CFD modeling of pressure drop caused by two-phase flow of oil/water emulsions through sudden expansions

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ABSTRACT

Pressure drop through sudden expansions are numerically investigated for two-phase flow of oil/water emulsions. Two-phase computational fluid dynamics (CFD) calculations, using Eulerian–Eulerian model are employed to calculate the velocity profiles and pressure drop across sudden expansion. Axial pressure drops have been determined by extrapolating the computed axial pressure profiles in the regions of fully developed pipe flow upstream and downstream of the pipe expansion, to the transitional cross section. The oil concentration is varied over a wide range of 0-97.3 % by volume. From the pressure-loss and velocity data, the loss coefficients are obtained. The loss coefficients for the emulsions are found to be independent of the concentration and type of emulsions. The numerical results are validated against experimental data from the literature and are found to be in good agreement.

Keywords- Two-phase flow, pressure drop, loss coefficient, velocity head, concentration, emulsion.

I.INTRODUCTION

Industrial piping systems are often charged with two-phase flows. In contrast to the well-known axial pressure profiles in the transitional region between the flow separation and reattachment for single-phase liquid flow, the pressure profiles and the shape of streamlines in two-phase flow through sudden change in flow area are still unknown. Due to inherent complexity of two-phase flows through such sections, from a physical as well as numerical point of view, generally applicable computational fluid dynamics (CFD) codes are non-existent. Two-phase flow of oil/water emulsions find application in a number of industries, such as petroleum, pharmaceutical, agriculture and food industries etc. In many applications, pumping of emulsions through pipes and pipe fittings is required. Since a detailed physical description of the flow mechanism is still not possible for two-phase flow, a considerable effort is generally needed to calculate the pressure drop along the flow path. Several papers have been published on flow of two-phase gas/liquid and liquid/liquid mixtures through pipe fittings. Hwang & Pal[1] studied experimentally the flow of oil/water emulsions through sudden expansions and contractions and found that the loss coefficient for emulsions is independent of the concentration and type of emulsions. Wadle[2] carried out a theoretical and experimental study on the pressure recovery in abrupt expansions. He proposed a formula for the pressure recovery based on the superficial velocities of the two phases and verified its predictive accuracy with measured experimental steam-water and air-water data.

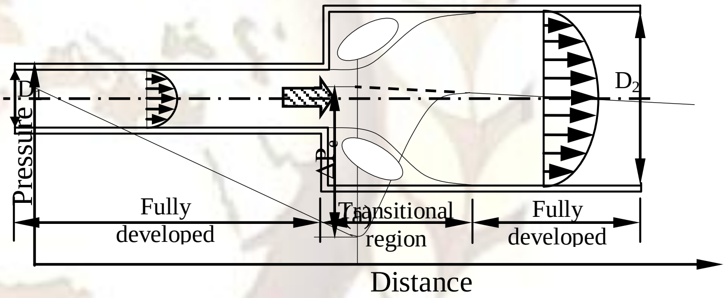


Fig. 1(a) Idealized course of boundary stream lines and (b) pressure profile for a sudden expansion.

Tapucu et al.[3] observed that emulsions can be treated as pseudo-homogeneous fluids with suitably averaged properties as the dispersed droplets of emulsions are small and are well dispersed. Consequently, the pressure loss for emulsion flow in expansion and contraction should be determinable in the same way as for single-phase fluid flow. Acrivos and Schrader[4] observed that significant velocity slip occurs at both sides of the enlargement for two phase flow mixtures. Attou et al.[5] developed a semi-analytical model for two-phase pressure drop in sudden enlargements, based on the solution of one-dimensional conservation equations downstream of the enlargement. They compared the predictions of three models (homogeneous flow; frozen flow; and bubbly flow) with experimental data, with the latter model providing the best agreement with data. Abdelall et al.[6] studied the pressure drop caused by abrupt flow area expansion and contraction in small channels and developed an empirical correlation for two-phase flow pressure drop through sudden area contraction. They indicated a significant velocity slip at the vicinity of the change of flow area. Salcudean et al.[7] studied the effect of various flow obstructions on pressure drops in horizontal two-phase flow of air-water mixtures and derived pressure loss coefficients and two-phase multipliers.

In the present study, an attempt has been made to simulate the flow through sudden expansion using two phase flow models in an Eulerian scheme. Fig. 1 shows a cross section of the test section. At this section there is a sudden, sharp edged expansion. Fig. 1(a) shows the schematic diagram of the boundary streamlines for the flow through a sudden expansion, while fig. 1(b) depicts the graph of the static pressure along the flow axis for a steady state flow of an incompressible fluid across an expansion.

II.Mathematical Formulations

The two-fluid or Euler - Euler technique is considered for the present formulation. The different phases are treated mathematically as interpenetrating continua, with each computational cell of the domain containing respective fractions of the continuous and dispersed phases. We have adopted the following assumptions in our study which are very realistic for the present situation.

1. The fluids in both phases are Newtonian, viscous and incompressible.
2. The physical properties remain constant.
3. No mass transfer between the two phases.
4. The pressure is assumed to be common to both the phases.
5. The realizable k-ε turbulent model is applied to describe the behavior of each phase.
6. The surface tension forces are neglected, therefore, the pressure of both phases are equal at any cross section.
7. The flow is assumed to be isothermal, so the energy equations are not needed.

With all the above assumptions the governing equations for phase q can be written as (Drew[8]):

Continuity equation:

$$\frac{\partial}{\partial t}\left(\alpha_q\rho_q\right)+\nabla\cdot\left(\alpha_q\rho_q\vec{v}_q\right)=0 \qquad (1)$$

The volume fractions are assumed to be continuous functions of space and time and their sum is equal to one.

$$\alpha_q+\alpha_p=1 \qquad (2)$$

Momentum equation:

$$\frac{\partial}{\partial t}(\alpha_q\rho_q\vec{v}_q)+\nabla\cdot\left(\alpha_q\rho_q\vec{v}_q\vec{v}_q\right)=-\alpha_q\nabla p+\nabla\cdot(\bar{\bar{\tau}}_q)+\alpha_q\rho_q\vec{g}+M_q \qquad (3)$$

$\bar{\bar{\tau}}_q$, is the qth phase stress tensor

$$\bar{\bar{\tau}}_q=\alpha_q\mu_q^{eff}\left(\nabla\vec{v}_q+\nabla\vec{v}_q^{T}\right) \qquad (4)$$

$$\mu_q^{eff}=\mu_q+\mu_{t,q} \qquad (5)$$

Where M_q is the interfacial momentum transfer term, which is given by:

$$M_q=M_q^d+M_q^{VM}+M_q^L \qquad (6)$$

Where the individual terms on the right hand side of Eq. (6) are, respectively, the drag force, virtual mass force and lift force. The drag force is expressed as,

$$M_q^d=\frac{3}{4d_p}\alpha_p\rho_qC_D\left|\vec{v}_p-\vec{v}_q\right|\left(\vec{v}_p-\vec{v}_q\right) \qquad (7)$$

The drag coefficient C_D depends on the particle Reynolds number as given below (Wallis[9]; Ishii and Zuber[10]):

$C_D = 24(1+0.15Re^{0.687})/Re$, $\quad Re \le 1000$
$\quad = 0.44$, $\quad Re > 1000 \qquad (8)$

Relative Reynolds number for primary phase q and secondary phase p is given by

$$Re=\frac{\rho_q\left|\vec{v}_q-\vec{v}_p\right|d_p}{\mu_q} \qquad (9)$$

The second term in Eq. (6) represents the virtual mass force, which can be described by the following expression (Drew[8]):

$$M_q^{VM}=-M_p^{VM}=C_{VM}\alpha_p\rho_q\left(\frac{d_q\vec{v}_q}{dt}-\frac{d_p\vec{v}_p}{dt}\right) \qquad (10)$$

where C_{VM} is the virtual mass coefficient, which for a spherical particle is equal to 0.5.

The third term in Eq. (6) is the lift force, and is given by (Drew and Lahey[11])

$$M_q^L=-M_p^L=C_L\alpha_p\rho_q\left(\vec{v}_p-\vec{v}_q\right)\times\left(\nabla\times\vec{v}_q\right) \qquad (11)$$

where C_L is the lift force coefficient, which for shear flow around a spherical droplet is equal to 0.5

2.1 Turbulence modeling

Here we considered the realizable per-phase $k-\varepsilon$ turbulence model.

Transport Equations for k (FLUENT 6.2 Manual[12]):

$$\frac{\partial}{\partial t}\left(\alpha_q\rho_q k_q\right)+\nabla.\left(\alpha_q\rho_q\vec{U}_q k_q\right)=\nabla.\left[\alpha_q\left(\mu_q+\frac{\mu_{t,q}}{\sigma_k}\right)\nabla k_q\right]+$$
$$\left(\alpha_q G_{k,q}-\alpha_q\rho_q\varepsilon_q\right)+K_{pq}\left(C_{pq}k_p-C_{qp}k_q\right)-$$
$$K_{pq}\left(\vec{U}_p-\vec{U}_q\right).\frac{\mu_{t,p}}{\alpha_p\sigma_p}\nabla\alpha_p+K_{pq}\left(\vec{U}_p-\vec{U}_q\right).\frac{\mu_{t,q}}{\alpha_q\sigma_q}\nabla\alpha_q \tag{12}$$

Transport Equations for ε :

$$\frac{\partial}{\partial t}\left(\alpha_q\rho_q\varepsilon_q\right)+\nabla.\left(\alpha_q\rho_q\vec{U}_q\varepsilon_q\right)=\nabla.\left[\alpha_q\left(\mu_q+\frac{\mu_{t,q}}{\sigma_\varepsilon}\right)\nabla\varepsilon_q\right]$$
$$+\alpha_q\rho_q C_1 S\varepsilon_q-C_2\alpha_q\rho_q\frac{\varepsilon_q^2}{k_q+\sqrt{\nu_{t,q}\varepsilon_q}}+C_{1\varepsilon}\frac{\varepsilon_q}{k_q}\left[K_{pq}\left(C_{pq}k_p-C_{qp}k_q\right)\right]$$
$$-C_{1\varepsilon}\frac{\varepsilon_q}{k_q}\left[K_{pq}\left(\vec{U}_p-\vec{U}_q\right)\cdot\frac{\mu_{t,p}}{\alpha_p\sigma_p}\nabla\alpha_p+K_{pq}\left(\vec{U}_p-\vec{U}_q\right)\cdot\frac{\mu_{t,q}}{\alpha_q\sigma_q}\nabla\alpha_q\right] \tag{13}$$

Where, $\vec{U}_q$ is the phase-weighted velocity. Here,

$C_1 = \max\left[0.43, \frac{\eta}{\eta+5}\right]$, $\eta = S\frac{k}{\varepsilon}$, $S=\left(2S_{ij}S_{ij}\right)^{0.5}$

The terms C_{pq} and C_{qp} can be approximated as

$$C_{pq}=2, C_{qp}=2\left(\frac{\eta_{pq}}{1+\eta_{pq}}\right) \tag{14}$$

Where η_{pq} is defined as

$$\eta_{pq}=\frac{\tau_{t,pq}}{\tau_{F,pq}} \tag{15}$$

Where, the Langrangian integral time scale ($\tau_{t,pq}$), is defined as

$$\tau_{t,pq}=\frac{\tau_{t,q}}{\sqrt{\left(1+C_\beta\xi^2\right)}} \tag{16}$$

Where, $\xi=\frac{\left|\vec{v}_{pq}\right|\tau_{t,q}}{L_{t,q}}$ (17)

Where $\tau_{t,q}$ is a characteristic time of the energetic turbulent eddies and is defined as:

$$\tau_{t,q}=\frac{3}{2}C_\mu\frac{k_q}{\varepsilon_q} \tag{18}$$

And $C_\beta = 1.8-1.35\cos^2\theta$ (19)

Where, θ is the angle between the mean particle velocity and the mean relative velocity. The characteristic particle relaxation time connected with inertial effects acting on a dispersed phase p is defined as

$$\tau_{F,pq}=\alpha_p\rho_q K_{pq}^{-1}\left(\frac{\rho_p}{\rho_q}+C_V\right) \tag{20}$$

Where, $C_V = 0.5$

The eddy viscosity model is used to calculate averaged fluctuating quantities. The Reynolds stress tensor for continuous phase *q* is given as:

$$\bar{\bar{\tau}}_q=-\frac{2}{3}\left(\rho_q k_q+\rho_q\mu_{t,q}\nabla\cdot\vec{U}_q\right)\bar{\bar{I}}+\rho_q\mu_{t,q}\left(\nabla\vec{U}_q+\nabla\vec{U}_q^T\right) \tag{21}$$

The turbulent viscosity $\mu_{t,q}$ is written in terms of the turbulent kinetic energy of phase q:

$$\mu_{t,q}=\rho_q C_\mu\frac{k_q^{\ 2}}{\varepsilon_q} \tag{22}$$

The production of turbulent kinetic energy, $G_{k,q}$ is computed from

$$G_{k,q}=\mu_{t,q}\left(\nabla\vec{v}_q+\nabla\vec{v}_q^T\right):\nabla\vec{v}_q \tag{23}$$

Unlike standard and RNG $k-\varepsilon$ models, C_μ is not a constant here. It is computed from:

$$C_\mu=\frac{1}{A_0+A_s\frac{kU^*}{\varepsilon}} \tag{24}$$

Where $U^*\equiv\sqrt{S_{ij}S_{ij}+\tilde{\Omega}_{ij}\tilde{\Omega}_{ij}}$ (25)

And $\tilde{\Omega}_{ij}=\Omega_{ij}-2\varepsilon_{ijk}\omega_k$

$$\Omega_{ij}=\overline{\Omega_{ij}}-\varepsilon_{ijk}\omega_k$$

Where, $\overline{\Omega_{ij}}$ is the mean rate of rotation tensor viewed in a rotating reference frame with the angular velocity $\overline{\omega_k}$ .The constants A_0 and A_s are given by

A_0= 4.04, A_s= $\sqrt{6}\cos\phi$

Where $\phi=\frac{1}{3}\cos^{-1}\left(\sqrt{6}W\right)$, $W=\frac{S_{ij}S_{jk}S_{ki}}{S^3}$,

$\tilde{S}=\sqrt{S_{ij}S_{ij}}$, $S_{ij}=\frac{1}{2}\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right)$

The constants used in the model are the following:
$C_{1\varepsilon}$ = 1.44; C_2 = 1.9; σ_k = 1.0; σ_ε = 1.2.

2.2 Boundary conditions

Velocity inlet boundary condition is applied at the inlet. A no-slip and no-penetrating boundary condition is imposed on the wall of the pipe. At the outlet, the boundary condition is assigned as outflow, which implies diffusion flux for the entire variables in exit direction are zero. Symmetry boundary condition is considered at the axis, which implies normal gradients of all flow variables are zero and radial velocity is zero at the axis.

III. NUMERICAL SOLUTION PROCEDURE

The objective of the present work is to simulate the flow through sudden expansion in pipes numerically by using two phase flow models in an Eulerian scheme. The flow field is assumed to be axisymmetric and solved in two dimensions. The two-dimensional equations of mass, momentum, volume fraction and turbulent quantities along with the boundary conditions have been integrated over a control volume and the subsequent equations have been discretized over the control volume using a finite volume technique to yield algebraic equations which are solved in an iterative manner for each time step. The finite difference algebraic equations for the conservation equations are solved using Fluent 6.2 double precision solver with an implicit scheme for all variables with a final time step of 0.001 for quick convergence. The discretization form for all the convective variables are taken to be first order up winding initially for better convergence. Slowly as time progressed the discretization forms are switched over to second order up winding and then slowly towards the QUICK scheme for better accuracy. The Phase-Coupled SIMPLE algorithm is used for the pressure-velocity coupling. The velocities are solved coupled by the phases, but in a segregated fashion. The block algebraic multigrid scheme is used to solve a vector equation formed by the velocity components of all phases simultaneously. Pressure and velocities are then corrected so as to satisfy the continuity constraint. The realizable per-phase k-ε model has been used as closure model for turbulent flow. Fine grids are used near the wall as well as near the expansion section to capture more details of velocity and volume fraction changes.

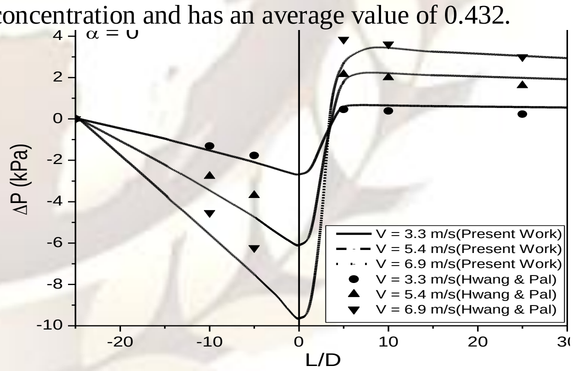
IV. RESULTS AND DISCUSSIONS

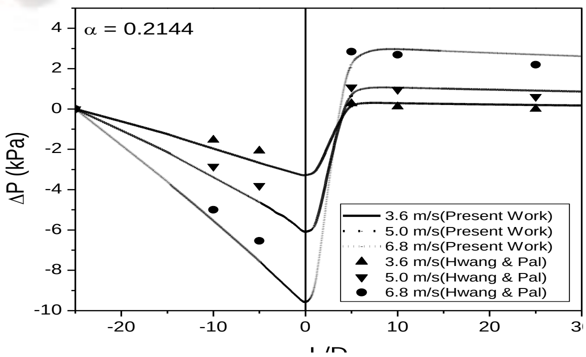
The sudden expansion considered in this work is made from two straight pipes having inner diameters of 2.037cm and 4.124cm. Axial static pressure profiles are computed both upstream and downstream from the expansion plane. By extrapolating these pressure profiles to the expansion plane the pressure drop is calculated. The pressure differentials are computed with respect to the reference pressure at $25D_1$ upstream position. The oil used in the present computational work is Bayol-35 (Esso Petroleum, Canada), which is a refined white mineral oil with a density of 780 kg/m^3 and a viscosity of 0.00272 Pa-s at 25°C. Density and viscosity of water are taken as 998.2 kg/m^3 and 0.001003 Pa-s, respectively. The volume fraction of oil is taken as 0, 0.2144, 0.3886, 0.6035, 0.6457, 0.6950, 0.8042, and 0.9728. The emulsions are considered as oil-in-water (O/W) type (water is taken as the continuous phase and oil as dispersed phase) up to an oil concentration of 62 % by volume and water-in-oil (W/O) type (oil is taken as the continuous phase and water as dispersed phase) beyond 64 % by volume (Hwang and Pal[1]).

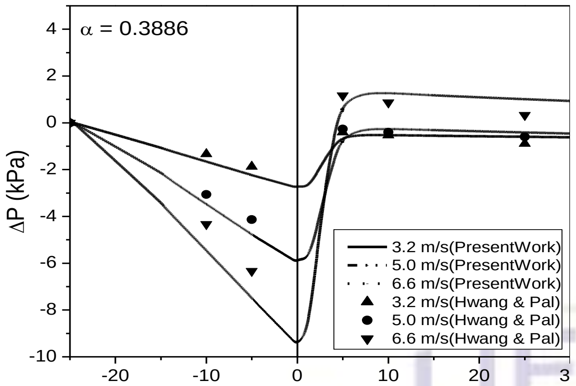
The streamlines and velocity vectors for α = 0.2144 and v = 6.8 m/s are depicted in figs. 4 and 5 respectively. The streamlines take a typical diverging pattern and a zone of recirculating flow with turbulent eddies near the wall of the larger pipe are created in the corner. This is due to the fact that the fluid particles near the wall due to their low kinetic energy can not overcome the adverse pressure hill in the direction of flow and hence follow up the reverse path under the favourable pressure gradient(since upstream pressure is lower than the downstream pressure as depicted in figs. 2 and 3).

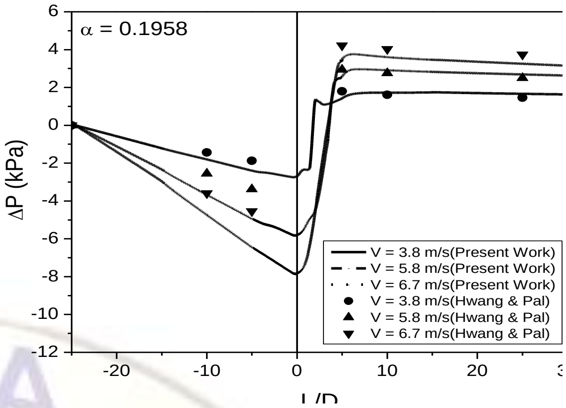
Figs. 6 and 7 show plot of $\Delta P_e/\rho$ versus velocity head $(V^2/2)$ for various differently concentrated oil-in-water and water-in-oil emulsions, respectively. It can be seen that $\Delta P_e/\rho$ versus $V^2/2$ data exhibit a linear relationship. K_2 is the slope of $\Delta P_e/\rho$ versus $V^2/2$ plots. Thus, the loss coefficient for expansion K_e which is equal to (K_1+K_2) is calculated for various differently concentrated oil-in-water and water-in-oil emulsions.

Here, $K_1=\left[1-(D_1/D_2)^4\right]$. The K_e values for different emulsions are plotted as a function of oil concentration in Fig. 8. Clearly, the expansion loss coefficient is found to be independent of oil concentration and has an average value of 0.432.









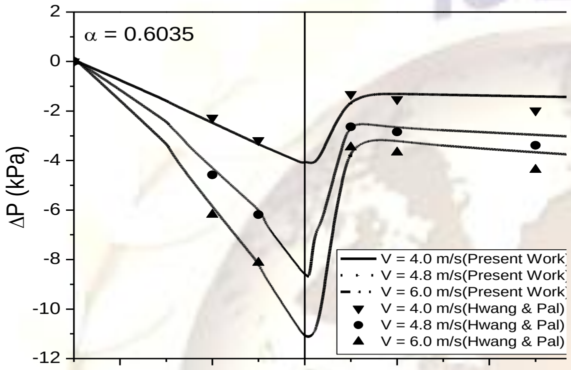


Fig. 2. Pressure profiles for oil-in-water emulsions flowing through a sudden expansion

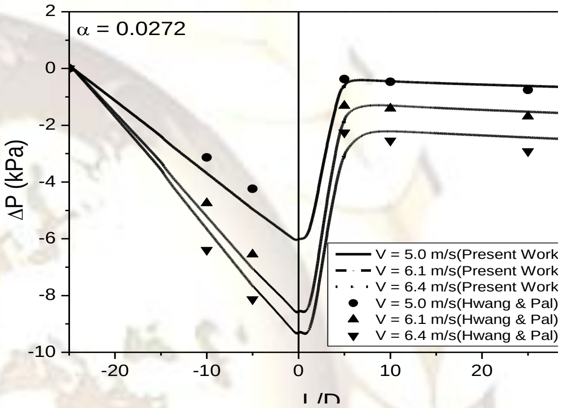
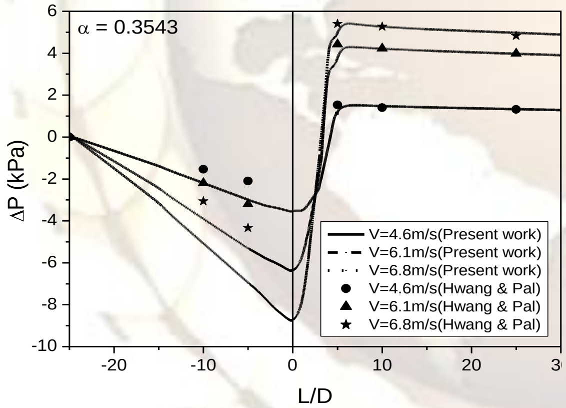


Fig. 3. Pressure profiles for water-in-oil emulsions flowing through a sudden expansion



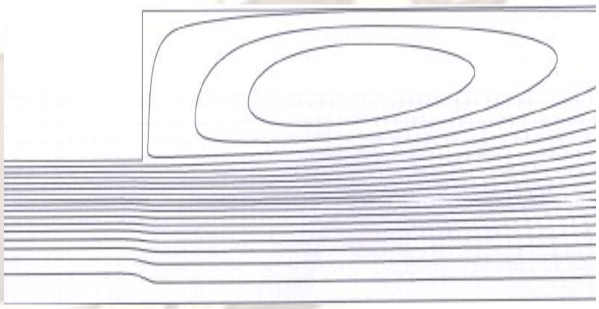
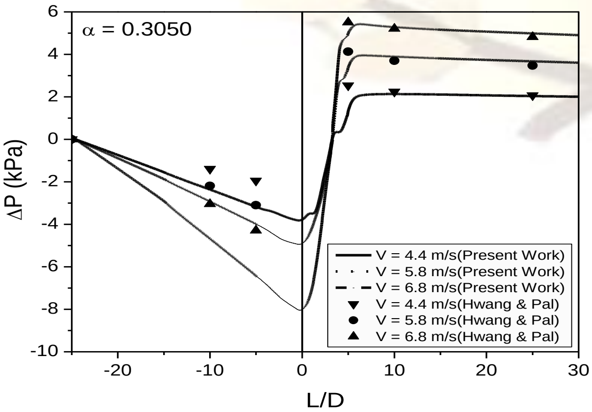


Fig. 4. Stream lines for α = 0.2144 and v = 6.8 m/s



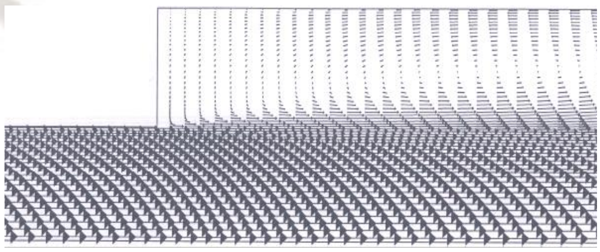


Fig. 5.Velocity vectors for α = 0.2144 and v = 6.8 m/s

Computed as well as experimental pressure profiles for oil-in-water and water-in-oil emulsions at various fluid velocities are shown in figs. 2 and 3 respectively. The matching between the computation and that of the experimental observation for the pressure drop seems to be pretty reasonable in all these cases. It can be observed that the frictional loss in the inlet section causes the decline in pressure. As the fluid reaches the transitional section, the fluid is decelerated in the enlarged pipe area and there occurs a sudden rise in pressure. The pressure change at the expansion plane (ΔP_e) is obtained by extrapolating the computed pressure profiles upstream and downstream of the pipe expansion (in the region of fully developed pipe flow) to the expansion plane.

The computed K_e values for emulsions are compared with the values obtained from the following equations:

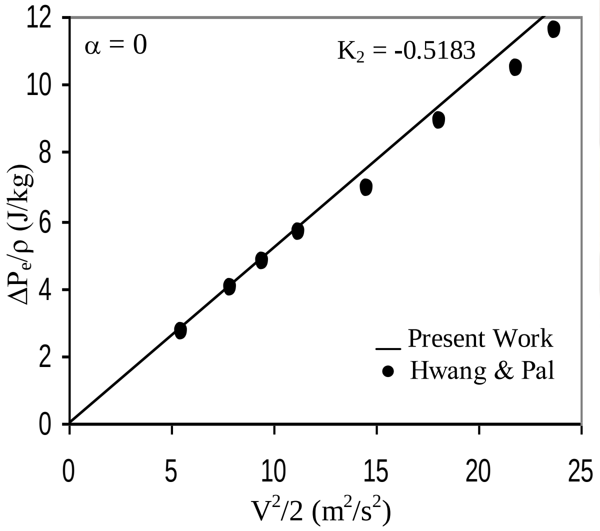
(i) Borda- Camot equation (Perry et al.[14]):

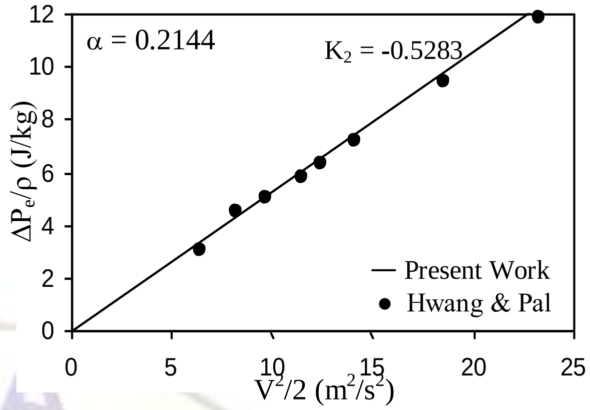
$$K_e = (1-\beta)^2 \qquad (26)$$

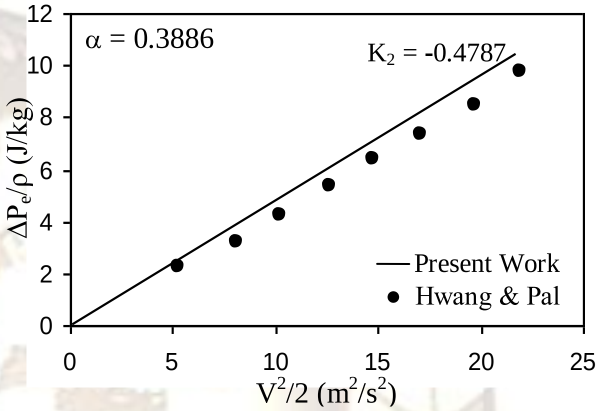
(ii) Equation of Wadle[2]:

$$K_e = 2\beta(1-\beta) \qquad (27)$$

Where β is the ratio of the cross sectional area of small pipe to that of large pipe. The β value for the expansion in the present work is 0.244. So the value of K_e obtained from Eqs. 26 and 27 are 0.5715 and 0.3689 respectively. The experimental value of K_e obtained by Hwang and Pal (1997) is 0.47. As shown in Fig. 8, the computed K_e values for all emulsions lie in between the two values obtained from Eqs. (26) and (27).







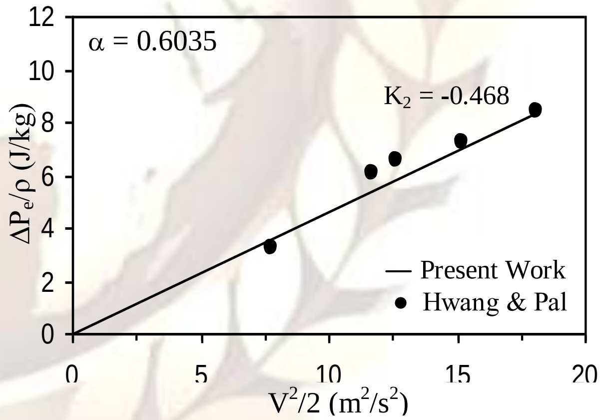


Fig. 6. $\Delta P_e/\rho$ versus $V^2/2$ for oil-in-water emulsions flowing through a sudden expansion

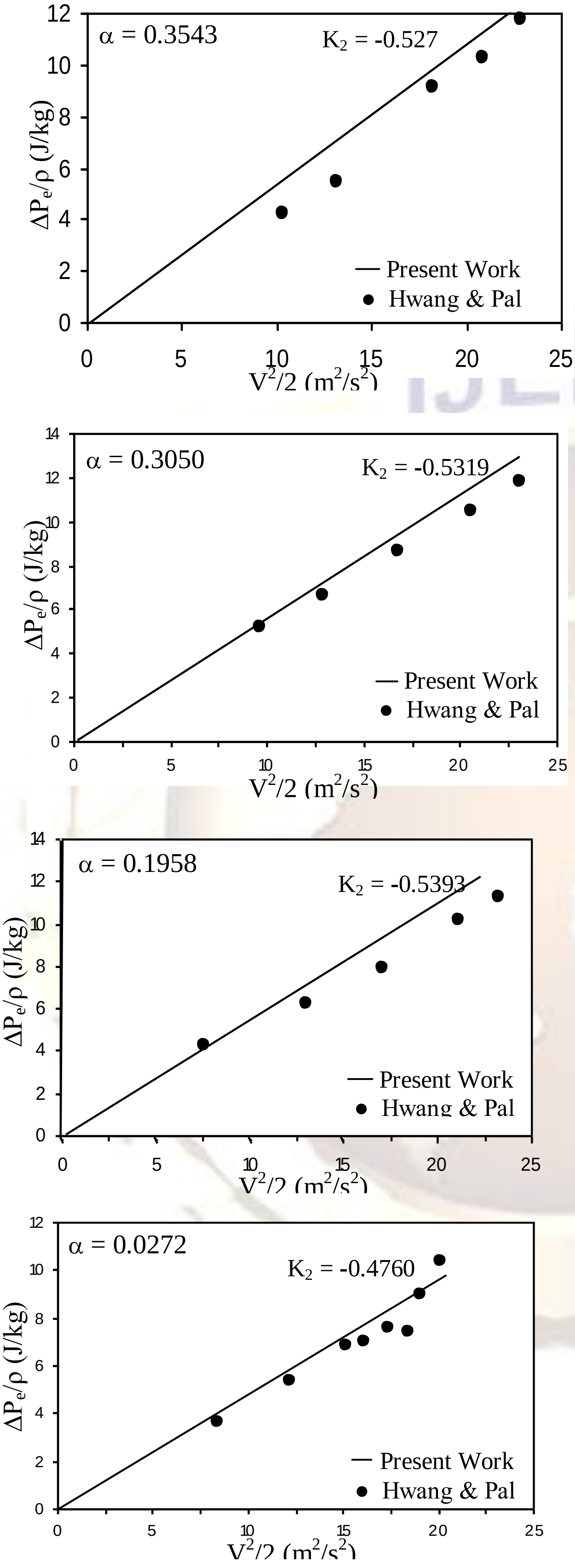


Fig.7. $\Delta P_e/\rho$ versus $V^2/2$ data for water-in-oil emulsions flowing through a sudden expansion

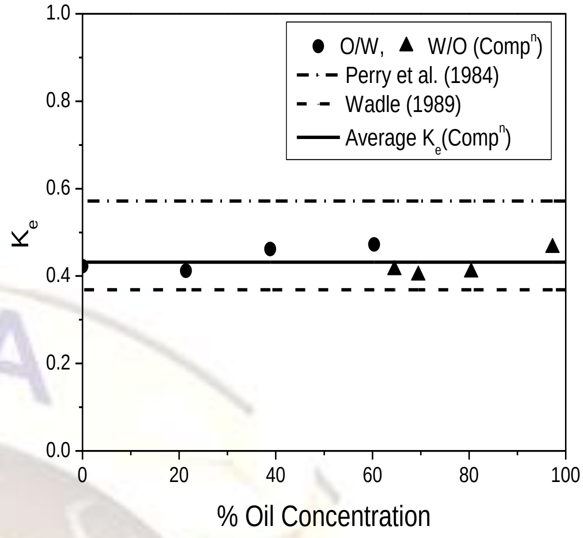


Fig.8. Expansion loss coefficient as a function of oil concentration

V. CONCLUSIONS

The flow through sudden expansion has been numerically investigated with oil-water emulsions by using two-phase flow model in an Eulerian scheme in this study. The major observations made relating to the pressure drop in the process of flow through sudden expansion can be summarized as follows:

1. The expansion loss coefficient is found to be independent of the velocity and hence Reynolds number.
2. The loss coefficient is not significantly influenced by the type and concentration of oil-water emulsions flowing through sudden expansion.
3. Effect of viscosity is negligible on the pressure drop through sudden expansion.
4. The computed expansion loss coefficient is found to lie in between the two values obtained from Borda- Carnot equation (Perry et al.[14]) and equation of Wadle[2]. It is in relatively close agreement with the predictions of Wadle[2].
5. The pressure drop increases with higher inlet velocity and hence with higher mass flow rate.
6. The satisfactory agreement between the numerical and experimental results indicates that the model may be used as a simple, efficient tool for engineering analysis of two-phase flow through sudden flow area expansions and contractions.

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