Improving Natural Frequency Of Vibration In Overhead Transmission Lines With Concentrated Masses

Sule, S and Nwofor T.C

Department of Civil and Environmental Engineering, University of Port Harcourt,
Rivers State, Nigeria. P.M.B 5323

ABSTRACT

In this paper, the natural frequencies of vibration of a transmission line subjected to three unequal and equal concentrated masses at equal interval are compared. The natural frequencies of vibration in both cases are predicted based on the assumption that the total kinetic and potential energy of the vibrating masses is constant in the course of the system's oscillation. The natural frequencies of vibration of a transmission line subjected to unequal masses (control) were compared with those of equal concentrated masses. The natural vibration frequency was found to increase by 29% in the first vibration mode, 27% in the second vibration mode but deceased by 4.96% in the third vibration mode when the transmission line was subjected to equal concentrated masses.

Keywords: Natural frequencies, transmission line, degree of freedom, concentrated masses, vibrating masses.

1.0 INTRODUCTION

An overhead transmission line is the medium through which electricity moves from the point of generation to the points of utilization. The distribution system moves electricity from the transmission line to where it is used by customers at home and business areas. Transmission lines are made from cables of aluminium alloy which are suspended by towers in a row.

Vibration of transmission lines in due to wind excitation causes oscillation of large amplitude in overhead transmission lines [1-8]. This large amplitude vibration is a very dangerous phenomenon that causes instability of the overhead transmission lines [9-13]. For example, the large amplitude displacement of transmission lines resulting from wind excitation normally occurs when one of the natural frequencies of vibration is excited leading to resonance. This short circuits the overhead transmission lines as a result of entanglement of lines. Dynamic analysis of a transmission line subjected to wind induced forces is of paramount importance to engineers as the end result is devastating to human lives.

The lumping of concentrated masses on the transmission line as vibration dampers results in discretizing the transmission line in to segments. The concentrated masses are assumed to undergo vertical oscillation about their mean positions. The degree of freedom of the transmission line is equal to the number of concentrated masses.

In this paper, the dynamic analysis of a single span transmission line subjected to unequal and equal concentrated masses is carried out using energy approach. The results of the dynamic analysis of a transmission line subjected to unequal concentrated masses are serving as the control points. The formulated energy model is computationally simple and can be handled manually most specially when fewer number of concentrated masses are involved.

2.0 MODEL DEVELOPMENT

Consider a transmission line of span L suspended between two transmission towers and carrying concentrated masses $m_1,\ m_2,...,m_n$ as shown in Figure 1. The tension T in the transmission line is assumed to be constant in the course of the system's oscillation.

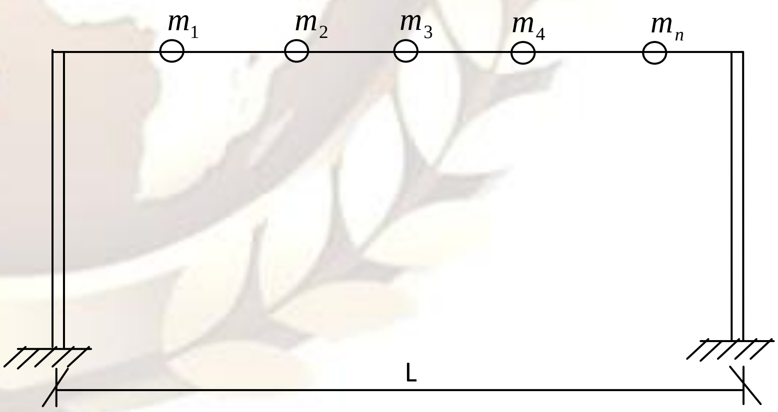

Figure 1: A transmission line and subjected to n concentrated masses $m_1,\ m_2,...,m_n$.

Let $X_i, i=1,2,3,\ldots,N$ represents the chosen coordinate that describes the configuration of the transmission line in Figure 1. X_i is assumed to be zero at equilibrium position. During self-excited vibration, the various parts of the above transmission line undergo instantaneous velocities given by:

$$V_i = \left(\dot{X}_1, \dot{X}_2, \ldots, \dot{X}_N\right)^T \qquad (1)$$

For small amplitude vibration of the transmission line, the potential and kinetic energy of vibration can be approximated by a quadratic surface given by:

$$K.E = \sum_{i}^{n}\sum_{j}^{n} a_{ij}\,\dot{X}_i\,\dot{X}_j \qquad (2)$$

$$P.E = \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\,X_i\,X_j \qquad (3)$$

In matrix form, equation (2) and (3) can be written as:

$$K.E = \dot{X}^T A\dot{X} \qquad (4)$$

$$P.E = X^T AX \qquad (5)$$

where

$a_{ij,}\, b_{ij}$ represent the elements in the ith row and jth column of the matrix respectively.

A, B = n X n symmetric matrices corresponding to kinetic and potential energies of the oscillating masses.

Applying the conservation of mass, the total energy of the oscillating masses is constant.

Therefore,

$$K.E + P.E = \text{constant} \qquad (6)$$

The rate of change of total energy given by equation (6) w.r.t. time is zero.

$$\Rightarrow \frac{d}{dt}(K.E + P.E) = 0 \qquad (7)$$

Substituting for K.E. and P.E. in equation (7) using equations (4) and (5) transforms equation (7) to:

$$\frac{d}{dt}\left(\dot{X}^T A\dot{X} + X^T BX\right) = 0 \qquad (8)$$

Differentiating equation (8) w.r.t. t gives:

$$\frac{d}{dt}\left(\dot{X}^T A\dot{X} + X^T BX\right) = \ddot{X}^T A\dot{X} + \dot{X}^T A\ddot{X} + \dot{X}^T BX + X^T B\dot{X} = 0 \qquad (9)$$

Without loss of generality, it is assumed that:

$\dot{X} = \dot{X}^T, X^T = X, \text{ and } \ddot{X}^T = \ddot{X}$ and simplifying transforms equation (9) to :

$$\frac{d}{dt}\left(\dot{X}^T A\dot{X} + X^T BX\right) = 2\dot{X}^T A\ddot{X} + 2\dot{X}^T BX = 2\dot{X}^T\left(A\ddot{X} + BX\right) = 0 \qquad (10)$$

Therefore,

$$2\dot{X}^T = 0 \qquad (11)$$

or

$$A\ddot{X} + BX = 0 \qquad (12)$$

Let

$$X(t) = y_j \cos\omega t \qquad (13)$$

be the solution of equation (12).

where:

y_j = Amplitude of displacement of a particular concentrated mass.

ω = Natural vibration frequency of a transmission line.

$$\dot{X}(t) = -\omega y_j \sin\omega t \qquad (14)$$

$$\ddot{X}(t) = -\omega^2 y_j \cos\omega t \qquad (15)$$

Equations (14) and (15) represent the velocity and acceleration of a particular concentrated mass on the transmission line.

Substituting equations (14) and (15) into equation (12) and factorizing gives:

$$\left(-\omega^2 Ay_j + By_j\right)\cos\omega t = 0 \qquad (16)$$

Therefore,

$$\cos\omega t = 0 \qquad (17)$$

or

$$-\omega^2 Ay_j + By_j = 0 \qquad (18)$$

From equation (18), we have:

$$\left(B - \omega^2 A\right)y_j = 0 \qquad (19)$$

For non-trivial solution, the determinant of equation (19) must be zero.

Therefore,

$$\left|B - \omega^2 A\right| = 0 \qquad (20)$$

Equation (20) is the frequency equation of a transmission line under self-excited oscillation.

For a transmission line carrying n concentrated masses (Figure 2),

$$B = \frac{1}{2}Ty_1^2 + \frac{1}{2}T\left(y_1 - y_2\right)^2 + \frac{1}{2}T\left(y_2 - y_3\right)^2 + \ldots + \frac{1}{2}T\left(y_{N-1} - y_N\right)^2 \qquad (21)$$

and

$$A = \frac{1}{2}m_1\dot{y}_1^2 + \frac{1}{2}m_2\dot{y}_2^2 + \frac{1}{2}m_3\dot{y}_3^2 + \ldots + \frac{1}{2}m_n\dot{y}_n^2 \qquad (22)$$

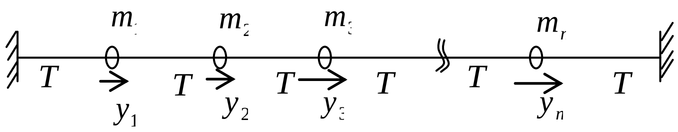

Figure 2: A Transmission line with constant tension carrying n concentrated masses.

3.0 RESULTS OF DYNAMIC ANALYSES

A transmission line subjected to three unequal and equal concentrated masses for numerical study.

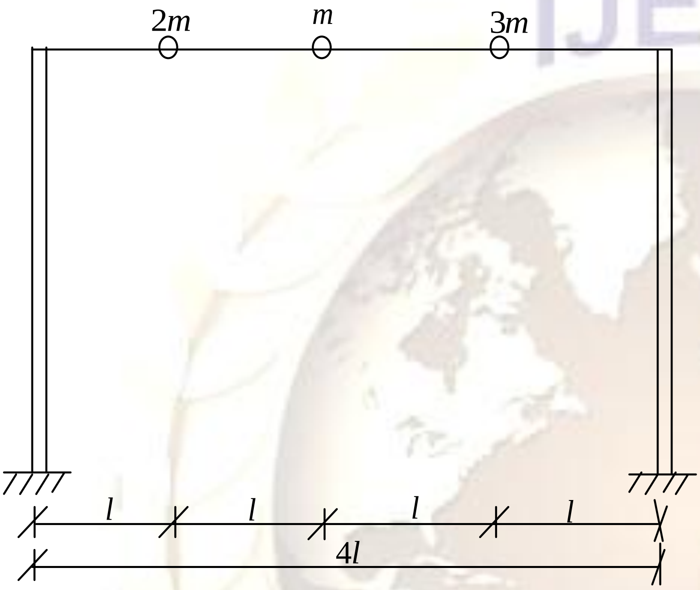

Figure 3: An overhead transmission carrying unequal concentrated masses for numerical study (control).

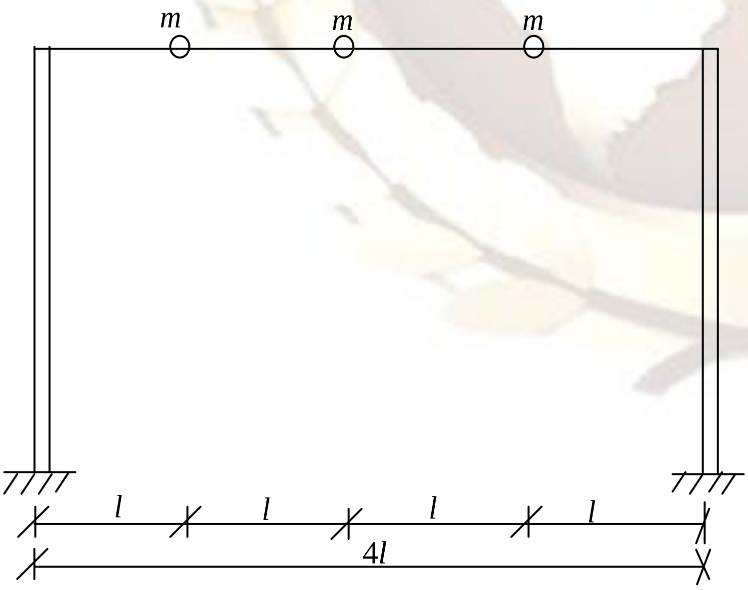

Figure 4: An overhead transmission carrying equal concentrated masses for numerical study.

From Figure 3, the equations of motion of the vibrating unequal concentrated masses are;

$$2m\ddot{y}_1 = -Ty_1 - T\left(y_1 - y_2\right) \tag{23}$$

$$m\ddot{y}_2 = T\left(y_1 - y_2\right) - T\left(y_2 - y_3\right) \tag{24}$$

$$3m\ddot{y}_3 = T\left(y_2 - y_3\right) - Ty_3 \tag{25}$$

Simplifying and arranging equations (23) – (25) gives:

$$2m_1\ddot{y}_1 + 2Ty_1 - Ty_2 = 0 \tag{26}$$

$$m_2\ddot{y}_2 + 2Ty_2 - Ty_1 - Ty_3 = 0 \tag{27}$$

$$3m\ddot{y}_3 + 2Ty_3 - Ty_2 = 0 \tag{28}$$

In matrix form, equations (26) – (28) can be written as:

$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 3m \end{bmatrix}\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix} + T\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{29}$$

From equations (21) and (22) and using equation (29) the kinetic and potential energy of symmetric matrices A and B are given by:

$$A = \frac{ml}{2}\begin{bmatrix} 2m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 3m \end{bmatrix} \tag{30}$$

and

$$B = \frac{T}{2}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \tag{31}$$

Substituting for A and B in equation (20) gives:

$$\left| \frac{T}{2}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} - \frac{\omega^2 ml}{2}\begin{bmatrix} 2m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 3m \end{bmatrix} \right| = 0 \tag{32}$$

Let

$$\lambda = \frac{ml\omega^2}{T} \tag{33}$$

Equation (32) now transforms to:

$$\left| \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \right| = 0 \tag{34}$$

$$\Rightarrow\begin{vmatrix}2-2\lambda & -1 & 0\\ -1 & 2-\lambda & -1\\ 0 & -1 & 2-3\lambda\end{vmatrix}=0 \qquad (35)$$

From equation (35),

$$\lambda^3-4.67\lambda^2+5.33\lambda-1.33=0 \qquad (36)$$

Using Newton Raphson approximation, the roots of equation (36) are:

$\lambda_1=0.352,\ \lambda_2=1.24,\ \lambda_3=3.78$

From equation (33),

$$\lambda=\frac{ml\omega^2}{T}$$

For $\lambda_1=0.352,\ \omega_1=0.5932\sqrt{\frac{T}{ml}}\ Rad/\text{sec.}$

For $\lambda_2=1.24,\ \omega_2=1.1135\sqrt{\frac{T}{ml}}\ Rad/\text{sec.}$

For $\lambda_3=3.78,\ \omega_3=1.9442\sqrt{\frac{T}{ml}}\ Rad/\text{sec.}$

From Figure 4, $m_1=m_2=m_3=m$

The frequency equation is:

$$\left|\frac{T}{2}\begin{pmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{pmatrix}-\frac{ml\omega^2}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\right|=0 \qquad (37)$$

Again, using equation (33),

$$\left|\begin{pmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{pmatrix}-\lambda\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\right|=0 \qquad (38)$$

$$\Rightarrow\begin{vmatrix}2-\lambda & -1 & 0\\ -1 & 2-\lambda & -1\\ 0 & -1 & 2-\lambda\end{vmatrix}=0 \qquad (39)$$

Expansion and evaluation of the above determinant gives:

$$\lambda^3-6\lambda^2+10\lambda-4=0 \qquad (40)$$

From equation (40),

$\lambda_1=0.586,\ \lambda_2=2,\ \lambda_3=3.414$

For $\lambda_1=0.586,\ \omega_1=0.7655\sqrt{\frac{T}{ml}}\ Rad/\text{sec.}$

For $\lambda_2=2,\ \omega_2=1.4142\sqrt{\frac{T}{ml}}\ Rad/\text{sec.}$

For $\lambda_3=3.414,\ \omega_3=1.8477\sqrt{\frac{T}{ml}}\ Rad/\text{sec.}$

4.0 DISCUSSION OF RESULTS

Table 1: Comparison of results of dynamic analysis of a transmission line subjected to unequal and equal concentrated masses.

	NATURAL FREQUENCY (RAD/SEC)		
	ω_1	ω_2	ω_3
Unequal concentrated masses(control)	$0.5932\left(\frac{T}{ml}\right.$	$1.1135\left(\frac{T}{ml}\right.$	$1.9442\left(\frac{T}{ml}\right.$
Equal concentrated masses	$0.7655\left(\frac{T}{ml}\right.$	$1.4142\left(\frac{T}{ml}\right.$	$1.8477\left(\frac{T}{ml}\right.$

The dynamic analyses of a transmission subjected to unequal and equal concentrated masses has been presented. From Table 1, it can be seen that for a transmission line subjected to unequal concentrated masses (control), the natural frequency of vibration is improved by 29% in the first vibration mode, 27% in the second vibration mode but decreased by 4.96% in the third vibration mode when unequal concentrated masses are replaced by equal concentrated masses.

5.0 CONCLUSION

The dynamic analyses of a transmission line subjected to unequal and equal concentrated masses using energy approach has been presented. From the results, it can be concluded that to improve the natural frequency of vibration, equal concentrated masses at equal spacing should be lumped on the transmission line at equal interval as the obtained results showed a significant improvement compared with those of the control, most especially in the first and second vibration modes. The damping characteristic of the transmission line is thus improved.

The formulated model can be used in the dynamic analysis of a multistory building having irregular floor masses and column stiffnesses.

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