Five Steps Block Method For The Solution Of Fourth Order Ordinary Differential Equations.

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Abstract

We considered method of collocation of the differential system and interpolation of the approximate solution to generate a continuous linear multistep method, which is solved for the independent solution to yield a continuous block method. The resultant method is evaluated at selected grid points to generate a discrete block method. The basic properties of the method was investigated and found to be zero stable, consistent and convergent. The method was tested on numerical examples solved by the existing method, our method was found to give better approximation.

Keywords: collocation, differential system, interpolation, approximate solution, independent solution, zero stable, consistent, convergent
AMS Subject Classification (2010): 65L05, 65L06, 65D30

1.0 Introduction

This paper considered the approximate solution to the general fourth order initial value problems of the form

$$y^{(iv)} = f(x, y, y', y'', y'''),\ y^k(x) = y_0^k \qquad (1)$$

$where\ \ k = 1(1)3\ and\ \ f$ iscontinuouswithin the interval of integration.

In most application, (1) is solved by reduction to an equivalent system of first order ordinary differential equation and appropriate numerical method could be employed to solve the resultant system. This approach had been reported by scholars, among them are: Bun andVasil'yer (1992), Awoyemi (2001), Adesanya*et. al* (2008). In spite of the success of this approach, the setback of the method is in writing computer program which is often complicated especially when subroutine are incorporated to supply the starting values required for the method. The consequence is in longer computer time and human effort. Furthermore, according to Vigor-Aguiar and Ramos (2006), this method does not utilize additional information associated with specificordinary differential equation, such as the oscillatory nature of the solution. In addition, Bun and Vasil'yer (1992) reported that more serious disadvantage of the method of reduction is the fact that the given system of equation to be solved cannot be solved explicitly with respect to the derivatives of the highest order. For these reasons, this method is inefficient and not suitable for general purpose.

Scholars later developed method to solve (1) directly without reducing to systems of first order ordinary differential equations and concluded that direct method is better than method of reduction, among these authors are Twizel and Khaliq (1984), Awoyemi and Kayode (2004), Vigo-Aguiar and Ramos (2006), Adesanya*et.al* (2009).

Scholars developed linear multistep method for the direct solution of higher order ordinary differential equation using power series approximate solution adopting interpolation and collocation method to derived continous linear multistep method. According to Awoyemi (2001), continous linear multistep method have greater advantages over the discrete method is that they give better error estimate, provide a simplified form of coefficient for further analytical work at different points and guarantee easy approximation of solution at all interior points of the integration interval. Among the authors that proposed the linear multistep method are Kayode (2004), Adesanya*et.al* (2009), Awoyemi and Kayode (2004) to mention few. They developed an implicit linear multistep method which was implemented in predictor corrector mode and adopted Taylor's series expansion to provide the starting value.

In spite of the advantages of the linear multistep method, they are usually applied to initial value problems as a single formula and this has some inherent disadvantages, for instance the implementation of the method in predictor-corrector mode is very costly and subroutine are very complicated to write because they require special technique to supply starting values.

In order to cater for the above mentioned setback, researchers came up with block methods which simultaneously generate approximation at different grid points within the interval of integration. Block method is less expensive in terms of the number of function evaluations compared to the linear multistep method or Ringe-Kuttamethod; above all, it does not require predictor or starting values. Among these authors are Ismail *et. al.* (2009), Adesanya*et. al.* (2012), Anake*et. al.* (2012), Awoyemi*et. al.* (2011).

In this paper, we proposed block method for the solution of general fourth order initial value problem of ordinary differential equation. This method is an extension of the work of Olabode (2009) and Mohammed (2010) who both worked on special fourth order initial value problems of ordinary differential equation.

2.0 Methodology

We consider a power series approximate solution in the form

$$y(x)=\sum_{d=0}^{s+r-1} a_j x^j \tag{2}$$

The fourth derivatives of (2) gives

$$y^{(iv)}=\sum_{d=0}^{s+r-1} j(j-1)(j-2)(j-3)a_j x^{j-4} \tag{3}$$

Substituting (3) into (1) gives

$$f(x,y,y',y'',y''')=\sum_{j=0}^{s+r-1} j(j-1)(j-2)(j-3)a_j x^{j-4} \tag{4}$$

where s and r are the number of interpolation and collocation points respectively.

Interpolating (2) at $x_{n+j}, s=1(1)4$ and collocating (4) at $x_{n+r}, r=0(1)5$ gives a system of nonlinear equation

$$AX=U \tag{5}$$

$$where\ X=\left[a_0\ a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7\ a_8\ a_9\ \right]^T$$

$$U=\left[y_{n+1}\ y_{n+2}\ y_{n+3}\ y_{n+4}\ f_n\ f_{n+1}\ f_{n+2}\ f_{n+3}\ f_{n+4}\ f_{n+5}\right]^T$$

$$A=\begin{bmatrix}
1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 & x_{n+1}^9\\
1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 & x_{n+2}^8 & x_{n+2}^9\\
1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & x_{n+3}^7 & x_{n+3}^8 & x_{n+3}^9\\
1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 & x_{n+4}^6 & x_{n+4}^7 & x_{n+4}^8 & x_{n+4}^9\\
0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & 1680x_n^4 & 3024x_n^4\\
0 & 0 & 0 & 0 & 24 & 120x_{n+1} & 360x_{n+1}^2 & 840x_{n+1}^3 & 1680x_{n+1}^4 & 3024x_{n+1}^4\\
0 & 0 & 0 & 0 & 24 & 120x_{n+2} & 360x_{n+2}^2 & 840x_{n+2}^3 & 1680x_{n+2}^4 & 3024x_{n+2}^4\\
0 & 0 & 0 & 0 & 24 & 120x_{n+3} & 360x_{n+3}^2 & 840x_{n+3}^3 & 1680x_{n+3}^4 & 3024x_{n+3}^4\\
0 & 0 & 0 & 0 & 24 & 120x_{n+4} & 360x_{n+4}^2 & 840x_{n+4}^3 & 1680x_{n+4}^4 & 3024x_{n+4}^4\\
0 & 0 & 0 & 0 & 24 & 120x_{n+5} & 360x_{n+5}^2 & 840x_{n+5}^3 & 1680x_{n+5}^4 & 3024x_{n+5}^4
\end{bmatrix}$$

Solving (5) for the a_j's using Gaussian elimination method, given a continuous linear multistep method in the form

$$y(x)=\sum_{j=1}^{4}\alpha_j(x)y_{n+j}+h^4\sum_{j=0}^{5}\beta_j(x)f_{n+j} \tag{6}$$

Where:

$$y_{n+j}=y(x_n+jh)$$

$$f_{n+j}=f(x_n+jh, y_n+jh, y'_n+jh, y''_n+jh, y'''_n+jh,)$$

The coefficient y_{n+i} and f_{n+i} are given as

$$\alpha_1 = -\frac{1}{6}(t^3 - 9t^2 + 26t - 24)$$

$$\alpha_2 = \frac{1}{2}(t^3 - 8t^2 + 19t - 12)$$

$$\alpha_3 = -\frac{1}{2}(t^3 - 7t^2 + 14t - 8)$$

$$\alpha_1 = \frac{1}{6}(t^3 - 6t^2 + 11t - e)$$

$$\beta_0 = -\frac{1}{1814400}(5t^9 - 135t^8 - 1530t^7 - 9450t^6 - 34524t^5 - 75600t^4 - 95335t^3 - 60975t^2 + 12346t + 2520)$$

$$\beta_1 = \frac{1}{362880}(5t^9 - 126t^8 + 127t^7 - 6468t^6 + 15120t^5 - 84383t^4 + 192318t^2 - 180240t + 62496)$$

$$\beta_2 = -\frac{1}{1814400}(5t^9 - 117t^8 + 1062t^7 - 4494t^6 + 7560t^5 - 13603t^3 + 127941t^2 - 229770t + 117448)$$

$$\beta_3 = -\frac{1}{1814400}(5t^9 - 108t^8 + 882t^7 - 3276t^6 + 5040t^5 - 12647t^3 + 36396t^2 - 57540t + 31248)$$

$$\beta_4 = -\frac{1}{362880}(5t^9 - 99t^8 + 738t^7 - 2562t^6 + 3780t^5 - 5633t^3 + 4677t^2 - 1410t + 504)$$

$$\beta_5 = -\frac{1}{1814400}(5t^9 - 90t^8 + 630t^7 - 2100t^6 + 3024t^5 - 4415t^3 + 340t^2 - 340t)$$

$$t = \frac{x - x_n}{h}$$

Solving (6) for the independent solution $y_{n+j},\ j = 0(1)5$

given a continuous block method in the form

$$y_{n+j} = \sum_{m=0}^{3} \frac{(jh)^m}{m!} y^{(m)}{}_n + h^4 \sum_{j=0}^{5} \sigma_j f_{n+j} \tag{7}$$

where:

$$\sigma_0 = -\frac{1}{1814400}(5t^9 - 135t^8 - 1530t^7 - 9450t^6 - 34524t^5 - 75600t^4)$$

$$\sigma_1 = \frac{1}{362880}(5t^9 - 126t^8 + 1278t^7 - 6468t^6 + 15120t^5)$$

$$\sigma_2 = -\frac{1}{1814400}(5t^9 - 117t^8 + 1062t^7 - 4494t^6 - 7560t^5)$$

$$\sigma_3 = -\frac{1}{1814400}(5t^9 - 108t^8 + 882t^7 - 3276t^6 + 5040t^5)$$

$$\sigma_4 = -\frac{1}{362880}(5t^9 - 99t^8 + 738t^7 - 2562t^6 - 3780t^5)$$

$$\sigma_5 = -\frac{1}{1814400}(5t^9 - 90t^8 + 630t^7 - 2100t^6 + 3024t^5)$$

Evaluating (7), the first, second and third derivatives at t = 1(1)5 gives a discrete block formula in the form

$$A^0 Y_m^{(i)} = \sum_{i=0}^{3-i} h^i e_i y_n^{(i)} + h^{4-i} df(y_n) + h^{4-i} bF(Y_m) \tag{8}$$

where:

$$Y_m = [y_{n+1}, y_{n+2}, y_{n+3}, y_{n+4}, y_{n+5}]^T$$

$$F(Y_m) = [f_{n+1}, f_{n+2}, f_{n+3}, f_{n+4}, f_{n+5}]^T$$

$$f(y_n) = [f_{n-1}, f_{n-2}, f_{n-3}, f_{n-4}, f_{n-5}]^T$$

$$y_n^{(i)} = \left[y_{n-1}^{(i)}, y_{n-2}^{(i)}, y_{n-3}^{(i)}, y_{n-4}^{(i)}, y_n^{(i)}\right]$$

$A^0 = 5\times 5$ identity matrix

$$e_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ e_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix},$$

$$e_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & \frac{9}{2} \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & \frac{25}{2} \end{bmatrix}, \ e_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 0 & \frac{27}{2} \\ 0 & 0 & 0 & 0 & \frac{32}{3} \\ 0 & 0 & 0 & 0 & \frac{125}{6} \end{bmatrix},$$

when $i = 0$,

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{49126}{1814400} \\ 0 & 0 & 0 & 0 & \frac{4264}{14175} \\ 0 & 0 & 0 & 0 & \frac{25488}{22400} \\ 0 & 0 & 0 & 0 & \frac{40448}{14175} \\ 0 & 0 & 0 & 0 & \frac{418250}{72576} \end{bmatrix} \quad b = \begin{bmatrix} \frac{49045}{1814400} & \frac{-40160}{1814400} & \frac{25430}{1814400} & \frac{-9310}{1814400} & \frac{1469}{1814400} \\ \frac{7960}{14175} & \frac{-4910}{14175} & \frac{3080}{14175} & \frac{-1120}{14175} & \frac{176}{14175} \\ \frac{63315}{22400} & \frac{-26460}{22400} & \frac{19170}{22400} & \frac{-7020}{22400} & \frac{1107}{22400} \\ \frac{116480}{14175} & \frac{-29440}{14175} & \frac{33280}{14175} & \frac{-11360}{14175} & \frac{1792}{14175} \\ \frac{1315625}{72576} & \frac{-175000}{72576} & \frac{418750}{72576} & \frac{-106250}{72576} & \frac{18625}{72576} \end{bmatrix}$$

when $i = 1$,

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{3929}{40320} \\ 0 & 0 & 0 & 0 & \frac{317}{630} \\ 0 & 0 & 0 & 0 & \frac{5181}{4480} \\ 0 & 0 & 0 & 0 & \frac{712}{315} \\ 0 & 0 & 0 & 0 & \frac{29125}{8064} \end{bmatrix} \quad b = \begin{bmatrix} \frac{4975}{40320} & \frac{-3862}{40320} & \frac{2422}{40320} & \frac{-883}{40320} & \frac{139}{40320} \\ \frac{734}{630} & \frac{-380}{630} & \frac{244}{630} & \frac{-89}{630} & \frac{14}{630} \\ \frac{16119}{4480} & \frac{-4374}{4480} & \frac{4230}{4480} & \frac{-1539}{4480} & \frac{243}{4480} \\ \frac{2336}{315} & \frac{-224}{315} & \frac{704}{315} & \frac{-200}{315} & \frac{32}{315} \\ \frac{101875}{8064} & \frac{1250}{8064} & \frac{38250}{8064} & \frac{-4375}{8064} & \frac{1375}{8064} \end{bmatrix}$$

when $i = 2$,

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{2462}{10080} \\ 0 & 0 & 0 & 0 & \frac{355}{630} \\ 0 & 0 & 0 & 0 & \frac{984}{1120} \\ 0 & 0 & 0 & 0 & \frac{376}{315} \\ 0 & 0 & 0 & 0 & \frac{3050}{2016} \end{bmatrix} \quad b = \begin{bmatrix} \frac{4315}{10080} & \frac{-3044}{10080} & \frac{1882}{10080} & \frac{-682}{10080} & \frac{107}{10080} \\ \frac{1088}{630} & \frac{-370}{630} & \frac{272}{630} & \frac{-101}{630} & \frac{16}{630} \\ \frac{3501}{1120} & \frac{-72}{1120} & \frac{870}{1120} & \frac{-288}{1120} & \frac{45}{1120} \\ \frac{1424}{315} & \frac{176}{315} & \frac{608}{315} & \frac{-80}{315} & \frac{16}{315} \\ \frac{11875}{2016} & \frac{2500}{2016} & \frac{6250}{2016} & \frac{1250}{2016} & \frac{275}{2016} \end{bmatrix}$$

When $i=3$

$$d=\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{475}{1440} \\ 0 & 0 & 0 & 0 & \frac{28}{90} \\ 0 & 0 & 0 & 0 & \frac{51}{160} \\ 0 & 0 & 0 & 0 & \frac{14}{45} \\ 0 & 0 & 0 & 0 & \frac{95}{280} \end{bmatrix} b=\begin{bmatrix} \frac{1427}{1440} & \frac{-798}{1440} & \frac{482}{1440} & \frac{-173}{1440} & \frac{27}{1440} \\ \frac{129}{90} & \frac{14}{90} & \frac{14}{90} & \frac{-6}{90} & \frac{1}{90} \\ \frac{219}{160} & \frac{114}{160} & \frac{114}{160} & \frac{-21}{160} & \frac{3}{160} \\ \frac{14}{45} & \frac{64}{45} & \frac{24}{45} & \frac{64}{45} & \frac{14}{45} \\ \frac{375}{288} & \frac{250}{288} & \frac{250}{288} & \frac{375}{288} & \frac{95}{288} \end{bmatrix}$$

3.0 Analysis of the basic properties of the method

(3.1) **Order of the method**

Let the linear operator $L\{y(x):h\}$ associated with the block formula (8) be defined as:

$$L\{y(x):h\}=A^0 y_m^{(i)}-\sum_{i=0}^{3-i} h^i e_i y_n^{(i)} - h^{4-i}\left(df(y_n)+bF(y_m)\right) \tag{9}$$

Expanding (9) in Taylor series and comparing the coefficients of h gives

$$L\{y(x):h\}=C_0 y(x)+C_1 hy'(x)+\ldots+C_p h^p y^p(x)+C_{p+1}h^{p+1}y^{p+1}(x)+C_{p+2}h^{p+2}y^{p+2}(x)+\ldots$$

Definition:-*the linear operator* L *and associated linear multistep method are said to be of order* p *if* $C_0=C_1=C_2=C_p=C_{p+1}=0$ *and* $C_{p+2}\neq 0$, C_{p+2} *is called the error constant and implies that the local truncation error is given by* $t_{n+k}=C_{p+2}h^{p+2}y^{p+2}(x)+0(h^{p+3})$.

For our method, expanding (8) at $i=0$ in Taylor series gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(h)^j}{j!}y^j{}_n - y_n - hy'_n - \frac{h^2}{2!}y''_n - \frac{h^3}{3!}y'''_n - \frac{49126}{1814400}h^4 y^{iv}{}_n - \sum_{j=0}^{\infty}\frac{h^{j+4}}{1814400\,j!}y^{j+4}{}_n\left[49045(1)^j-40160(2)^j+25430(3)^j-9310(4)^j-1469(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(2h)^j}{j!}y^j{}_n - y_n - 2hy'_n - \frac{(2h)^2}{2!}y''_n - \frac{(2h)^3}{3!}y'''_n - \frac{4264}{14175}h^4 y^{iv}{}_n - \sum_{j=0}^{\infty}\frac{h^{j+4}}{14175(j!)}\left[7960(1)^j-4910(2)^j-3080(3)^j-1120(4)^j-176(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(3h)^j}{j!}y^j{}_n - y_n - 3hy'_n - \frac{(3h)^2}{2!}y''_n - \frac{(3h)^3}{3!}y'''_n - \frac{25488}{22400}h^4 y^{iv}{}_n - \sum_{j=0}^{\infty}\frac{h^{j+4}}{22400(j!)}\left[63315(1)^j-26460(2)^j+19170(3)^j-7020(4)^j+1107(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(4h)^j}{j!}y^j{}_n - y_n - 4hy'_n - \frac{(4h)^2}{2!}y''_n - \frac{(4h)^3}{3!}y'''_n - \frac{40448}{14175}h^4 y^{iv}{}_n - \sum_{j=0}^{\infty}\frac{h^{j+4}}{14175(j!)}\left[116480(1)^j-\frac{29400}{14175}(2)^j+\frac{33280}{14175}(3)^j-\frac{11360}{14175}(4)^j-\frac{1792}{14175}(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(5h)^j}{j!}y^j{}_n - y_n - 5hy'_n - \frac{(5h)^2}{2!}y''_n - \frac{(5h)^3}{3!}y'''_n - \frac{418250}{72576}h^4 y^{iv}{}_n - \sum_{j=0}^{\infty}\frac{h^{j+4}}{72576(j!)}\left[1315625(1)^j-175000(2)^j+418750(3)^j-10625(4)^j+18625(5)^j\right] \end{bmatrix}$$

comparing the coefficient of h given $C_0=C_1=C_2=C_3=C_4=C_5=C_6=0=C_7=C_8=C_9=0$

and the error constant $C_{10}=\begin{bmatrix} \frac{-2323}{3628800} & \frac{-137}{14175} & \frac{-1737}{44800} & \frac{-1408}{14175} & \frac{-29375}{145152} \end{bmatrix}$

(3.2) **Zero stability**

A block method is said to be zero stable if as $h\to 0$, the root $r_j\, j=1(1)k$ of the first characteristic polynomial $\rho(R)=0$ that is $\rho(R)=\det\left[\sum A^0 R^{K-1}\right]=0$ satisfying $|R|\leq 1$ and for those root with $|R|\leq 1$ must have multiplicity equal to unity (*see Fatunla (1991) for detail).*

$$\rho(R)=\left[R\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\right]=0$$

$$\rho(R)=R^4(R-1)=0, R=0,0,0,0,1$$

Hence the block is zero stable

4.0 Numerical examples

Problem I: We consider a special fourth order initial value problem

$$y^{iv}=x,\ y(0)=0,\ y'(0)=1,\ y''(0)=y'''(0)=0,\ h=0.1$$

Exact solution: $y(x)=\dfrac{x^5}{120}+x$

This problem was solved by Olabode (2009) using block method developed for special fourth order odes of order six with h=0.1We compare our result with their result as shown in Table1.

Problem II: We consider a linear fourth order initial value problem

$$y^{iv}+y''=0\ \ 0\le x\le \tfrac{\pi}{2}$$

$$y(0)=0,\ y'(0)=\tfrac{-1.1}{72-50\pi},\ y''(0)=\tfrac{1}{144-100\pi},\ y'''(0)=\tfrac{1.2}{144-100\pi}$$

Exact solution: $y(x)=\dfrac{1-x-\cos x-1.2\sin x}{144-100\pi}$

This problem was solved by Kayode (2008) adopting predictor corrector method, where the corrector of order six was proposed with step length $h=\dfrac{1}{320.}$. We solve this problem using our method for step length $h=0.01$. We compare our result with Kayode (2008) as shown in table II.

Problem III: We consider a nonlinear initial value problem

$$y^{iv}=(y')^2-y(y'')-4x^2+e^x(1-4x+x^2)\ 0\le x\le 1$$

$$y(0)=1,\ y'(0)=1,\ y''(0)=3,\ y'''(0)=1$$

Exact solution: $y(x)=x^2+e^x$

We compare our result with Kayode (2008) who adopted predictor corrector method where a method of order six as proposed with step length $h=\dfrac{1}{320.}$.We solve this problem for step length $h=0.01$. Our result is shown in table III.

$$y^{iv}=(y')^2-yy^{(3)}-4x^2+e^x(1-4x+x^2),\ 0\le x\le 1$$

$$y(0)=1,\ y'(0)=1,\ y''(0)=3,\ y'''(0)=1$$

$$y(x)=x^2+e^x$$

Error = $\left| Exact\ result - Computed\ result \right|$

Table 1 Showing result of problem **I, h= 0.1**

X	EXACT RESULT	COMPUTED RESULT	ERROR	Error in Olabode(2009)
0.1	0.100000083333333	0.100000083333333	0.0000+00	7.0000(-10)
0.2	0.200002666666667	0.200002666666667	0.0000+00	8.9999(-10)
0.3	0.300020250000000	0.300020250000000	0.0000+00	2.0999(-09)
0.4	0.400085333333334	0.400085333333334	0.0000+00	5.1000(-09)
0.5	0.500260416666667	0.500260416666667	0.0000+00	7.7999 (-09)
0.6	0.600648000000000	0.600648000000000	0.0000+00	1.1800 (-08)
0.7	0.701400583333333	0.701400583333333	0.0000+00	1.2400 (-08)
0.8	0.802730666666667	0.802730666666667	0.0000+00	1.4100 (-08)
0.9	0.904920750000001	0.904920750000001	0.0000+00	1.8800 (-08)
1.0	1.008333333333333	1.008333333333333	0.0000+00	2.6000 (-08)

Table II Showing result of problem **II h=0.01**

X	EXACT RESULT	COMPUTED RESULT	ERROR	ERROR IN Kayode (2008)
0.1	0.0012623718420566	0.0012623718420566	6.5052(-19)	4.8355(-17)
0.2	0.0024592829189318	0.0024592829189318	1.3010(-18)	1.3933(-16)
0.3	0.0035846460422381	0.0035846460422381	4.7704(-18)	6.6893(-16)
0.4	0.0046330889070900	0.0046330889070900	1.7347(-17)	2.0129(-15)
0.5	0.0056000077703975	0.0056000077703976	4.3368(-17)	4.6736 (-15)
0.6	0.0064816134499462	0.0064816134499463	9.5409(-17)	9.1874 (-15)
0.7	0.0072749691846527	0.0072749691846529	1.8127(-16)	1.6069 (-14)
0.8	0.0079780199777112	0.0079780199777115	3.1571(-16)	2.5407 (-14)
0.9	0.0085896131294417	0.0085896131294422	5.1868(-16)	3.8108 (-14)
1.0	0.0091095097546848	0.0091095097546856	8.0491(-16)	5.4051 (-14)

Table III Showing result of problem **III**

X	EXACT RESULT	COMPUTED RESULT	ERROR	ERROR IN Kayode (2008)
0.1	1.1151709180756477	1.1151709180747431	9.0460(-13)	6.6613(-15)
0.2	1.2614027581601699	1.2614027581580183	2.1516(-12)	9.7477(-14)
0.3	1.4398588075760033	1.4398588075722483	3.7549(-12)	4.6185(-13)
0.4	1.6518246976412703	1.6518246976355817	5.6885(-12)	1.4224(-12)
0.5	1.8987212707001282	1.8987212706922463	7.8819(-12)	3.4254 (-12)
0.6	2.1821188003905090	2.1821188003802967	1.0212(-11)	6.8736 (-12)
0.7	2.5037527074704768	2.5037527074579797	1.2497(-11)	1.2636 (-11)
0.8	2.8655409284924680	2.8655409284779814	1.4486(-11)	2.1388 (-11)
0.9	3.2696031111569508	3.2696031111411017	1.5849(-11)	3.3989 (-11)
1.0	3.7182818284590464	3.7182818284428873	1.6159(-11)	5.1402 (-11)

5.0 CONCLUSION

We have proposed a five steps numerical integrator for the solution of fourth order initial value problems which was implemented in continuous block method. Continuous block method has advantage of evaluation at all selected points within the interval of integration. It must be noted that the continuous block method has the same advantages as the continuous linear multistep method which is extensively discussed by Awoyemi (1991). The results show that our method gave better approximation than the existing methods that we compared our results with.

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