Unsteady MHD Flow Induced by a Porous Flat Plate in a Rotating System

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ABSTRACT

The unsteady magnetohydrodynamic flow of a viscous incompressible electrically conducting fluid bounded by an infinite porous flat plate in a rotating system in the presence of a uniform transverse magnetic field has been analyzed. Initially ($t' = 0$) the fluid at infinity moves with uniform velocity U_0. At time $t' > 0$, the plate suddenly starts to move with uniform velocity U_0 in the direction of the flow. The velocity field and the shear stresses at the plate have been derived using the Laplace transform technique. Solutions are also obtained for small time. It is observed that the primary velocity increases whereas the secondary velocity decreases with an increase in either rotation parameter or magnetic parameter. It is also found that the shear stress at the plate due to the primary flow decreases with an increase in either rotation parameter or magnetic parameter. On the other hand, the shear stress at the plate due to the secondary flow decreases with an increase in rotation parameter while it increases with an increase in magnetic parameter.

Keywords: **Magnetohydrodynamic, rotation, inertial oscillation and porous plate.**

I. INTRODUCTION

When a vast expanse of viscous fluid bounded by an infinite flat plate is rotating about an axis normal to the plate, a layer is formed near the plate where the coriolis and viscous forces are of the same order of magnitude. This layer is known as Ekman layer. The study of flow of a viscous incompressible electrically conducting fluid induced by a porous flat plate in rotating system under the influence of a magnetic field has attracted the interest of many researchers in view of its wide applications in many engineering problems such as oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics. The unsteady flow of a viscous incompressible fluid in a rotating system has been studied by Thornley[2], Pop and Soundalgeker[3], Gupta and Gupta[4], Deka et al.[5] and many other researchers. Flow in the Ekman layer on an oscillating plate has been studied by Gupta et al.[6]. Guria and Jana[7] have studied the hydromagnetic flow in the Ekman layer on an oscillating porous plate in the presence of a uniform transverse magnetic field. Recently, Das et al.[8] have studied the unsteady viscous incompressible flow induced by a porous plate in a rotating system.

This paper is devoted to study the effect of the magnetic field on the unsteady MHD flow of a viscous incompressible electrically conducting fluid induced by an infinite porous flat plate in a rotating system. Initially $(t' = 0)$, the fluid at infinity moves with uniform velocity U_0. At time $t' > 0$, the plate suddenly starts to move with a uniform velocity U_0 in the direction of the flow. The velocity distributions and the shear stresses at the plate due to the primary and secondary flows are also obtained for small time t. To demonstrate the effects of rotation and applied magnetic field on the flow field, the the velocity distributions and shear stresses due to the primary and the secondary flows are depicted graphically. It is observed that the primary velocity increases whereas the secondary velocity decreases with an increase in either rotation parameter K^2 or magnetic parameter M^2. Further, it is found that the series solution obtained for small time converges more rapidly than the general solution.

II. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider the flow of a viscous incompressible electrically conducting fluid filling the semi infinite space $z \geq 0$ in a cartesian coordinate system. Initially, the fluid flows past an infinitely long porous flat plate when both the plate and the fluid rotate in unison with an uniform angular velocity Ω about an axis normal to the plate. Initially $(t' = 0)$, the fluid at infinity moves with uniform velocity U_0 along x-axis. A uniform magnetic field H_0 is imposed along z-axis [See Fig.1] and the plate is taken electrically non-conducting. At time $t' > 0$, the plate suddenly starts to move with the same uniform velocity as that of the free stream velocity U_0.

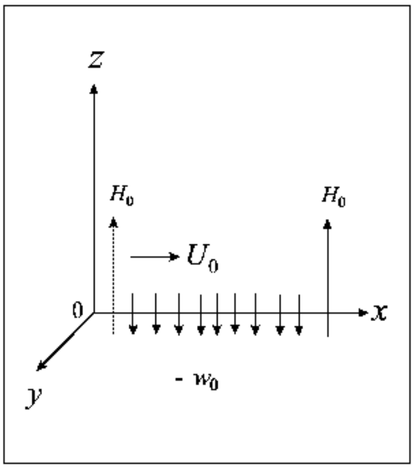

Fig.1: Geometry of the problem

At time $(t' = 0)$, the equations of motion are

$$-w_0 \frac{d\hat{u}}{dz} - 2\Omega\hat{v} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2\hat{u}}{dz^2} + \frac{\sigma H_0^2}{\rho} j_y, \qquad (1)$$

$$-w_0 \frac{d\hat{v}}{dz} + 2\Omega\hat{u} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{d^2\hat{v}}{dz^2} - \frac{\sigma H_0^2}{\rho} j_x, \qquad (2)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z}, \qquad (3)$$

where $(\hat{u}, \hat{v}, -w_0)$ are the velocity components along x, y and z-directions, p the pressure including centrifugal force, ρ the fluid density, ν the kinematic coefficient of viscosity, σ the electrical conductivity and w_0 being the suction velocity at the plate.

The boundary conditions for $\hat{u}$ and $\hat{v}$ are

$$\hat{u} = 0,\ \hat{v} = 0,\ \hat{w} = -w_0 \text{ at } z = 0$$
$$\hat{u} \to U_0,\ \hat{v} \to 0 \text{ as } z \to \infty. \qquad (4)$$

The Ohm's law is

$$\vec{j} = \sigma(\vec{E} + \mu_e \vec{q} \times \vec{H}), \qquad (5)$$

where $\vec{H}$ is the magnetic field vector, $\vec{E}$ the electric field vector, μ_e the magnetic permeability.

We shall assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. The solenoidal relation $\nabla.\vec{H} = 0$ for the magnetic field gives $H_z = H_0 =$ constant everywhere in the fluid where $\vec{H} \equiv (H_x, H_y, H_z)$. The equation of conservation of the charge $\nabla \cdot \vec{j} = 0$ gives $j_z =$ constant where $\vec{j} \equiv (j_x, j_y, j_z)$. This constant is zero since $j_z = 0$ at the plate which is electrically non-conducting. Thus $j_z = 0$ everywhere in the flow. Since the induced magnetic field is neglected, the Maxwell's equation $\nabla \times \vec{E} = -\mu_e \frac{\partial \vec{H}}{\partial t}$ becomes $\nabla \times \vec{E} = 0$ which in turn gives $\frac{\partial E_x}{\partial z} = 0$ and $\frac{\partial E_y}{\partial z} = 0$. This implies that $E_x =$ constant and $E_y =$ constant everywhere in the flow.

In view of these conditions, equation (5) yields

$$j_x = \sigma(E_x + \mu_e H_0 \hat{v}), \qquad (6)$$
$$j_y = \sigma(E_y - \mu_e H_0 \hat{u}). \qquad (7)$$

As the magnetic field is uniform in the free stream, we have from $\vec{j} = \nabla \times \vec{H}$ that $j_x \to 0$, $j_y \to 0$ as $z \to \infty$. Further, $\hat{u} \to 0$, $\hat{v} \to 0$ as $z \to \infty$ gives $E_x = 0,\ E_y = \mu_e H_0 U_0$.

Substituting the values of E_x and E_y, equations (6) and (7) become

$$j_x = \sigma\mu_e H_0 \hat{v},\ j_y = -\sigma\mu_e H_0(\hat{u} - U_0). \qquad (8)$$

On the use of (8) and usual boundary layer approximations, equations (1) and (2) become

$$-w_0 \frac{d\hat{u}}{dz} - 2\Omega\hat{v} = \nu\frac{d^2\hat{u}}{dz^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}\left(\hat{u} - U_0\right), \qquad (9)$$

$$-w_0 \frac{d\hat{v}}{dz} + 2\Omega\left(\hat{u} - U_0\right) = \nu\frac{d^2\hat{v}}{dz^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}\hat{v}. \qquad (10)$$

Introducing the non-dimensional variables

$$\eta = \frac{U_0 z}{\nu},\ \hat{F} = \frac{(\hat{u} + i\hat{v})}{U_0},\ i = \sqrt{-1}, \qquad (11)$$

equations (9) and (10) become

$$\frac{d^2\hat{F}}{d\eta^2} + S\frac{d\hat{F}}{d\eta} - \left(M^2 + 2iK^2\right)\hat{F} = -\left(M^2 + 2iK^2\right), \qquad 12)$$

where $S = \frac{w_0}{U_0}$ is the suction parameter, $K^2 = \frac{\Omega\nu}{U_0^2}$ the rotation parameter and $M^2 = \frac{\sigma\mu_e^2 H_0^2 \nu}{\rho U_0^2}$ the magnetic parameter.

The corresponding boundary conditions for $\hat{F}(\eta)$ are

$$\hat{F} = 0 \text{ at } \eta = 0 \text{ and } \hat{F} \to 1 \text{ as } \eta \to \infty. \qquad (13)$$

The solution of (12) subject to the boundary conditions (13) can be obtained, on using (11), as

$$\frac{\hat{u}}{U_0} = 1 - e^{-a_1\eta}\cos b_1\eta, \qquad (14)$$

$$\frac{\hat{v}}{U_0} = e^{-a_1\eta}\sin b_1\eta, \qquad (15)$$

where

$$a_1 = \frac{S}{2} + \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+M^2\right)^2 + 4K^4\right\}^{\frac{1}{2}} + \left(\frac{S^2}{4}+M^2\right)\right]^{\frac{1}{2}},$$

$$b_1 = \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+M^2\right)^2 + 4K^4\right\}^{\frac{1}{2}} - \left(\frac{S^2}{4}+M^2\right)\right]^{\frac{1}{2}}. \quad (16)$$

The solutions given by (14) and (15) are valid for both suction $(S > 0)$ and blowing $(S < 0)$ at the plate. If $M^2 = 0$, the equations (14) and (15) are identical with the equation(12) of Gupta[9].

At time $t' > 0$, the plate suddenly starts to move with a uniform velocity U_0 along x-axis in the direction of the flow. Assuming the velocity components $(u, v, -w_0)$ along the coordinate axes, we have the equations of motion as

$$\frac{\partial u}{\partial t'} - w_0\frac{\partial u}{\partial z} - 2\Omega\, v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}(u - U_0), \quad (17)$$

$$\frac{\partial v}{\partial t'} - w_0\frac{\partial v}{\partial z} + 2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}v, \quad (18)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z}. \quad (19)$$

The initials and boundary conditions are

$$u = \hat{u},\ v = \hat{v} \text{ at } t' = 0 \text{ for } z \ge 0,$$
$$u = U_0,\ v = 0 \text{ at } z = 0,\ t' > 0$$
$$u \to U_0,\ v \to 0 \text{ as } z \to \infty,\ t' > 0 \quad (20)$$

It is observed from the equation (19) that the pressure p is independent of z. Using equations (17) and (18) together with conditions $u \to U_0$ and $v \to 0$ as $z \to \infty$, we have

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \text{ and } -\frac{1}{\rho}\frac{\partial p}{\partial y} = 0. \quad (21)$$

On the use of (20), equations (17) and (18) become

$$\frac{\partial u}{\partial t'} - w_0\frac{\partial u}{\partial z} - 2\Omega v = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}(u - U_0), \quad (22)$$

$$\frac{\partial v}{\partial t'} - w_0\frac{\partial v}{\partial z} + 2\Omega(u - U_0) = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}v. \quad (23)$$

Equations (22) and (23) can be written in combined form as

$$\frac{\partial F}{\partial t'} - w_0\frac{\partial F}{\partial z} + 2i\,\Omega F = \nu\frac{\partial^2 F}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}F, \quad (24)$$

where

$$F = \frac{(u + iv)}{U_0} - 1. \quad (25)$$

On the use of (11) together with $t = \frac{U_0^2 t'}{\nu}$, equation (24) yields

$$\frac{\partial F}{\partial t} - S\frac{\partial F}{\partial \eta} = \frac{\partial^2 F}{\partial \eta^2} - (M^2 + 2iK^2)F. \quad (26)$$

The corresponding initial and the boundary conditions for $F(\eta, \tau)$ are

$$F(\eta, 0) = \hat{F}(\eta),\ \eta \ge 0, \quad (27)$$
$$F(0, t) = 0 \text{ for } t > 0,\ F(\infty, t) = 0 \text{ for } t > 0, \quad (28)$$

where $\hat{F}(\eta)$ is given by (11).

By defining

$$F(\eta, t) = H(\eta, t)e^{-\lambda t}, \quad (29)$$

equation (26) becomes

$$\frac{\partial H}{\partial t} - S\frac{\partial H}{\partial \eta} = \frac{\partial^2 H}{\partial \eta^2}, \quad (30)$$

where

$$\lambda = M^2 + iK^2, \quad (31)$$

with initial and boundary conditions

$$H(\eta, 0) = \hat{F}(\eta) \text{ for } \eta \ge 0, \quad (32)$$

$$H(0, t) = 0 \text{ for } t > 0,\ H(\infty, t) = 0 \text{ for } t > 0. \quad (33)$$

On the use of Laplace transform, equation (30) becomes

$$\frac{d^2\bar{H}}{d\eta^2} + s\frac{d\bar{H}}{d\eta} - s\bar{H} = e^{-(\alpha + i\beta)\eta}, \quad (34)$$

where

$$\bar{H}(\eta, s) = \int_0^\infty H(\eta, t)e^{-st}dt. \quad (35)$$

The boundary condition for $\bar{H}(\eta, s)$ are

$$\bar{H}(0, s) = 0 \text{ for } t > 0 \text{ and } \bar{H}(\infty, s) = 0 \text{ for } t > 0. \quad (36)$$

The solution of the equation (34) subject to the boundary conditions (36) is

$$\bar{H}(\eta, s) = \frac{e^{-\left(\frac{S}{2} + \sqrt{\frac{S^2}{4} + s}\right)\eta}}{s - \lambda} - \frac{e^{-(a_1 + ib_1)\eta}}{s - \lambda}. \quad (37)$$

The inverse Laplace transform of (37) and on the use of (29) and (25), we get

$$\frac{u + iv}{U_0} = 1 - e^{-(a_1 + ib_1)\eta} + \frac{1}{2}e^{-\frac{S}{2}\eta}$$
$$\times\left[e^{(a_2 + ib_2)\eta}\operatorname{erfc}\left\{\frac{\eta}{2\sqrt{t}} + (a_2 + ib_2)\sqrt{t}\right\}\right.$$
$$\left. + e^{-(a_2 + ib_2)\eta}\operatorname{erfc}\left\{\frac{\eta}{2\sqrt{t}} - (a_2 + ib_2)\sqrt{t}\right\}\right], \quad (38)$$

where

$$a_2, b_2 = \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+M^2\right)^2+4K^4\right\}^{\frac{1}{2}} \pm \left(\frac{S^2}{4}+M^2\right)\right]^{\frac{1}{2}}. \quad (39)$$

On separating into a real and imaginary parts one can easily obtain the velocity components $\frac{u}{U_0}$ and $\frac{v}{U_0}$ from equation (38). The solution given by (38) is valid for both suction $(S>0)$ and blowing $(S<0)$ at the plate. If $M^2=0$, then the equation (38) is identical with equation (28) of Das et al. [8].

III. DISCUSSIONS

To study the flow situations due to the impulsive starts of a porous plate for different values of rotation parameter K^2, magnetic parameter M^2, suction parameter S and time t, the velocities are shown in Figs.2-5. The primary velocity $\frac{u}{U_0}$ and the secondary velocity $\frac{v}{U_0}$ are shown in Figs.2 and 3 against the distance η from the plate for several values of K^2 and M^2 with $t=0.2$ and $S=1.0$. It is observed that the primary velocity $\frac{u}{U_0}$ increases whereas the secondary velocity $\frac{v}{U_0}$ decreases with an increase in either rotation parameter K^2 or magnetic parameter M^2. It is observed that for $M^2=2$ and $K^2 \geq 5$, the secondary velocity $\frac{v}{U_0}$ shows incipient flow reversal near the plate although the primary flow does not. Figs.4 and5 show the effect of time t on the primary and the secondary velocities with $M^2=5$, $K^2=2$ and $S=1$. It is found that the primary velocity increases whereas the secondary velocity decreases with an increase in time t.

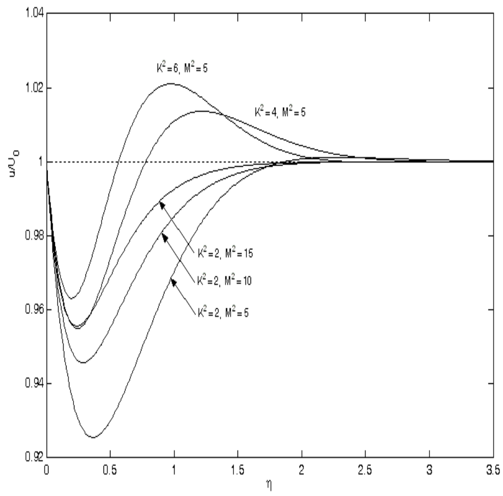

Fig.2: Primary velocity for different K^2 and M^2 when $t=0.2$ and $S=1$

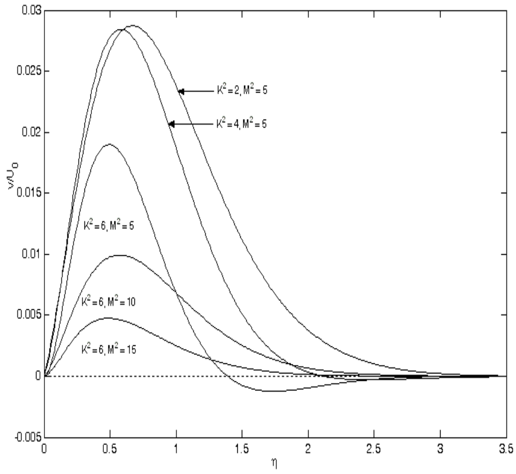

Fig.3: Secondary velocity for different K^2 and M^2 when $t=0.2$ and $S=1$

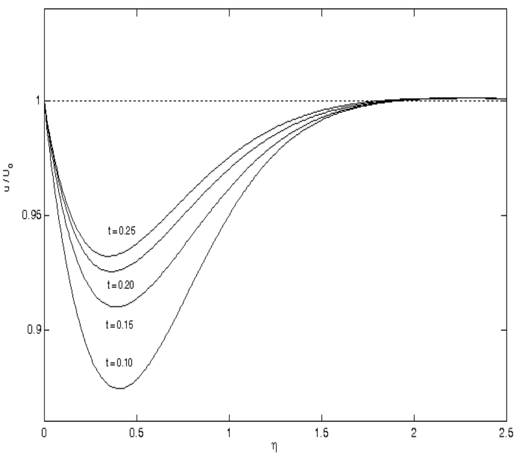

Fig.4: Primary velocity for different time t when $M^2=5$, $K^2=2$ and $S=1$

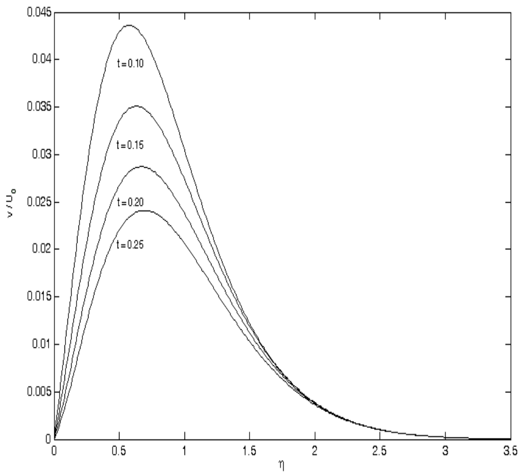

Fig.5: Secondary velocity for different time t when $M^2=5$, $K^2=2$ and $S=1$

We now consider the case when time t is small which correspond to large $s(\gg 1)$. In this case, method used by Carslaw and Jaeger [9] is used since it converges rapidly for small times. Hence, for small times, the inverse Laplace transform of the equation (37) gives

$$H(\eta,t)=e^{-\frac{S}{2}\eta-\frac{1}{4}S^2 t}\sum_{n=0}^{\infty}\left(\frac{S^2}{4}+\lambda\right)^n (4t)^n i^{2n}$$
$$\times \operatorname{erfc}\left(\frac{\eta}{2\sqrt{t}}\right)-e^{-(\alpha+i\beta)\eta+\lambda t}, \qquad (40)$$

where λ is given by (31).

On the use of (29), equation (40) yields

$$F(\eta,t)=e^{-\frac{S}{2}\eta-\frac{S^2 t}{4}-\lambda t}\sum_{n=0}^{\infty}\left(\frac{S^2}{4}+\lambda\right)^n (4t)^n i^{2n}$$
$$\times \operatorname{erfc}\left(\frac{\eta}{2\sqrt{t}}\right)-e^{-(\alpha+i\beta)\eta}, \qquad (41)$$

where $i^n\operatorname{erfc}(.)$ denotes the repeated integrals of the complementary error function given by

$$i^n\operatorname{erfc}(x)=\int_x^{\infty} i^{n-1}\operatorname{erfc}(\xi)\,d\xi,\ n=0,1,2,\cdots,$$
$$i^0\operatorname{erfc}(x)=\operatorname{erfc}(x),\ i^{-1}\operatorname{erfc}(x)=\frac{2}{\sqrt{\pi}}e^{-x^2}. \qquad (42)$$

On the use of (25) and separating into a real and imaginary parts, equation (41) gives

$$\frac{u}{U_0}=1-e^{-a_1\eta}\cos b_1\eta+e^{-\left\{\frac{S}{2}\eta+\left(\frac{S^2}{4}+M^2\right)t\right\}}$$
$$\times\left[A(\eta,t)\cos 2K^2 t+B(\eta,t)\sin 2K^2 t\right], \qquad (43)$$

$$\frac{v}{U_0}=e^{-a_1\eta}\sin b_1\eta+e^{-\left\{\frac{S}{2}\eta+\left(\frac{S^2}{4}+M^2\right)t\right\}}$$
$$\times\left[B(\eta,t)\cos 2K^2 t-A(\eta,t)\sin 2K^2 t\right], \qquad (44)$$

where

$$A(\eta,t)=T_0+\left(\frac{S^2}{4}+M^2\right)(4t)T_2$$
$$+\left\{\left(\frac{S^2}{4}+M^2\right)^2-4K^4\right\}(4t)^2 T_4$$
$$+\left\{\left(\frac{S^2}{4}+M^2\right)^3-12K^4\left(\frac{S^2}{4}+M^2\right)\right\}(4t)^3 T_6+\cdots,$$
(45)

$$B(\eta,t)=2K^2(4t)T_2+4K^2\left(\frac{S^2}{4}+M^2\right)(4t)^2 T_4$$
$$+\left\{6K^2\left(\frac{S^2}{4}+M^2\right)^2-8K^6\right\}(4t)^3 T_6+\cdots. \qquad (46)$$

with $T_{2n}=i^n\operatorname{erfc}\left(\frac{\eta}{2\sqrt{t}}\right),\ n=0,2,4,\cdots.$

Equations (43) and (44) show that the effects of rotation on the unsteady part of the secondary flow become important only when terms of order t is taken into account.

For small values of time, we have drawn the velocity components $\frac{u}{U_0}$ and $\frac{v}{U_0}$ on using the exact solution given by equation (38) and the series solutions given by equations (43) and (44) in Figs.6 and 7. It is seen that the series solutions given by (43) and (44) converge more rapidly than the exact solution given by (38) for small times. Hence, we conclude that for small times, the numerical values of the velocity components can be evaluated from the equations (43) and (44) instead of equation (38).

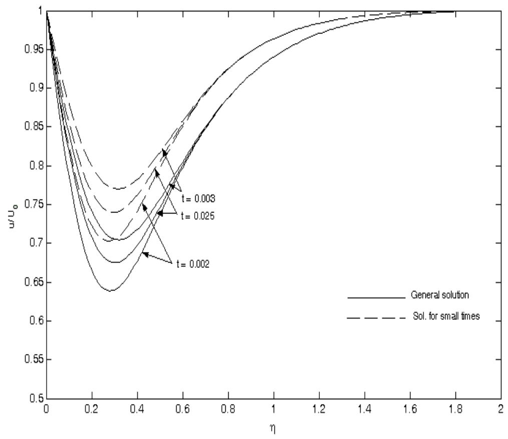

Fig.6: Primary velocity for general solution and solution for small time with $M^2 = 5, K^2 = 2$ and $S = 1$

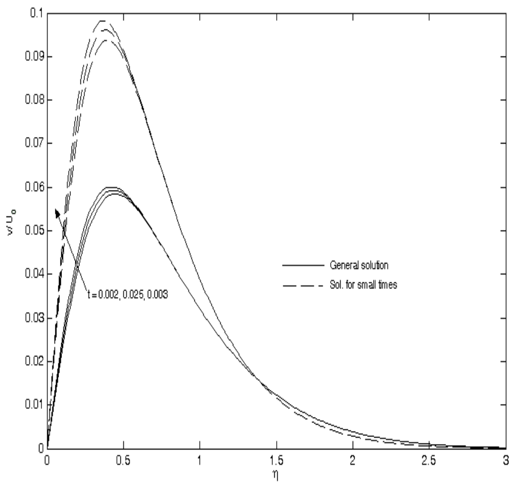

Fig.7: Secondary velocity for general solution and solution for small time with $M^2 = 5$, $K^2 = 2$ and $S = 1$

The non-dimensional shear stresses at the plate $(\eta = 0)$ due to the primary and the secondary flows are given by [from equation (38)]

$$\tau_{x_0} + i\tau_{y_0} = \frac{u' + iv'}{U_0} = a_1 + ib_1 - \left[\frac{S}{2} + (a_2 + ib_2)\,\mathrm{erf}\,(a_2 + ib_2)\sqrt{t} + \frac{1}{\sqrt{\pi t}} e^{-(a_2+ib_2)^2 t}\right]. \quad (47)$$

The numerical results of the shear stresses τ_{x_0} and τ_{y_0} at the plate $(\eta = 0)$ are shown in Figs.8 and 9 against the magnetic parameter M^2 for several values of the rotation parameter K^2, time t with $S = 1$. It is seen that both the shear stresses at the plate $(\eta = 0)$ due to the primary and the secondary flows decrease with an increase in K^2. Fig.9 reveals that for fixed value of K^2, both the shear stresses τ_{x_0} and τ_{y_0} decrease with an increase in time t. Further, it is observed that τ_{x_0} steadily decreases while τ_{y_0} steadily increases with an increase in M^2.

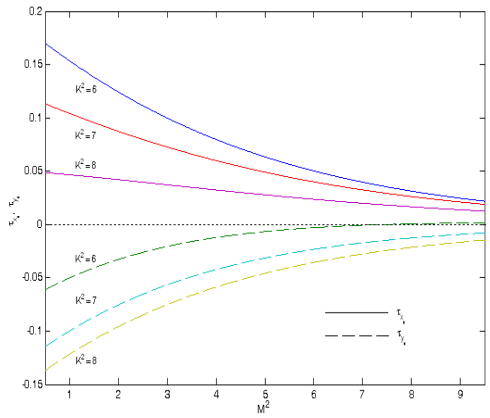

Fig.8: Shear stress τ_{x_0} and τ_{y_0} when $t = 0.2$ and $S = 1$

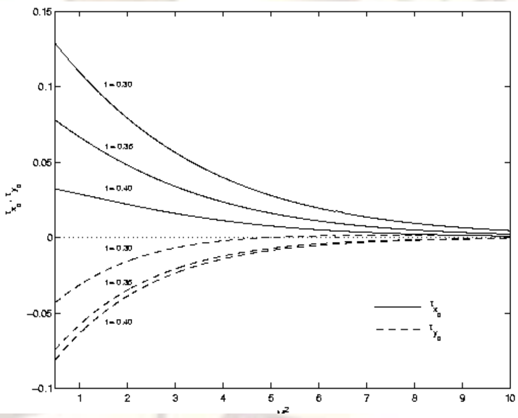

Fig.9: Shear stress τ_{x_0} and τ_{y_0} when $K^2 = 2$ and $S = 1$

The steady state shear stress components at the plate $(\eta = 0)$ are obtained by letting $t \to \infty$ as

$$\tau_x + i\tau_y = (a_1 + ib_1) - \frac{S}{2} + (a_2 + ib_2). \quad (48)$$

Estimation of the time which elapses from the starting of the impulsive motion of the plate till the steady state is reached can be obtained as follows. It is observed from (47) that the steady state is reached after time t_0 when $\mathrm{erf}\,(a_2 + ib_2)\sqrt{t} = 1$. Since $\mathrm{erf}\,(a_2 + ib_2) = 1$ when $|a_2 + ib_2|\sqrt{t_0} = 2$, it follows that

$$t_0 = 4\left[\left(\frac{S^2}{4}+M^2\right)^2+4K^4\right]^{-\frac{1}{2}}. \qquad (49)$$

It is seen that t_0 decreases with increase in either S or K^2 or M^2. This means that the system with suction or rotation or magnetic field takes less time to reach the steady state than the case without suction or rotation or magnetic field.

For small time, the shear stresses at the plate $(\eta = 0)$ due to primary and the secondary flows can be obtained as

$$\tau_{x_0} = \frac{u'(0,t)}{U_0} = \alpha - \frac{1}{2}e^{-\left(\frac{S^2}{4}+M^2\right)t} \times\left[P(0,t)\cos 2K^2t + Q(0,t)\sin 2K^2t\right], \qquad (50)$$

$$\tau_{y_0} = \frac{v'(0,t)}{U_0} = \beta - \frac{1}{2}e^{-\left(\frac{S^2}{4}+M^2\right)t} \times\left[Q(0,t)\cos 2K^2t - P(0,t)\sin 2K^2t\right], \qquad (51)$$

where

$$P(\eta,t) = \left(ST_0 + Y_{-1}/\sqrt{t}\right) + \left(\frac{S^2}{4}+M^2\right)(4t)\left(ST_2+Y_1/\sqrt{t}\right) + \left\{\left(\frac{S^2}{4}+M^2\right)^2 - 4K^4\right\}(4t)^2\left(ST_4+Y_3/\sqrt{t}\right)$$
$$+\left\{\left(\frac{S^2}{4}+M^2\right)^3 - 12K^4\left(\frac{S^2}{4}+M^2\right)\right\}\times(4t)^3\left(ST_6+Y_5/\sqrt{t}\right)+\cdots \qquad (52)$$

$$Q(\eta,t) = 2K^2(4t)\left(ST_2+\frac{Y_1}{\sqrt{t}}\right) + 4K^2\left(\frac{S^2}{4}+M^2\right)(4t)^2\left(ST_4+\frac{Y_3}{\sqrt{t}}\right) + \left\{6K^2\left(\frac{S^2}{4}+M^2\right)^2 - 8K^6\right\}(4t)^3\left(ST_6+\frac{Y_5}{\sqrt{t}}\right)+\cdots, \qquad (53)$$

with

$$\frac{dT_{2n}}{d\eta} = -\frac{Y_{2n-1}}{2\sqrt{t}}, \quad \text{where} \quad Y_{2n-1} = i^{2n-1}\operatorname{erfc}\left(\frac{\eta}{2\sqrt{t}}\right). \qquad (54)$$

Table 1. Shear stress at the plate $(\eta = 0)$ due to primary flow when $M^2 = 5$ and $S = 1$

	$-\tau_{x_0}$ (For General solution)			$-\tau_{x_0}$ (Solution for small times)		
$K^2 \backslash t$	0.002	0.004	0.006	0.002	0.004	0.006
0	10.456620	6.816017	5.220609	10.45661	6.816015	5.220608
4	10.026410	6.386783	4.792615	10.02646	6.387048	4.793337
8	9.426691	5.789964	4.199518	9.426891	5.791084	4.202537
12	8.891527	5.259642	3.675391	8.891985	5.262150	3.682057

Table 2. Shear stress at the plate $(\eta = 0)$ due to secondary flow when $M^2 = 5$ and $S = 1$

	τ_{y_0} (For General Solution)			τ_{y_0} (Solution for small times)		
$K^2 \backslash t$	0.002	0.004	0.006	0.002	0.004	0.006
0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
4	1.268350	1.186032	1.123545	1.268379	1.186199	1.124013
8	2.004932	1.840343	1.715468	2.004992	1.840708	1.716536
12	2.504465	2.257698	2.070635	2.504561	2.258326	2.072551

For small time, the numerical values of the shear stresses calculated from equations (47), (50) and (51) are entered in Tables 1 and 2 for several values of rotation parameter K^2 and time t. It is observed that for small time the shear stresses calculated from the equations (50) and (51) give better result than those calculated from the equation (47). Hence, for small time, one should derived the numerical results of the shear stresses from the equations (50) and (51) instead of equation (47).

We also consider the case when t is large. For large times, the asymptotic formula for the complementary error function with complex argument z

$$\operatorname{erfc}(z) \approx \frac{e^{-z^2}}{\sqrt{\pi}z} \quad \text{as } |z| \to \infty, \qquad (55)$$

with $\operatorname{erfc}(-z) = 2 - \operatorname{erfc}(z)$ enables us to derive the asymptotic solution from the equation (38) in the following form

$$\frac{u+iv}{U_0}=1+\frac{1}{2}e^{-\frac{S}{2}\eta}\left[e^{(a_2+ib_2)\eta}\operatorname{erfc}\left\{(a_2+ib_2)\sqrt{t}+\frac{\eta}{2\sqrt{t}}\right\}\right.$$

$$\left.-e^{-(a_2+ib_2)\eta}\operatorname{erfc}\left\{(a_2+ib_2)\sqrt{t}-\frac{\eta}{2\sqrt{t}}\right\}\right]. \qquad (56)$$

Further, if $\eta \gg 2\sqrt{t}$, $t \gg 1$ then the equation (56) yields

$$\frac{u}{U_0}=1+\frac{e^{-\frac{S}{2}\eta-(a_2^2-b_2^2)t}}{(a_2^2+b_2^2)\sqrt{\pi t}}$$

$$\times\left[\left(a_2\cos 2K^2t-b_2\sin 2K^2t\right)\sinh a_2\eta\cos b_2\eta\right.$$

$$\left.+\left(b_2\cos 2K^2t+a_2\sin 2K^2t\right)\cosh a_2\eta\sin b_2\eta\right], \quad (57)$$

$$\frac{v}{U_0}=\frac{e^{-\frac{S}{2}\eta-(a_2^2-b_2^2)t}}{(a_2^2+b_2^2)\sqrt{\pi t}}$$

$$\times\left[\left(a_2\cos 2K^2t-b_2\sin 2K^2t\right)\cosh a_2\eta\sin b_2\eta\right.$$

$$\left.-\left(b_2\cos 2K^2t+a_2\sin 2K^2t\right)\sinh a_2\eta\cos b_2\eta\right].$$

(58)

Equations (57) and (58) show the existence of inertial oscillations. The frequency of these oscillations is $2K^2$. It is observed that the rotation not only induced a cross flow but also occurs inertial oscillations of the fluid velocity. The frequency of these oscillations increases with an increase in the rotation parameter K^2. It may be noted that the inertial oscillations do not occur in the absence of the rotation. It is interesting to note that the frequency of these oscillations is independent of the magnetic field as well as suction/blowing at the plate.

IV. CONCLUSION

An unsteady MHD flow of an incompressible electrically conducting viscous fluid bounded by an infinitely long porous flat plate in a rotating system has been studied. Initially, at time $(t'=0)$, the fluid at infinity moves with a uniform velocity U_0. After time $t'>0$, the plate suddenly starts to move with the same uniform velocity U_0 as that of the fluid at infinity in the direction of the flow. It is observed that the primary velocity increases with an increase in magnetic parameter M^2. On the other hand, the secondary velocity decreases with an increase in magnetic parameter which is expected as the magnetic field has a retarding influence on the flow field. It is interesting to note that the series solution obtained for small time converges more rapidly than the general solution. Further, for large values of the rotation parameter $K^2 \geq 5$, the secondary flow shows an incipient flow reversal near the plate although the primary flow does not. It is seen that the shear stress components decrease with an increase in rotation parameter.

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