

Image Denoising Based On Averaging Of Two Wavelet Transformed Images

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ABSTRACT

When the signal in the form of image is processed, it gets distorted and further processing does not provide good results. Hence it is very important to get back the image in its original noise free condition. In this paper we present an image denoising method for noise removal. In this work, a wavelet-based multiscale linear minimum mean square-error estimation (LMMSE) scheme for image denoising is proposed. In order to achieve this undecimated wavelet transforms (UWT) have been applied to the image. We use two wavelet filters and generate two wavelet transformed images for each filter. Average of these two transformed images is used to generate a new image which is visually pleasing as compared to individual filtered image. It is also shown quantitatively by PSNR and MSE value. Experiments show that the proposed scheme outperforms some existing denoising methods.

Keywords - bior1.3, db2, image processing, LMMSE, UWT

1. INTRODUCTION

Each imaging system suffers with a common problem of "Noise". Unwanted data which may reduce the contrast deteriorating the shape or size of objects in the image and blurring of edges or dilution of fine details in the image may be termed as noise. It may be due to one or more of the following reasons, physical nature of the system, shortcomings of image acquisition devices, image developing mechanism and due to environment. Mathematically there are two basic models of Noise, additive and multiplicative. Additive noise is systematic in nature and can be easily modeled and hence removed or reduced easily. Whereas multiplicative noise is image dependent, complex to model and hence difficult to reduce. Suppressing such noise is, thus, the usual first step. Thus, denoising is often a necessary and the first step to be taken before the images data is analyzed. It is necessary to apply an efficient denoising technique to compensate for such data corruption. Image denoising still remains a challenge for researchers because noise removal introduces artifacts and causes blurring of the images. Most image

processing techniques involve treating the image as a two-dimensional signal and applying standard signal-processing techniques to it. There are applications in image processing that require the analysis to be localized in the spatial domain. The term spatial domain refers to the image plane itself, and approaches in this category are based on direct manipulation of pixels in an image. Transform domain processing techniques are based on modifying the Fourier or wavelet transform of an image. The classical way of doing this is through what is called Windowed Fourier Transform. Central idea of windowing is reflected in Short Time Fourier Transform (STFT). The STFT conveys the localized frequency component present in the signal during the short window of time. The same concept can be extended to a two-dimensional spatial image where the localized frequency components may be determined from the windowed transform. This is one of the bases of the conceptual understanding of wavelet transforms. Hence, wavelet transforms have been kept as the main consideration in this paper. Denoising of natural images corrupted by noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet denoising scheme thresholds the wavelet coefficients arising from the wavelet transform. The wavelet transform yields a large number of small coefficients and a small number of large coefficients. Simple denoising algorithms that use the wavelet transform consist of three steps.

- Calculate the wavelet transform of the noisy signal.
- Modify the noisy wavelet coefficients according to some rule.
- Compute the inverse transform using the modified coefficients.

The problem of Image de-noising can be summarized as follows. Let $A(i, j)$ be the noise-free image and $B(i, j)$ the image corrupted with noise $Z(i, j)$, $B(i, j) = A(i, j) + \sigma Z(i, j)$. The problem is to estimate the desired signal as accurately as possible according to some criteria. In the wavelet domain, the problem can be formulated as $Y(i, j) = W(i, j) + N(i, j)$. Where $Y(i, j)$ is noisy wavelet coefficient,

$W(i, j)$ is true coefficient and $N(i, j)$ noise. The wavelet transform has proved to be very successful in making signal and noise components of the signal distinct. As wavelets have compact support the wavelet coefficients resulting from the signal are localized, whereas the coefficients resulting from noise in the signal are distributed. Thus the energy from the signal is directed into a limited number of coefficients which 'stand out' from the noise. Wavelet denoising must not be confused with smoothing; smoothing only removes the high frequencies and retains the lower ones. Wavelet shrinkage denoising then consists of identifying the magnitude of wavelet coefficients one can expect from the noise (the threshold), and then shrinking the magnitudes of all the coefficients by this amount. What remain of the coefficients should be valid signal data, and the transform can then be inverted to reconstruct an estimate of the signal. Wavelet shrinkage depends heavily on the choice of a thresholding parameter and the choice of this threshold determines, to a great extent the efficacy of denoising. Researchers have developed various techniques for choosing denoising parameters and so far there is no "best" universal threshold determination technique. Two types of wavelet transforms have been used: Discrete wavelet transform (DWT) and Undecimated Discrete wavelet transform (UWT). In threshold-based denoising schemes, a threshold is set to distinguish noise from the structural information. Thresholding can be classified into soft and hard ones, in which coefficients less than the threshold will be set to 0 but those above the threshold will be preserved (hard thresholding) or shrunk (soft thresholding). Donoho (1995) first presented the Wavelet Shrinkage scheme with universal threshold based on orthonormal wavelet bases. It is generally accepted that in each subband the image wavelet coefficients can be modeled as independent identically distributed (i.i.d.) random variables with generalized Gaussian distribution (GGD). So by choosing different type of threshold we can improve the quality of the images. Although wavelet transform well decorrelates signals, strong intrascale and interscale dependencies between wavelet coefficients may still exist. The denoising schemes benefited from intrascale models. Wavelet interscale models are also used in many other applications. If a coefficient at a coarser scale has small magnitude, its descendants at finer scales are very likely to be small too. Shapiro (1993) exploited this property and developed the well-known embedded zero tree wavelet image compression scheme. In another viewpoint, if a wavelet coefficient generated by true signal has large magnitude at a finer scale; its ascendants at coarser scales will likely be significant as well. But for those coefficients caused by noise, the magnitudes may decay rapidly along the scales. With this observation, it is expected that multiplying

the wavelet coefficients at adjacent scales would strengthen the significant structures while diluting noise. Such a property has been exploited for denoising.

In this paper we have taken two wavelet filter biorthogonal (bior1.3) and Daubechies (db2) and reconstructed the image individual filter. Now the average of two reconstructed images and generate the new image which is superior then individually generated images. The LMMSE based denoising technique with the assistance of UWT (undecimated wavelet transforms) adopted in this paper.

2. THE DENOISING METHOD

This paper discusses how to remove the additive white Gaussian noise (AWGN) with a zero mean and reconstructed images also analyzed for other mean values. Decimation of the wavelet coefficients is an intrinsic property of the discrete wavelet transform (DWT). The decimation step removes every other of the coefficients of the current level. Thus the computation of the wavelet transform is faster and more compact in terms of storage space. More importantly, the transformed signal can be perfectly reconstructed from the remaining coefficients. Unfortunately, the decimation is causing shift variance of the wavelet transform. In order to achieve shift-invariance, researches from different fields and having various goals have invented several wavelets transform algorithms. This type of transforms is known under the common name undecimated wavelet transform (UWT). Unlike the discrete wavelet transform (DWT), which downsamples the approximation coefficients and detail coefficients at each decomposition level, the undecimated wavelet transform (UWT) does not incorporate the down sampling operations. This algorithm is based on the idea of no decimation. It applies the wavelet transform and omits both down sampling in the forward and up sampling in the inverse transform. More precisely, it applies the transform at each point of the image and saves the detail coefficients and uses the low-frequency coefficients for the next level. The size of the coefficients array does not diminish from level to level. Thus, the approximation coefficients and detail coefficients at each level are the same length as the original signal. By using all coefficients at each level, we get very well allocated high-frequency information. The resolution of the UWT coefficients decreases with increasing levels of decomposition. By comparing the UWT with the DWT, the UWT has some unique features, Translation invariance, better denoising capability, better peak detection capability.

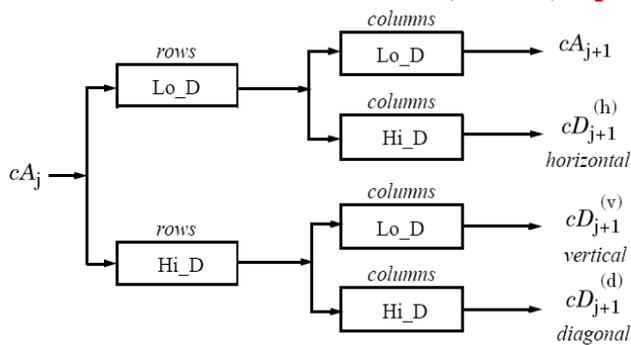


Fig.2: a 2-dimensional UWT - decomposition step

2.1 Wavelet Thresholding

The term wavelet thresholding is explained as decomposition of the data or the image into wavelet coefficients, comparing the detail coefficients with a given threshold value, and shrinking these coefficients close to zero to take away the effect of noise in the data. The image is reconstructed from the modified coefficients. Thresholding distinguishes between the coefficients due to noise and the ones consisting of important signal information. The choice of a threshold is an important point of interest. It plays a major role in the removal of noise in images because denoising most frequently produces smoothed images, reducing the sharpness of the image. Care should be taken so as to preserve the edges of the denoised image. There exist various methods for wavelet thresholding, which rely on the choice of a threshold value. Some typically used methods for image noise removal include VisuShrink, SureShrink and BayesShrink. For all these methods the image is first subjected to a wavelet transform, which decomposes the image into various sub-bands. The sub-bands $HH_m, HL_m, LH_m, m = 1, 2, \dots, j$ are called the details, where m is the scale and j denotes the largest or coarsest scale in decomposition. Note, LL_m is the low-resolution component. Thresholding is now applied to the detail components of these sub bands to remove the unwanted coefficients, which contribute to noise. And as a final step in the denoising algorithm, the inverse wavelet transform is applied to build back the modified image from its coefficients. In threshold-based (hard or soft) denoising schemes, the wavelet coefficients whose magnitudes are below a threshold will be set to 0. These pixels are generally noise predominated and thus the thresholding of these coefficients is safely a structure preserving denoising process. We apply the LMMSE only to those coefficients above a threshold and shrink those below the threshold to 0. Here in this paper threshold applied to w_j (wavelet coefficients) is set as $T_H = 1.8 * \sigma_j$.

Suppose the original signal f is corrupted with additive white Gaussian noise n .

$$g = f + n \quad (1)$$

where $n \in N(0, \sigma^2)$, Applying the UWT to the noisy signal g , at scale yields

$$w_j = x_j + v_j \quad (2)$$

where w_j is wavelet coefficients at scale j , x_j and v_j are the expansions of f and n respectively. In this thesis, the LMMSE of wavelet coefficients is employed instead of soft thresholding. Suppose the variance of v_j and x_j is σ_j^2 and $\sigma_{x_j}^2$ respectively. Since v_j and x_j are both zero mean, the LMMSE of x_j is: $x'_j = k' \cdot w_j$

$$\text{where } k' = \frac{\sigma_{x_j}^2}{\sigma_{x_j}^2 + \sigma_j^2} \quad (3)$$

This leads to the energy shrinkage of the restored signal. After applying the LMMSE, approximate \bar{x}'_j is obtained and x'_j is extracted, and this is nearly approximate coefficients of the real image.

2.2 Wavelet Filter Used

Orthogonal filter banks with symmetric FIR filters are of great interest in certain applications of image and video processing. The symmetry property of the filters is important for handling boundary distortions of finite length signals effectively. The orthogonality property in filter banks preserves the energy of the input signal in the subbands, which guarantees that errors arising from quantization or transmission will not be amplified. In addition, orthogonality usually leads to high energy compaction. Thus, it is desirable to design filter banks that are both symmetric and orthogonal. Daubechies (1992)

Ingrid Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets, thus making discrete wavelet analysis practicable. The names of the Daubechies family wavelets are written dbN, where N is the order, and db the "surname" of the wavelet. The db1 wavelet is the same as Haar wavelet. In this paper we used db2 wavelet filter of this family which is used in programming by this instruction.

`[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('wname')`

This computes four filters associated with the orthogonal or biorthogonal wavelet named in the string 'wname'. Here we signify db2 wavelet filter name.

Biorthogonal Filter

For image denoising, symmetric wavelet filters are more desirable because a symmetrical extension at the image edges can be used, providing less distortion and higher denoising. The most common type of symmetrical wavelet filter is called a biorthogonal filter. FASTMAN has implemented a biorthogonal wavelet filter pair as a mega function. This mega function implements one stage of the wavelet filter pyramid. It takes an input signal at a clock frequency f_0 , computes the decimated high-

pass and low-pass outputs (at clock frequencies $f_0/2$) and then interleaves them to produce a single output containing alternating lowpass and highpass coefficients. Wavelet coefficients are computed by applying multiple stages of a lowpass and highpass filter pair, called a quadrature mirror filter pair, to the data signal. At each stage (or scale) of the pyramid, the low-pass filter computes a smoother version of the signal and the high-pass filter brings out the signal's detail information at that scale. At the first stage, the filters are applied to the original, full-length signal. Then, at the next stage, the filter pair is applied to the smoothed and decimated low-pass output of the first stage. The wavelet coefficients consist of the accumulated detail components and the final smooth component.

Biorthogonal wavelets with FIR filters: These wavelets can be defined through the two scaling filters w_r and w_d , for reconstruction and decomposition respectively. The Biorthogonal wavelet family is a predefined family of this type such as *bior1.1*, *bior1.3*.

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. In this family two wavelets are used, one for decomposition and the other for reconstruction. In this thesis second filter we used *bior1.3* wavelet filter which is used in programming by this instruction.

`[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('wname')`
'wname' we specify here is *bior1.3*.

2.3 Image Denosing using proposed algorithm

Image denosing based on averaging of two wavelets transformed images algorithm is as follows:

1. Read the input image.
2. Corrupt the image with Additive White Gaussian noise (AWGN) with different noise intensity.
3. Perform multiscale decomposition of the image corrupted by Gaussian noise using Undecimated wavelet transform (UWT) with the help of 'bior1.3' wavelet filter.
4. Calculate the wavelet coefficients of noisy image for 3 different scales.
5. Select only those coefficients above the threshold and shrink those below the threshold to 0.
6. We combine the wavelet coefficients with the same spatial location across adjacent scales as a vector, to which the LMMSE is then applied.
7. Invert the *multiscale* decomposition to reconstruct the denoised image.
8. Repeat the step from 3 to 6 for the 'db2' wavelet filter and reconstruct the image.
9. Averaging the matrices of two reconstructed images.
10. Generate a new reconstructed image matrix and get the denoised image which is better

than individually generated denoised images.

3. PERFORMANCE MEASURE

In this work the performance of image denoising is computed in terms of Mean square error (MSE) and Peak signal to noise ratio (PSNR).

(i) Mean Square Error (MSE)

$$MSE = \frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N [f(x,y) - f'(x,y)]^2 \quad (4)$$

$f(x,y)$: Original image

$f'(x,y)$: reconstructed image after denoising

M, N: dimensions of the image

For better performance MSE value should be minimum.

(ii) Peak Signal to Noise Ratio (PSNR)

It is the measure of the peak error in the signal and is expressed mathematically by the following equation:

$$PSNR = \frac{\sum_{x=1}^M \sum_{y=1}^N [f'(x,y)]^2}{MSE} \quad (5)$$

The higher the value of peak signal to noise ratio means the ratio of the significant signal to noise is better.

4. EXPERIMENTAL RESULTS

The proposed image denoising algorithm is implemented in matrix laboratory (MATLAB). This part presents the results, obtained after following the wavelet denoising algorithm. The results have been demonstrated in the form of plots and reconstructed images. The proposed technique has been tested for assumed standard test images *Lena* and *Barbara*. The size of images which we have examined is 512×512 . These images are widely used by researchers in image processing applications. The qualitative judgment made visually based on the results obtained for Gaussian noise variance 5, 10, 15, 20, 25, 30, with zero mean. For making the method general the images were examined for different mean values of noise also and verified the results. In Fig. 4.1 and Fig. 4.2 it has been clearly seen that the MSE value of reconstructed image generated by averaging of two wavelets transformed images is below the MSE value of individually generated image by single filter. It is well known that the performance of the image quality increases with decrement in MSE value. So from Fig. 4.1 and Fig. 4.2 it can be easily concluded that the PSNR value obtained for proposed method is better than the PSNR value of individual filter.

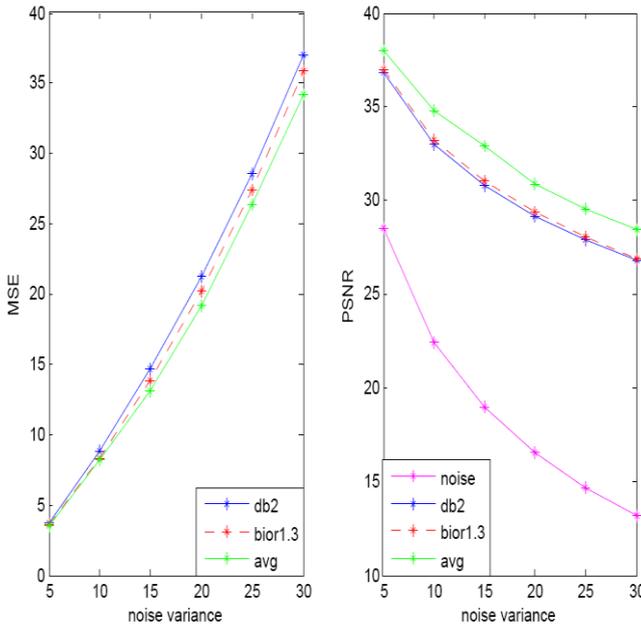


Fig.4.1: Variation of MSE and PSNR for different filter for *Lena* image

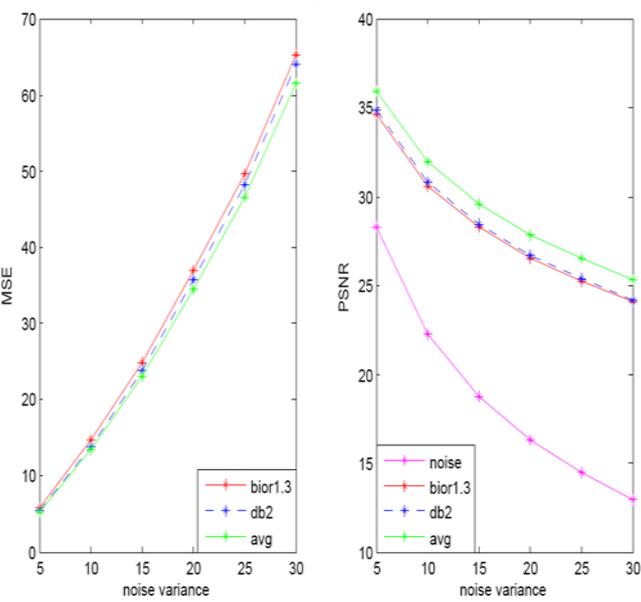


Fig.4.2: variation of MSE and PSNR for different filter for *Barbara* image



(a): noisy image ($\sigma=5$)



(b): Denoised image ($\sigma=5$)



(c): noisy image ($\sigma=10$)



(d): Denoised image ($\sigma=10$)



(e): noisy image ($\sigma=15$)



(f): Denoised image ($\sigma=15$)



(g): noisy image ($\sigma=20$)



(h): Denoised image ($\sigma=20$)



(i): noisy image ($\sigma=25$)



(j): Denoised image ($\sigma=25$)



(k): noisy image ($\sigma=30$)



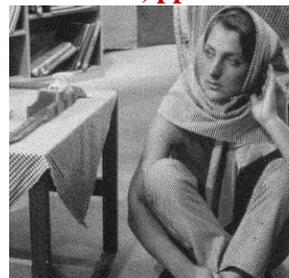
(l): Denoised image ($\sigma=30$)



(a): noisy image ($\sigma=5$)
 image ($\sigma=5$)



(b): Denoised image



(k): noisy image ($\sigma=30$)
 image ($\sigma=30$)



(l): Denoised image

Fig.4.3: comparison between noisy and denoised images

In summary, the proposed scheme permits more smoothing away from edges and lesser smoothing near edges, resulting in restoring the noisy image without significantly degrading its edges. However, visually, the edges of the image appear sharp and most of the background and other low activity areas of the image have been overly smoothed.

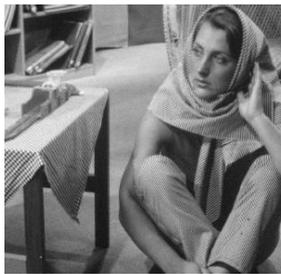
As of now the results of various images for different values of noise variance with zero mean have been observed. It has been seen that, experimentally this method gives good results for zero mean. Now the proposed method has been examined for two mean values 1, 2 using standard image *Lena*.

Table 4: Comparison of quantitative parameters for Bior1.3, Db2 and Averaging filter with different mean for *Lena* image

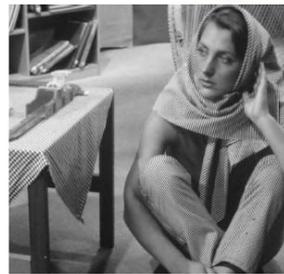
(a) Additive White Gaussian noise with mean=1

Variance	MSE		
	Bior 1.3	Db2	Averaging filter
15	14.9996	15.8271	14.644
20	21.1894	22.21	20.557
25	28.4396	29.5641	27.4408

Variance	PSNR(db)		
	Bior 1.3	Db2	Averaging filter
15	30.7656	30.5343	31.8706
20	29.2633	29.0615	30.3958
25	27.9866	27.8212	29.143



(c): noisy image ($\sigma=10$)
 ($\sigma=10$)



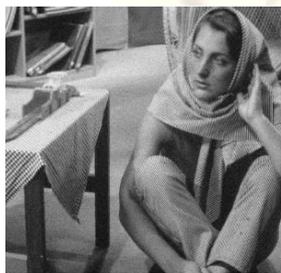
(d): Denoised image



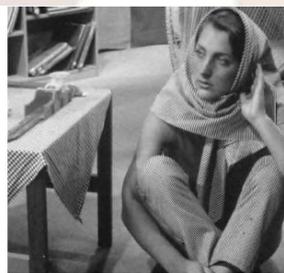
(e): noisy image ($\sigma=15$)



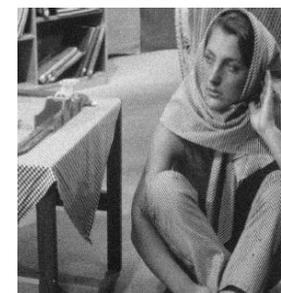
(f): Denoised image ($\sigma=15$)



(g): noisy image ($\sigma=20$)
 ($\sigma=20$)



(h): Denoised image



(i): noisy image ($\sigma=25$)



(j): Denoised image ($\sigma=25$)

(b) Additive White Gaussian noise with mean=2

represented by black line. This is providing the best results to all other methods of denoising.

5. Conclusion

In this paper, Averaging of two wavelet transformed images algorithm for denoising natural images were investigated and their performance was comparatively assessed. Its performance was shown to be competitive with or exceeding the performance of other algorithms. In addition, it has been shown to enjoy the advantage of implementation simplicity.

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Variance	MSE		
	Bior 1.3	Db2	Averaging filter
15	17.8939	18.6266	17.4962
20	24.1054	25.1874	23.5045
25	31.1594	32.3935	30.2098

Variance	PSNR		
	Bior 1.3	Db2	Averaging filter
15	30.0593	29.8868	31.1576
20	28.7642	28.576	29.8748
25	27.6495	27.4838	28.785

From above tables it has been observed that as mean value of AWGN increases, MSE value also increases which degrades the performance of image in terms of PSNR. But the proposed method gives better results as compared to single filter reconstructed image for different mean values. So it can be seen from Table 4 that the proposed method gives better results for any value of mean with different variance.

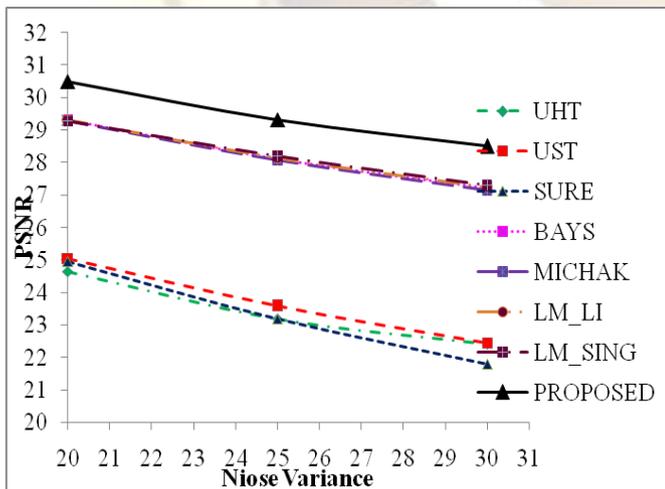


Fig.4.4: comparison of PSNR (dB) for different image denoising technique for Lena image corrupted by AWGN

The above graph show that the different denoising response for three different variance 15, 20, 30. These denoising schemes clearly differentiated by their respective colors. The graph compares the value of PSNR (dB) for different type of denoising schemes. The proposed scheme is

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