ABSTRACT

The connecting rod is a structural component cyclic loaded during the Internal Combustion Engines (ICE) operation, it means that fatigue phenomena should be taken into account during the development, in order to guarantee the connecting rod required lifetime. Numerical tools have been extremely used during the connecting rod development phase, therefore, the complete understand of the mechanisms involved as well as the reliability of the numerical methodology are extremely important to take technological advantages, such as, to reduce project lead time and prototypes cost reduction. The present work shows the complete connecting rod Finite Element Analysis (FEA) methodology. It was also performed a fatigue study based on Stress Life (SxN) theory, considering the Modified Goodman diagram.

Keywords – Crank System Kinematics, Fatigue analysis, FEA

I. INTRODUCTION

During the ICE operation, the connecting rod is cyclic loaded due to the engine physics behavior. Basically, the tensile and compressive forces are applied on the connecting rod during the engine cycle. The tensile force is applied during the exhaust stroke, while the compression occurs at the power stroke. Depending on these loads magnitudes and its combination, localized cracks can be nucleated. Adding the fact of the high cycles presented on the ICE, premature and catastrophic failures can occur. The intention is to present the connecting rod FEA methodology as well as the fatigue treatment based on SxN assumption.

II. FINITE ELEMENT METHODOLOGY

The FEA is divided in three different steps:
- Pre processor: Includes the 3D model preparation, loads and boundary condition definition, to select the appropriated element type and shape function and Finite Element (FE) mesh generation.
- Solver: Definition of the numerical method to solve the linear system of equations, convergence criteria, error estimation and strain stress calculation.
- Post Processor: Engineers analysis and judgment phase based on stablished design criteria.
Where:
L = Connecting rod length
r = Crank radius
θ = Crank angle
β = Connecting rod angle
x = Piston instantaneous position
TDC = Top dead center

The piston instantaneous position can be calculated according to the scheme presented in Figure 2.

\[ x = (r + L) - (r \cos \theta + L \cos \beta) \]  

Rewriting the expression:

\[ x = r (1 - \cos \theta) + L (1 - \cos \beta) \]  

The equation 2 has two degrees of freedom, but it is possible to obtain the angle \( \beta \) in terms of angle \( \theta \).

\[ L \sin \beta = r \sin \theta \]

Therefore:

\[ \sin \beta = \frac{r \sin \theta}{L} \]

Denominating \( \lambda \) as the relation between the crank radius \( r \) and the connecting rod length \( L \) we have:

\[ \sin \beta = \lambda \sin \theta \]

Through the first trigonometry law we have:

\[ \cos \beta = \sqrt{1 - \sin^2 \beta} = \sqrt{1 - \lambda^2 \sin^2 \theta} \]

Therefore:

\[ x = r (1 - \cos \theta) + L (1 - \sqrt{1 - \lambda^2 \sin^2 \theta}) \]

The equation 4 can be approximated by the relation:

\[ (a+b)^n = a^n + m a^{m-1} b + m (m-1) a^{m-2} b^2 + \frac{m (m-1) (m-2) a^{m-3} b^3}{3!} + \cdots \]

Therefore:

\[ \cos \beta = (1 - \lambda^2 \sin^2 \theta)^{1/2} \]

Naming \( a=1, \ b=-\lambda^2 \sin^2 \theta, \ m=1/2 \) we will have:

\[ \cos \beta = 1 - \frac{1}{2} \lambda^2 \sin^2 \theta - \frac{1}{8} \lambda^4 \sin^4 \theta - \frac{1}{16} \lambda^6 \sin^6 \theta + \cdots \]

Performing the transformations we will have:

\[ \cos \beta = 1 - \frac{\lambda^2}{4} + \frac{\lambda^2}{4} \cos 2\theta - \frac{\lambda^4}{64} \cos 4\theta + \frac{\lambda^5}{512} \cos 6\theta + \cdots \]

The values of \( \lambda \) are usually small, therefore the series high order terms can be neglected, and thus the final equation that represents the piston displacement is:

\[ x = r (1 - \cos \theta) + L \frac{\lambda^2}{4} (1 - \cos 2\theta) \]

The instantaneous velocity can be directly obtained by differentiating the equation 5

\[ v = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{dx}{d\theta} \]

Thus, we have:

\[ v = \omega r \left(\sin \theta + \frac{1}{2} \sin 2\theta\right) \]

The piston acceleration can be determined by:

\[ a = \frac{dv}{dt} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{dv}{d\theta} \]

Therefore:

\[ a = \omega^2 r \left(\cos \theta + \lambda \cos 2\theta\right) \]

IV. DYNAMIC LOADS

The connecting rod dynamic load calculation is determined based on the cylinder gas pressure versus crank angle curve (Figure 3), and the inertia forces generated due to the reciprocating masses and the engine speed (Figure 4). The combination of the gas force, generated due to the cylinder gas pressure, and the inertia force, provide the resultant force applied on the connecting rod (Figure 5). The conventional connecting rod structural analysis has been performed considering the over load and over speed operational conditions.

Historically, these regimes are the responsible for the maximum tensile force and maximum compressive force respectively. However, a combination of cylinder gas pressure and engine speed for the intermediate operational conditions may provide critical loads for the connecting rod. Therefore, the complete engine operational conditions map evaluation is recommended in order do not omit important loads.

![Cylinder gas pressure versus crank angle](image-url)
The gas force can be calculated for each crank angle according to the expression below:

\[ F_{\text{gas}}(\theta) = P_{\text{gas}}(\theta) \cdot \frac{\pi D^2}{4} \]  

(8)

Where:
- \( F_{\text{gas}}(\theta) = \) Gas force at crank angle \( \theta \)
- \( P_{\text{gas}}(\theta) = \) Cylinder gas pressure at crank angle \( \theta \)
- \( D = \) Piston diameter

The inertia force is calculated by the following expression:

\[ F_{\text{inertia}} = m_{rsc} \cdot a \]  

(9)

Where
- \( F_{\text{inertia}} = \) Inertia force
- \( m_{rsc} = \) Reciprocating masses
- \( a = \) Piston acceleration

\[ F_{rod} = \left( F_{\text{gas}} + F_{\text{inertia}} \right) \cdot \frac{1}{\cos \beta} \]  

(10)

V. FINITE ELEMENT MODEL

A FEA was performed in order to obtain the connecting rod strain and stress results. In order to simulate the connecting rod structural behavior, the complete connecting rod assembly should be taken into account, therefore, the piston pin, bushing, bearings, crank pin and bolts should be considered in the FE model.

Historically, most connecting rod fatigue failures occur at the small end region, due to this fact, the present study will be concentrated in this region. Therefore, the connecting rod joint surface will be considered completely bonded without bolts and the bearings are also omitted in this analysis.

The FEA was performed using ANSYS software.

Figure 4 – Inertia force versus crank angle.

Figure 5 – Connecting rod resultant force.

After calculating the gas and inertia forces, the resultant connecting rod force can be directly determined by the expression:

\[ F_{rod} = \left( F_{\text{gas}} + F_{\text{inertia}} \right) \cdot \frac{1}{\cos \beta} \]  

Table 1 – FEA model information.

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Number of elements</th>
</tr>
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<tbody>
<tr>
<td>SOLID</td>
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</tr>
<tr>
<td>TETRAHEDRAL</td>
<td></td>
</tr>
<tr>
<td>CONTACT</td>
<td>77</td>
</tr>
</tbody>
</table>

VI. BOUNDARY CONDITIONS

As described previously, there are two critical loads for each engine operating condition, the tensile force generated due to inertia, and compressive force due to the gas load. The work presents the FEA results performed for over load @ 1800 rpm and over speed @ 2625 rpm conditions.
Figure 8 – Maximum principal stress @ 1800 rpm.

Table 2 - Load cases summary

<table>
<thead>
<tr>
<th>Over Load @1800 rpm</th>
<th>Over Speed @2625 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{tensile} = 32303 N$</td>
<td>$F_{tensile} = 74143 N$</td>
</tr>
<tr>
<td>$F_{compress} = 298505 N$</td>
<td>$F_{compress} = -245334 N$</td>
</tr>
</tbody>
</table>

VII. STRESS RESULTS

The connecting rod assembly was numerically simulated by FEA in order to evaluate the maximum and minimum loads, according to described previously. The prestress due to press fit assembly was also considered in all simulated cases.

Figure 9 – Maximum principal stress @ 2625 rpm.

The figures 8 and 9 show the stress distribution for the compressive load @ 1800 rpm and the tensile load @2625 rpm respectively.

VIII. FATIGUE LIFE PREDICTION

The Stress Life (SxN) theory was employed to evaluate the connecting rod fatigue life. It implicates that the component will have infinite life for a number of cycles over to $10^7$, according to.

Figure 10 – Typical SxN diagram

In order to perform the fatigue study, the finite element results should be combined to obtain the alternate and mean stresses for each operating condition, according to the definition bellow:

Figure 11 – Nomenclature for constant amplitude cyclic loading.
Based on the figure above, we can define:

\[ \sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]  (11)

\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \]  (12)

Where

- \( \sigma_a \) = Alternative stress
- \( \sigma_m \) = Mean stress
- \( \sigma_{\text{max}} \) = Maximum stress
- \( \sigma_{\text{min}} \) = Minimum stress

The alternate and mean stresses were calculated for each operating condition, combining the finite element results according to explained below:

\[ \sigma_a = \frac{\sigma_{\text{compression}} - \sigma_{\text{tensile}}}{2} \]

\[ \sigma_m = \frac{\sigma_{\text{compression}} + \sigma_{\text{tensile}}}{2} \]

After calculating the alternate and mean stresses, we can plot the Modified Goodman diagram.

According to table 4, the lowest fatigue factor obtained was 1.32 (90° small end region). The KMCL fatigue factor acceptable criteria is 1.3. By analyzing the numerical results and established acceptable criteria, we can conclude that no connecting rod fatigue failures are expected for these loads level.

**IX. CONCLUSIONS**

According to obtained results, the highest stresses were observed in the small end region and fatigue factors calculated for most critical nodes at three different positions at the small end. Lowest fatigue factor obtained was 1.32 in acceptable range, So we can verified that the proposed numerical methodology to evaluate the connecting rod structural and fatigue lifetime. Therefore the methodology presented in this work, showed to be an important tool to be applied during the connecting rod development phase.

**REFERENCES**