# High Incidence Supersonic Similitude For Planar Wedge

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## ABSTRACT

A similitude has been obtained for a planar wedge with attached bow shock at high incidence in supersonic flow. A strip theory in which flow at a span wise location is two dimensional developed by Ghosh is been used. This combines with the similitude to lead to a piston theory which gives closed form of solutions for unsteady derivatives in pitch. Substantially the same results as the theory of Liu and Hui are obtained with remarkable computational ease.

**Key Words: Planar Wedge, Quasi-wedge, Supersonic**

## 1. INTRODUCTION

Sychev's [1] high incidence hypersonic similitude is applicable to a wing provided it has an extremely small span in addition to small thickness. The unsteady infinite span case has been analysed, but mostly for small flow deflections. The piston theory of Lighthill [2] neglects the effects of secondary wave reflection. Appleton [3] and McInthosh [4] have included these effects. Hui's [5] theory is valid for wedges of arbitrary thickness oscillating with small amplitude provided the bow shock remains attached. Erricsson's [6] theory covers viscous and elastic effects for airfoils with large flow deflection. Orlik-Ruckemann [7] has included viscous effect and Mandl[8] has addressed small surface curvature effect for oscillating thin wedges. Ghosh's [9] similitude and piston theory for the infinite span case with large flow deflection is valid for airfoils with planar surfaces. In the present work the Ghosh similitude has been extended for supersonic flows past a planar wedge.
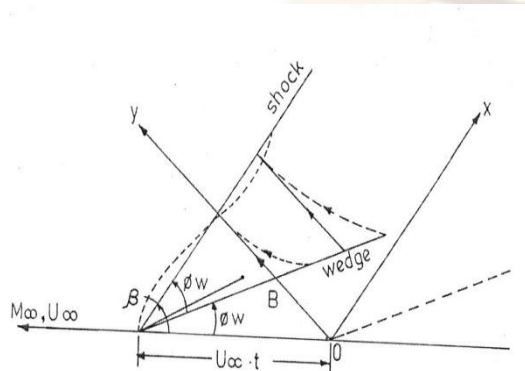
## 2.STEADY WEDGE



Fig. 1 shows the wedge at time t.

Fig. 1 shows that upper half of a steady wedge with attached bow shock in rectilinear flight from right to left in stationary air, at time t. Dimensional analysis indicates that the flow is conical in nature, i.e., at a given instant $\frac{\partial}{\partial r} = 0$ where r is the distance along a ray from the apex. Hence the bow shock must coincide with array. The space-fixed co-ordinate system (x, y) is so chosen that the x-axis coincides with the bow shock at time t=0. Conicality of the flow implies that the instantaneous streamlines have the same slope where they intersect a ray from the apex. Since the shock sets the fluid particles in motion normal to itself, the instantaneous streamlines intersect the shock at right angles. The dashed lines in fig. 1 are probable streamline shapes. We tentatively assume that the streamlines are straight shown by firm lines in Fig.1, if this leads to $\frac{\partial}{\partial r} = 0$ then it is a solution. Consider the plane flow on stream surface x=0. At time t the shock location on x=0 is

$$y_s = U_\infty t \sin \beta \qquad (1)$$

And the body location can be shown to be

$$Y_b = OB = U_\infty t \sin \theta_w / \cos \theta_w. \qquad (2)$$

Since the flow in plane x=0 is independent of the flow in a neighboring parallel plane, it can be taken as a piston driven fluid motion where the piston Mach number $M_p = M_\infty \sin \theta_w / \cos(\beta - \theta_w)$ and the shock Mach number $M_s = M_\infty \sin \beta$. Since $M_p$ is independent of t, pressure remains constant in this 1D space.

Therefore, $\partial p / \partial y = 0$. Since the streamlines are straight there is no centrifugal force; thus $\partial p / \partial x = 0$. Hence, $\partial p / \partial r = 0$. Thus the wedge flow is exactly equivalent to 1D piston motion normal to the shock. It can be shown that the relation between $M_S \& M_P$ yields the well known oblique shock relation giving the shock relation giving the shock angle in terms of $\theta_w$.

## 2.1 QUASI-WEDGE OR OSCILLATING WEDGE:

Fig. 1 shows the probable shape of the bow shock in dotted when the wedge is either oscillating

or replaced by a quasi-wedge. The slope of the curved shock with x-axis remains small, say of order $\phi$. Let the Mach number behind the shock, in body-fixed coordinate, be $M_2$. The characteristics make an angle $\delta = (\sin^{-1} 1/M_2 - \phi_W)$ with the x-axis. It can be shown that this angle remains small for fairly large values of $\theta_W$ even for moderate Mach numbers. For example for $M_\infty = 2, \theta_W = 15°$, we have $\delta = 13°$.

Again for $M_\infty = 3$, $\theta_W = 20°$ we get $\delta = 12.5°$. we stipulate (see Fig.1) that $\delta \leq 0.3$, and then $\phi \,\&\, \delta$ are of same order. Since the gradient is normal to the characteristics we have,

$$\frac{\partial}{\partial x} = 0(\phi.\frac{\partial}{\partial y}) \tag{3}$$

Also the net perturbation introduced by the shock and Mach waves will chiefly be in the y-direction. Thus
$$u=0(\phi.v) \tag{4}$$

Where, u and v are velocity components in x & y directions. Equations. (3) & (4) suggest transformations

$$u^1 = \phi^{-1}.u \,\&\, x^1 = \phi.x. \tag{5}$$

we apply these transformations to the equation of continuity to get $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial y} = -\phi^2 \frac{\partial(\rho u^1)}{\partial x^1}$.

Similarly, applying these transformations to the rest of the equations of motion and boundary conditions and neglecting terms of $0(\phi^2)$, we get equivalence with a 1D piston motion in y-direction and unified supersonic/hypersonic similitude.

## 2.2  PISTON THEORY:

A thin strip of the wing, parallel to the centerline, can be considered independent of the z dimension when the velocity component along the z direction is small. This has been discussed by Ghosh's [9]. The strip theory combined with Ghosh's large incidence similitude leads to the "piston analogy" and pressure P on the surface can be directly related to equivalent piston mach no. Mp. In this case both Mp and flow deflections are permitted to be large. Hence light hill piston theory[2] or miles strong shock piston theory cannot be used but Ghosh's piston theory will be applicable.

$$\frac{P}{P_\infty} = 1 + AM_P^2 + AM_P(B + M_P^2)^{\frac{1}{2}}$$

, Where $P_\infty$ is free stream pressure         (6)

Since strips at different span wise location are assumed independent of each other, the strip can be considered as a flat plate at an angle of attack. The

angle of incidence is same as that of wing. Angle $\phi$ is the angle between the shock and the strip. A piston theory which has been used in equation (7) has been extended to supersonic flow. The expression is given below.

$$\frac{p}{p_\infty} = 1 + A(\frac{M_p}{\cos\phi})^2 + A(\frac{M_p}{\cos\phi})(B + (\frac{M_p}{\cos\phi})^2)^{\frac{1}{2}} \tag{7}$$

Where $p_\infty$ is free stream pressure, $A = \frac{(\gamma+1)}{4}, B = (4/(\gamma+1)^2, \gamma$ is the specific heat ratio and $M_p = $ the local piston Mach number normal to the wedge surface.

### 2.3 Pitching moment derivatives

Let the mean incidence be $\alpha_0$ for the wing oscillating in pitch with small frequency and amplitude about an axis $X_0$. The piston velocity and hence pressure on the windward surface remains constant on a span wise strip of length 2L at x, the pressure on the lee surface is assumed zero. Therefore, the nose up moment is

$$m = -\int_0^L p..(x - x_0)dx \tag{8}$$

The Stiffness and damping derivatives are respectively

$$-C_{m_\alpha} = \frac{2}{\frac{1}{2}\rho_\infty U_\infty^2 C^2}\left(-\frac{\partial m}{\partial \alpha}\right)_{\alpha=\alpha_0, q=0} \tag{9}$$

$$-C_{m_q} = \frac{2}{\frac{1}{2}\rho_\infty U_\infty^2 C^3}\left(-\frac{\partial m}{\partial q}\right)_{\alpha=\alpha_0, q=0} \tag{10}$$

Piston Mach number is given by

$$M_p = \frac{[u_\infty \sin\alpha + (x-x_\circ)q]}{a_\infty} \tag{11}$$

Where $\rho_\infty, a_\infty$ are density and velocity of sound in the free stream. Combining (8) through (11), differentiation under the integral sign is performed. Defining $x_0 = hL\cos^2\alpha_0, c = L\cos\alpha_0$,

$$S_1 = \frac{M_\infty \sin\alpha_O}{\cos\phi}$$, the derivatives in pitch of a planar wedge become equal to

$$-C_{m_\alpha} = \left[\frac{(\gamma+1)}{M_\infty \cos\alpha \cdot \cos\emptyset}\right]F(S_1)\left(\frac{1}{2} - h\cos^2\alpha_\circ\right) \tag{12}$$

$$-C_{m_q} = \frac{(\gamma+1)}{M_\infty} \left\{ \frac{F(S_1)}{cos\phi cos^3\alpha \cdot} \right\} \left\{ \frac{1}{3} - hcos^2\alpha \cdot + \right.$$
$$\left. h2cos4\alpha \cdot \right\} \tag{13}$$

Where

$$F(S_1) = 2S_1 + \frac{B + 2S_1{}^2}{\sqrt{B + S_1{}^2}}$$

## 2. RESULTS AND DISCUSSIONS

In the present work an attempt is made to estimate the Stability Derivatives for planar wedges for a wide range of mach numbers and angle of attack for attached shock cases.(Fig 2 to Fig 5). Stiffness derivative in pitch calculated by present theory has been compared (Fig. 6) with Hui [5] and it shows a good agreement. The Damping derivatives in pitch is calculated and compared (Fig. 7 and Fig. 8,Fig. 9) with Hui and Lui [10] and Hui [5].The difference in the damping derivative is attributed to the present theory being a quasi-steady one where as Lui et al give an unsteady theory which predicts Cm$_{q..}$ .The present theory is simpler than Lui and Hui [10] and brings out the explicit dependence of the derivatives on the similarity parameter S$_1$.
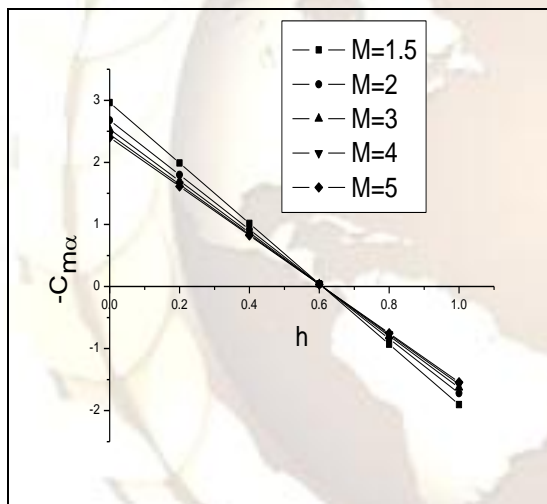


Fig.2: Variation of stiffness derivative with pivot position
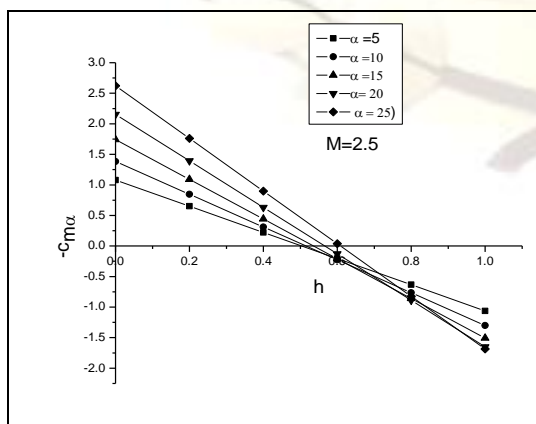


Fig3 : variation of stiffness derivative with pivot position
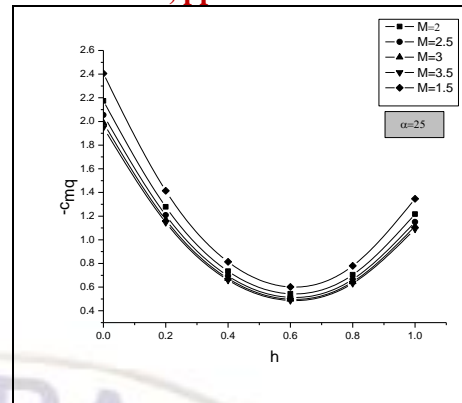


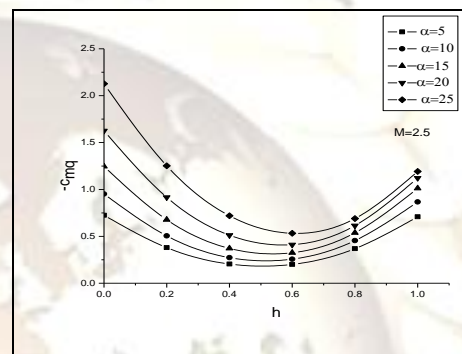Fig 4: variation of damping derivative with pivot position


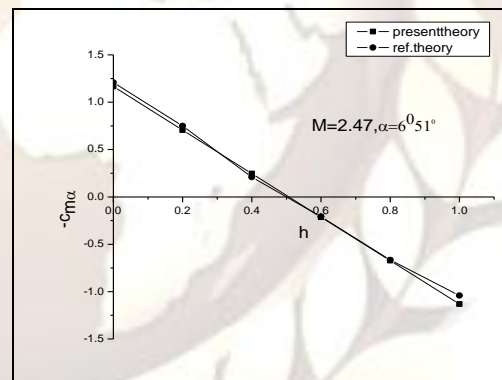
Fig5: Variation of damping derivative with pivot position



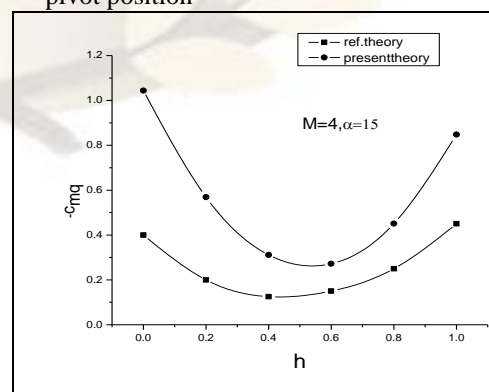Fig6: variation of stiffness derivative with pivot position



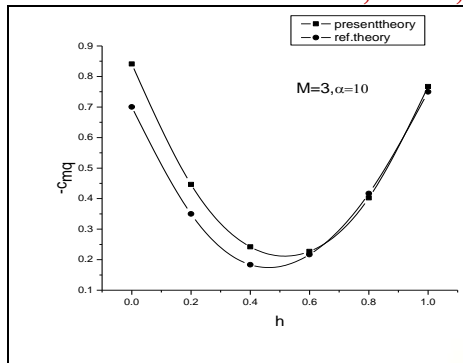Fig. 7: variation of damping derivative with pivot position

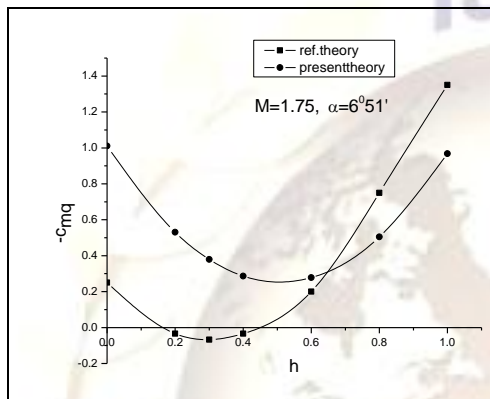Fig. 8 : variation of Damping derivative with pivot position



Fig. 9: variation of damping derivative with pivot Position

## 3. CONCLUSION

The present theory demonstrates its wide application range in angle of incidence and the Mach number. It is valid only when the shock wave is attached. The present theory is simple and yet gives good results with remarkable computational ease. Further extension of this theory can be done by taking into consideration the effect of viscosity and wave reflection.

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