.5).

Theorem (2.8): In a (12)-recurrent Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then the third also holds:

a) It is conformal (12)-recurrent,
b) It is conharmonic (12)-recurrent,
c) It is concircular (12)-recurrent.

Proof: Barring X and Y in equation (2.17), we get

$$C(\overline{X},\overline{Y},Z) = L(\overline{X},\overline{Y},Z) + \frac{n}{n-2}\{K(\overline{X},\overline{Y},Z) - V(\overline{X},\overline{Y},Z)\}.$$

(2.23)

Multiplying equation (2.23) by $A(T_1)$, barring X and then using equation (1.1)a in the resulting equation, we get

$$a^r A(T_1)C(X,\overline{Y},Z) = a^r A(T_1)L(X,\overline{Y},Z) + \frac{na^r A(T_1)}{(n-2)}\{K(X,\overline{Y},Z) - V(X,\overline{Y},Z)\}.$$

(2.24)

Differentiating equation (2.23) w.r.t T_1, using equation (2.23) and then barring X in the resulting equation, we get

$$a^r\nabla C(X,\overline{Y},Z,T_1) + C((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) + a^r C(X,(\nabla\phi)(Y,T_1),Z) = a^r\nabla L(X,\overline{Y},Z,T_1) + L((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) + a^r L(X,(\nabla\phi)(Y,T_1),Z) +$$

$$\frac{n}{(n-2)}\{a^r\nabla K(X,\overline{Y},Z,T_1) + K((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) +$$

$$a^r K(X,(\nabla\phi)(Y,T_1),Z) - a^r\nabla V(X,\overline{Y},Z,T_1) -$$

$$V((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) - a^r V(X,(\nabla\phi)(Y,T_1),Z)\}.$$

(2.25)

Subtracting equation (2.24) from equation (2.25), we get

$$a^r\nabla C(X,\overline{Y},Z,T_1) + C((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) +$$
$$a^r C(X,(\nabla\phi)(Y,T_1),Z) - a^r A(T_1)C(X,\overline{Y},Z) =$$
$$a^r\nabla L(X,\overline{Y},Z,T_1) + L((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) +$$
$$a^r L(X,(\nabla\phi)(Y,T_1),Z) - a^r A(T_1)L(X,\overline{Y},Z) +$$
$$\frac{n}{(n-2)}\{a^r\nabla K(X,\overline{Y},Z,T_1) + K((\nabla\phi)(X,T_1),\overline{Y},Z) +$$

$$a^r K(X,(\nabla\phi)(Y,T_1),Z) - a^r A(T_1)K(X,\overline{Y},Z) -$$

$$a^r\nabla V(X,\overline{Y},Z,T_1) - V((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) -$$
$$a^r V(X,(\nabla\phi)(Y,T_1),Z) + a^r A(T_1)V(X,\overline{Y},Z)\}.$$

(2.26)

If a (12)-recurrent Hsu-structure manifold is conformal (12)-recurrent and conharmonic (12)-recurrent for the same recurrence parameter then from equation (2.26) , we get

$$a^r\nabla V(X,\overline{Y},Z,T_1) + V((\nabla\phi)(\overline{X},T_1),\overline{Y},Z) +$$
$$a^r V(X,(\nabla\phi)(Y,T_1),Z) = a^r A(T_1)V(X,\overline{Y},Z)$$

(2.27)

Which shows, that the manifold is concircular-(12)-recurrent.

The proof of the remaining two cases follows similarly.

Theorem (2.9): In a (12)-symmetric Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then third also holds:

a) It is conformal (12)-symmetric,
b) It is conharmonic (12)-symmetric,
c) It is concircular (12)-symmetric.

Proof: The statement follows from the theorem (2.8) and definition (2.5).

Theorem (2.10): In a (123)-recurrent Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then the third also holds:

a) It is conformal (123)-recurrent,
b) It is conharmonic (123)-recurrent,
c) It is concircular (123)-recurrent.

Proof: Barring X, Y and Z in equation (2.17), we get

$$C(\overline{X},\overline{Y},\overline{Z}) = L(\overline{X},\overline{Y},\overline{Z}) + \frac{n}{n-2}\{K(\overline{X},\overline{Y},\overline{Z}) - V(\overline{X},\overline{Y},\overline{Z})\}.$$

(2.28)

Multiplying equation (2.28) by $A(T_1)$, barring X and then using equation (1.1)a in the resulting equation , we get

$$a^r A(T_1)C(X,\overline{Y},\overline{Z}) = a^r A(T_1)L(X,\overline{Y},\overline{Z}) +$$
$$\frac{na^r A(T_1)}{(n-2)}\{K(X,\overline{Y},\overline{Z}) - V(X,\overline{Y},\overline{Z})\}.$$

(2.29)

Differentiating equation (2.28) w.r.t T_1, using equation (2.28) and then barring X in the resulting equation, we get

$$a^r\nabla C(X,\overline{Y},\overline{Z},T_1) + C((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) +$$

$$a^r C(X,(\nabla\phi)(Y,T_1),\overline{Z}) + a^r C(X,\overline{Y},(\nabla\phi)(Z,T_1)) =$$

$$a^r\nabla L(X,\overline{Y},\overline{Z},T_1) + L((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) +$$
$$a^r L(X,(\nabla\phi)(Y,T_1),\overline{Z}) + a^r L(X,\overline{Y},(\nabla\phi)(Z,T_1)) +$$

$$\frac{n}{(n-2)}\{a^r\nabla K(X,\overline{Y},\overline{Z},T_1) + K((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) +$$

$$a^r K(X,(\nabla\phi)(Y,T_1),\overline{Z}) + a^r K(X,\overline{Y},(\nabla\phi)(Z,T_1)) -$$

$$a^r\nabla V(X,\overline{Y},\overline{Z},T_1) - V((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) -\}$$
$$a^r V(X,(\nabla\phi)(Y,T_1),\overline{Z}) - a^r V(X,\overline{Y},(\nabla\phi)(Z,T_1))\}$$

.

(2.30)

Subtracting equation (2.29) from equation (2.30), we get

$$a^r\nabla C(X,\overline{Y},\overline{Z},T_1) + C((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) +$$
$$a^r C(X,(\nabla\phi)(Y,T_1),\overline{Z}) + a^r C(X,\overline{Y},(\nabla\phi)(Z,T_1)) -$$

$$a^r A(T_1)C(X,\overline{Y},\overline{Z}) = a^r\nabla L(X,\overline{Y},\overline{Z},T_1) +$$
$$L((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) + a^r L(X,(\nabla\phi)(Y,T_1),\overline{Z}) +$$

$$a^r L(X,\overline{Y},(\nabla\phi)(Z,T_1)) - a^r A(T_1)L(X,\overline{Y},\overline{Z}) +$$
$$\frac{n}{(n-2)}\{a^r\nabla K(X,\overline{Y},\overline{Z},T_1) + K((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) +$$

$$a^r K(X,(\nabla\phi)(Y,T_1),\overline{Z}) + a^r K(X,\overline{Y},(\nabla\phi)(Z,T_1)) -$$

$$a^r A(T_1)K(X,\overline{Y},\overline{Z}) - a^r\nabla V(X,\overline{Y},\overline{Z},T_1) -$$
$$V((\nabla\phi)(\overline{X},T_1),\overline{Y},\overline{Z}) - a^r V(X,(\nabla\phi)(Y,T_1),\overline{Z}) -$$

$$a^r V\left(X, \overline{Y}, (\nabla\phi)(Z, T_1)\right) + a^r A(T_1) V\left(X, \overline{Y}, \overline{Z}\right)\}$$
. (2.31)

If a (123)-recurrent Hsu-structure manifold is conformal (123)-recurrent and conharmonic (123)-recurrent for the same recurrence parameter then from equation (2.31), we get

$$a^r \nabla V\left(X, \overline{Y}, \overline{Z}, T_1\right) + V\left((\nabla\phi)(\overline{X}, T_1), \overline{Y}, \overline{Z}\right) + a^r V\left(X, (\nabla\phi)(Y, T_1), \overline{Z}\right) + a^r V\left(X, \overline{Y}, (\nabla\phi)(Z, T_1)\right) = a^r A(T_1) V\left(X, \overline{Y}, \overline{Z}\right). \quad (2.32)$$

This shows, that the manifold is concircular (123)-recurrent Hsu-structure manifold.

The proof of the remaining two cases follows similarly.

Theorem (2.11): In a (123)-symmetric Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then third also holds:

a) It is conformal (123)-symmetric,
b) It is conharmonic (123)-symmetric,
c) It is concircular (123)-symmetric.

Proof: The statement follows from the theorem (2.8) and definition (2.5).

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